PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 25 pages and an Information Sheet of 2 pages (i–ii). Please check that your question paper is complete.

2. Read the questions carefully.

3. Answer ALL the questions on the question paper and hand this in at the end of the examination. Remember to write your examination number in the space provided.

4. Diagrams are not necessarily drawn to scale.

5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.

6. Round off your answers to one decimal digit where necessary, unless otherwise stated.

7. All the necessary working details must be clearly shown.

8. It is in your own interest to write legibly and to present your work neatly.

9. One blank page (page 25) is included at the end of the paper. If you run out of space for a question, use this page. Clearly indicate the question number of your answer should you use this extra space.
QUESTION 1

The picture below shows a side view design of a swing.

The diagram below, NOT drawn to scale, models the above swing design in a Cartesian plane such that AO = AB and OÂB = θ with A (100; 250) and B (200; 0).

Calculate:

1.1 the length of AB in simplified surd form

\[ AB = \sqrt{A^2 + B^2} = \sqrt{(100 - 200)^2 + (250 - 0)^2} = \sqrt{100^2 + 250^2} = \sqrt{10000 + 62500} = \sqrt{72500} = 269.1 \text{ cm} \]
1.2 the gradient of AB

(2)

1.3 the midpoint M of line OA

(2)

1.4 the size of $\theta$

(4)
QUESTION 2

2.1 The diagram below shows line LP, with equation \( y = -x + 2 \). NR \parallel LP \quad \text{and} \quad NL \perp LP. \quad M (-4; 4) \text{ is the midpoint of line LN. P is a point on the x-axis and R is a point on the y-axis.}

2.1.1 Determine the equation of line LN.

\[
\text{Equation of line LN: } y = \quad (3)
\]
2.1.2 Calculate the coordinates of N (show all working).

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(5)

2.1.3 Determine the coordinates of P.

__________________________________________________________________________
__________________________________________________________________________
__________________________________________________________________________
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__________________________________________________________________________

(2)

2.1.4 Determine the equation of the circle with midpoint O, passing through point P.

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__________________________________________________________________________

(2)
2.2 Draw the graph defined by

\[ \frac{x^2}{9} + \frac{y^2}{16} = 1 \]

Clearly show ALL the intercepts with the axes.
QUESTION 3

3.1 In the diagram below, \( P(2\sqrt{3}; -2) \) is a point in the Cartesian plane with origin \( O \). The angle of \( OP \) with the positive \( x \)-axis is \( \theta \).

\[ \theta \]

Determine the following:

3.1.1 the length of \( OP \)

3.1.2 the value of \( \theta \)

3.2 Calculate the numerical value of \( \sec(a - b) \) if \( a = 2,695 \) and \( b = 1,112 \).
3.3 Simplify without the use of a calculator and clearly show all working:

\[
\frac{\sin 210^\circ \tan 45^\circ \cos 315^\circ}{\sin 45^\circ \cos 60^\circ}
\]

3.4 Prove the following identity: \( \tan x \cdot \sin x = \sec x - \cos x \)

3.5 Solve for \( x \):

3.5.1 \( \cosec 2x = 2.114; \ 2x \in [0^\circ; 180^\circ] \)
3.5.2 Simplify: \[
\sin (360° - x) \cdot \cos (180° - x) \cdot \tan (180° + x) \\
\cos^2 x \cdot \sin \left( \frac{5}{6} \pi \right)
\]
QUESTION 4

Given: \( f(x) = 2 \cos x \) and \( g(x) = \sin (x - 30^\circ) \) for \( x \in [0^\circ;360^\circ] \)

4.1 Draw a sketch graph of \( f \) and \( g \) on the same set of axes on the grid below. Clearly indicate ALL turning points and intercepts with the axes.

4.2 Write down the amplitude of \( f \).

4.3 Give the period of \( g \).

4.4 Determine the values of \( x \) for which \( f(x) > g(x) \)

[12]
QUESTION 5

The diagram below shows a vertical pole AD with points C and B on the same horizontal plane as point A, the base of the pole. If $\angle CDA = 58^\circ$, $\angle CAB = 108^\circ$, $\angle CBA = 30^\circ$, $CD = 2$ m and $AB = 2.3$ m.

Calculate:

5.1 the length of AC

5.2 the area of $\triangle ABC$

5.3 the length of BC
5.4 the size of $\angle CDB$, if $BD = 2.5 \text{ m}$
QUESTION 6

In the figure below, O is the centre of circle SPR with PT = PR and \( \hat{O}_1 = 120^\circ \)

6.1 Calculate, with reasons, the size of the following angles:

6.1.1  \( \hat{P}_2 \)

6.1.2  \( \hat{R}_1 \)
6.2 ABCD is a cyclic quadrilateral. MK is a tangent of the circle at C. CA bisects BCD.

If AC and BD intersect at E and $\hat{B}CM = 50^\circ$ and $\hat{B}EA = 110^\circ$, calculate, with reasons, the size of each of the following:

6.2.1 $\hat{D}_2$

6.2.2 $\hat{B}_1$

6.2.3 $\hat{D}_1$
6.3 In the diagram, points P, Q, R and T lie on the circle.
MW is a tangent to the circle at P.
PT is produced to meet RU at U.
\[\widehat{MPR} = 78^\circ\]
\[\widehat{PQT} = 49^\circ\]
\[\widehat{QTR} = 32^\circ\]

Calculate, with reasons, the size of the following:

6.3.1 \(a\)

\[\text{Reasons:} \]

\[\text{Size of } a = \text{Angle calculation} \]

\[\text{Size of } a = \text{Calculation steps} \]

(2)

6.3.2 \(b\)

\[\text{Reasons:} \]

\[\text{Size of } b = \text{Angle calculation} \]

\[\text{Size of } b = \text{Calculation steps} \]

(3)
6.3.3 \( c \)

\[ \]
QUESTION 7

A picture of a shelf stand is shown below, which is modelled by the accompanying diagram.

In \( \triangle KLM \), C and Q are points on KM
B and P are points on KL
BC \( \parallel \) PQ \( \parallel \) LM and BQ \( \parallel \) PM
KM = 20 units, PL = 4 units and KQ : QM = 3 : 1

7.1 Determine, stating reasons, the lengths of each of the following:

7.1.1 QM

7.1.2 KP

(3)
7.1.3  KB

7.2  7.2.1  Prove that ∆KPM ||| ∆KBQ.

7.2.2  Hence, or otherwise, determine the length of BQ if PM = 10 units.
QUESTION 8

8.1 A wheel has a radius of 25 cm. The circumferential velocity of the rotating wheel is given as 8.75 cm per second. Calculate the angular velocity of the rotating wheel in radians per second.

\[
\text{Angular velocity} = \frac{\text{Circumferential velocity}}{\text{Radius}} = \frac{8.75 \text{ cm/s}}{25 \text{ cm}} = 0.35 \text{ rad/s}
\]
8.2 The picture and diagram below show one of the mechanisms of a machine in a textile factory where two pulleys with centres A and E are connected with a cross-belt. CP, CQ, CR and CT are tangents to the circles at points P, Q, R and T.

The reflex angles on both pulleys are 240°
The radius of the large pulley is 28 cm and the radius of the small pulley is 12 cm
AB = BC and CD = DE

8.2.1 Calculate the length of the section of the belt that is **in contact** with the large pulley.
8.2.2 Calculate the **total length** of the belt, if the length of the section of the belt that is in contact with the small pulley, is 50,3 cm.
**QUESTION 9**

9.1 A regular octahedron is given with lengths of all edges $3\sqrt{2}$ units. ABCD is a square.

9.1.1 Calculate the height of ΔABE, the left face of the octahedron.

9.1.2 Calculate the total outer surface area of the octahedron.
9.2 The picture and diagram below show a solid shape that consists of a right cylindrical section with a hemispherical section at one end, with a right cylindrical section removed.

Calculate the volume of this hollowed solid shape as illustrated in this diagram.

The following formulae may be used:

Volume of sphere = \( \frac{4}{3} \pi r^3 \)

Volume of cylinder = \( \pi r^2 h \)
9.3 The irregular figure as shown in the picture below has one straight side of 6.5 m long. It is divided into 5 equal parts with ordinates 0.8 m, 1.3 m, 1.1 m and 0.5 m given. Calculate the area of the irregular figure using the mid-ordinate rule.

\[
\begin{align*}
\text{Area} &= \frac{1}{2} \times (6.5 \times (0.8 + 1.3 + 1.1 + 0.5)) \\
&= \frac{1}{2} \times 6.5 \times 4.6 \\
&= 15.15 \\
\end{align*}
\]

TOTAL: 150 marks
ADDITIONAL SPACE (ALL questions)

REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.