NATIONAL SENIOR CERTIFICATE EXAMINATION
SUPPLEMENTARY PAPER 2014

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours | 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A

QUESTION 1

(a)
$$\frac{x}{3}=\frac{3}{x}$$
$$x^2=9$$
$$x=\pm 3$$
(2)

(b)
$$\left(2x-1\right)\left(5x+4\right)=\left(x+1\right)\left(x+5\right)-3$$
$$10x^2+3x-4=x^2+6x+5-3$$
$$9x^2-3x-6=0$$
$$3x^2-x-2=0$$
$$(3x+2)(x-1)=0$$
$$x=-\frac{2}{3} \text{ or } x=1$$
(4)

(c)
$$x(x+2)>3$$
$$x^2+2x-3>0$$
$$(x+3)(x-1)>0$$

-3 1

\+   0   -   0   +

$$x < -3 \text{ or } x > 1$$
(4)

(d)
$$\log_x 9 = -2$$
$$x^{-2} = 9$$
$$x^2 = \frac{1}{9}$$
$$x = \frac{1}{3}$$
(3)

[13]

QUESTION 2

(a) $T_n = \dfrac{n^2}{(n+1)^2}$ / $\left(\dfrac{n}{n+1}\right)^2$ (1)

(b) (1) $a = \dfrac{3}{8}$; $r = 2$

$T_{10} = ar^9$

$= \dfrac{3}{8}.2^9$

$= 192$ (3)

(2) $T_n = \dfrac{3}{8}.2^{n-1} = 12288$

$2^{n-1} = 32768$

$= 2^{15}$

$n - 1 = 15$

$n = 16$

i.e. T_{16} (4)

(c) $n = 21$; $a = 3$; $T_{21} = 53$

$S_n = \dfrac{n}{2}\left[a + l\right]$

$S_{21} = \dfrac{21}{2}\left[3 + 53\right]$

$= 588$ (3)

[11]

QUESTION 3

(a) (1) $f(x) = -2(x+4)^2 + 3$
has T.P. (– 4 ; 3)
$\therefore$ new T.P. (– 4 ; 2) (3)

(2) (– 5 ; 3) (2)

(3) (– 4 ; 12) (2)

(b) (1) $g(x) = b^x$
$K: 2 = b^{-1}$
$\therefore b = \dfrac{1}{2}$ (1)

(2) $g^{-1}(x) = \log_{\frac{1}{2}} x \;\; / \left(-\log_2 x\right)$ (2)

(3) $y_T = g\left(x_T\right)$
$= g(0,64)$
$= \left(\dfrac{1}{2}\right)^{0,64}$
$\approx 0,64$
ALTERNATIVELY
T lies on the line $y = x$
$\therefore y_T = x_T = 0,64$ (2)

(4) $0 < x < 1$ (2)

(5) $h(x) = \log_{\frac{1}{2}}(-x), \quad x < 0$ (2)

[16]

QUESTION 4

(a) $2P = P(1+i)^5$
$(1+i)^5 = 2$
$1 + i = \sqrt[5]{2}$
$i = 1,148698355 - 1$
$= 0,148698355$
$\approx 14,9\%$ (4)

(b) $635,37 \times 24 \quad = 12499(1+2i)$
$1 + 2i = 1,220008001...$
$2i = 0,22...$
$i = 0,110004...$
$\approx 11\%$ (5)

(c) $A \leftarrow 1 + \dfrac{0,095}{12}$
$P_v = \dfrac{6500\left[1 - A^{-180}\right]}{\dfrac{0,095}{12}}$
$= \text{R}622\ 471,40$ (3)

(d) $$550\,000 = \frac{6500\left[1 - A^{-n}\right]}{\frac{0,095}{12}}$$

$$1 - A^{-n} = 0,6698717949$$

$$A^{-n} = 0,3301282051$$

$$-n = \log_A(0,3301...)$$

$$n = 140,545...$$

i.e. 11 years 9 months (6)

[18]

QUESTION 5

(a) $$f(x) = \frac{x^2}{4}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{(x+h)^2}{4} - \frac{x^2}{4}}{h}$$

$$= \lim_{h \to 0} \frac{1}{4h}\left[x^2 + 2hx + h^2 - x^2\right]$$

$$= \lim_{h \to 0} \frac{h}{4h}\left[2x + h\right]$$

$$= \lim_{h \to 0} \left(\frac{x}{2} + \frac{h}{4}\right)$$

$$= \frac{x}{2}$$ (5)

(b) $$y = \frac{15x^2 + x - 2}{3x - 1}$$

$$= \frac{(5x+2)(3x-1)}{3x-1}$$

$$= 5x + 2$$

$$\frac{dy}{dx} = 5$$ (3)

(c) $$f(x) = \sqrt{x} + \frac{1}{x^2} - 3$$

$$= x^{\frac{1}{2}} + x^{-2} - 3$$

$$f'(x) = \tfrac{1}{2}x^{-\frac{1}{2}} - 2x^{-3}$$

$$f'(4) = \tfrac{1}{2}4^{-\frac{1}{2}} - 2.4^{-3}$$

$$= \frac{1}{2.2} - \frac{2}{64}$$

$$= \frac{7}{32} \qquad (0,2)$$ (5)

(d) $f(x) = x^3 - 3x^2 + kx + 8$

$f'(x) = 3x^2 - 6x + k$

$f'(1) = 3 - 6 + k = 0$

$k = 3$

(4)

[17]

75 marks

SECTION B

QUESTION 6

(a) $\frac{x^2}{2}-\frac{7x}{2}+3=-x+6$

$x^2-7x+6=-2x+12$

$x^2-5x-6=0$

$(x-6)(x+1)=0$

$x=6 \quad \text{or } x=-1$

$\text{At A}: x=-1$

$y=-(-1)+6$

$=7$

$\therefore \text{A}(-1\,;\,7)$ (5)

(b) $x\le -1$ or $x\ge 6$ (2)

(c) At C : $\frac{x^2}{2}-\frac{7x}{2}+3=0$

$x^2-7x+6=0$

$(x-1)(x-6)=0$

$x=1$ or $x=6$

$x_c=1$

$f'(x)=x-\frac{7}{2}$

$f'(1)=-\frac{5}{2}$

Eqn. of Tangent :

$y-0=-\frac{5}{2}(x-1)$

$y=-\frac{5x}{2}+\frac{5}{2}$ (8)

(d) Let x be the x-coordinate of P.

$\text{PQ}=-x+6-\left(\frac{x^2}{2}-\frac{7x}{2}+3\right)$

$=-\frac{x^2}{2}+\frac{5x}{2}+3$

$\frac{dPQ}{dx}=-x+\frac{5}{2}=0$

$x=\frac{5}{2}$

ALTERNATIVELY

$\frac{-b}{2a}=\frac{-5}{2}\div 2\left(-\frac{1}{2}\right)$

$=\frac{5}{2}$

Max PQ $=-\frac{1}{2}\left(\frac{5}{2}\right)^2+\frac{5}{2}\left(\frac{5}{2}\right)+3$

$=\frac{49}{8}$ (6,125) (6)

[21]

QUESTION 7

(a) $x \geq 2$

$y \geq 1$

$y \geq -2x+6$

$y \leq -x+9$ (6)

(b) When $x = 2\frac{1}{2}$

$-2x+6$

$= -2\left(\frac{5}{2}\right)+6$

$= 1$

$-x+9$

$= \frac{-5}{2}+9$

$= \frac{13}{2}$

$\therefore 1 \leq y \leq \frac{13}{2}$ (3)

(c) $W = 5x+2y$

$2y = -5x+W$

$y = -\frac{5x}{2}+\frac{W}{2}$

Max W at (4 ; 5)

i.e. 4 hours of netball and 5 hours of chess. (3)

(d) $W = 4\times 5+2\times 5$

$= 20+10$

$= 30$ (2)

[14]

QUESTION 8

(a) A.P. $a = 30$; $d = -1,5$

$T_5 = a+4d$

$= 30+4(-1,5)$

$= \text{R}24$

ALTERNATIVELY

30 ; 28,50 ; 27 ; 25,50 ; 24

$\therefore$ T_5 = R24 (3)

(b) $30+(n-1)(-1,5) = 0$

$-1,5(n-1) = -30$

$n-1 = 20$

$n = 21$

i.e. 20 GB are charged then 21st GB is free. (3)

(c) $S_{17} = \frac{7}{2}\left[2a+16d\right]$

$= \frac{17}{2}\left[2\times 30+16(-1,5)\right]$

$= \text{R}306$ (3)

(d) Charge for n GB

$$= \frac{n}{2}\left[2 \times 30 + (n-1)(-1,5)\right]$$
$$= \frac{n}{2}\left[60 - \frac{3n}{2} + \frac{3}{2}\right]$$
$$= -\frac{3n^2}{4} + \frac{123n}{4}$$
$$= -\frac{3}{4} \times 20^2 + \frac{123}{4} \times 20$$
$$= \text{R}315$$

(4)

[13]

QUESTION 9

(a) $m(t) = 500(0,92)^t$

(1) 500 g (1)

(2) 8% (1)

(3) $m(50) = 500(0,92)^{50}$

$= 7,7332...$

$\approx 7,7 \text{ g}$ (2)

(4) $500(0,92)^t < 1$

$$0,92^t < \frac{1}{500}$$
$$t > 74,5321...$$

i.e. 75 years (3)

(b) $f(\sqrt{4}) + f(\sqrt{8})$

$$= \frac{\sqrt{4}}{2} + \left(\sqrt{8}\right)^2$$
$$= 1 + 8$$
$$= 9$$

(3)

(c) $f(x) = \dfrac{2}{x-3} + 1$

(1) V.A. $x = 3$

H.A. $y = 1$ (2)

(2) $f(x+5) = \dfrac{2}{x+2} + 1$

Lines of symmetry :

$y = (x+2) + 1$

$= x + 3$

or

$y = -(x+2) + 1$

$= -x - 1$ (3)

[15]

QUESTION 10

(a) Domain : $x \in [-8\,;16]$
Range : $y \in [-6\,;10]$ (4)

(b) If $\left(x; g(x)\right)$ is on the graph of g,
Then $\left(-x; -g(x)\right)$
$= \left(-x; g(-x)\right)$ is also on the graph of g
ALTERNATIVELY
Let $(a;b)$ be a point on the graph of g.
i.e. $g(a) = b$
So $-b = -g(a)$
$= g(-a)$
proving that $(-a;-b)$ is also a point on the graph of g. (4)

(c) The turning point $(0;10)$ moves to $(0+p;10+q)$
and the turning point $(8;-6)$ moves to $(8+p;-6+q)$
So $8+p=-p$ and $-6+q=-10-q$
$\therefore \quad 2p=-8 \qquad 2q=-4$
$p=-4 \qquad q=-2$
ALTERNATIVELY
Do the same for the points $(16;8)$ and $(-8;-4)$
ALTERNATIVELY
$(4;2)$ must become origin of new curve
Move curve 4 left
2 down
$g(x)+2 = f(x+4)$
$g(x) = f(x+4)-2$
$p=-4 \qquad q=-2$ (4)

[12]

75 marks

Total: 150 marks