## TECHNICAL MATHEMATICS: PAPER II

## EXAMINATION NUMBER

$\square$
Time: 3 hours
150 marks

## PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 25 pages and an Information Sheet of 2 pages (i-ii). Please check that your question paper is complete.
2. Read the questions carefully.
3. Answer ALL the questions on the question paper and hand this in at the end of the examination. Remember to write your examination number in the space provided.
4. Diagrams are not necessarily drawn to scale.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Round off your answers to one decimal digit where necessary, unless otherwise stated.
7. All the necessary working details must be clearly shown.
8. It is in your own interest to write legibly and to present your work neatly.
9. One blank page (page 25) is included at the end of the paper. If you run out of space for a question, use this page. Clearly indicate the question number of your answer should you use this extra space.

FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

| Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | Q9 | TOTAL |
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## QUESTION 1

The picture below shows a side view design of a swing.


The diagram below, NOT drawn to scale, models the above swing design in a Cartesian plane such that $A O=A B$ and $O A \hat{B}=\theta$ with $A(100 ; 250)$ and $B(200 ; 0)$.


Calculate:
1.1 the length of $A B$ in simplified surd form
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$\qquad$
$\qquad$

## 1.2 the gradient of $A B$

$\qquad$
$\qquad$
$\qquad$
1.3 the midpoint M of line OA
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$\qquad$
1.4 the size of $\theta$
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## QUESTION 2

2.1 The diagram below shows line LP, with equation $y=-x+2$. NR \|| LP and NL $\perp$ LP. $M(-4 ; 4)$ is the midpoint of line $L N$. $P$ is a point on the $x$-axis and $R$ is a point on the $y$-axis.

2.1.1 Determine the equation of line $L N$.
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(3)
2.1.2 Calculate the coordinates of N (show all working).
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2.1.3 Determine the coordinates of $P$.
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2.1.4 Determine the equation of the circle with midpoint $O$, passing through point $P$.
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$\qquad$
2.2 Draw the graph defined by

$$
\frac{x^{2}}{9}+\frac{y^{2}}{16}=1
$$

Clearly show ALL the intercepts with the axes.

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$\qquad$

## QUESTION 3

3.1 In the diagram below, $P(2 \sqrt{3} ;-2)$ is a point in the Cartesian plane with origin O . The angle of OP with the positive $x$-axis is $\theta$.


Determine the following:
3.1.1 the length of OP
$\qquad$
$\qquad$
$\qquad$

### 3.1.2 the value of $\theta$

$\qquad$
$\qquad$
$\qquad$
3.2 Calculate the numerical value of $\sec (a-b)$ if $a=2,695$ and $b=1,112$.
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$\qquad$
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3.3 Simplify without the use of a calculator and clearly show all working:

```
    sin 210矢 tan 45 cos 315'
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3.4 Prove the following identity: $\tan x \cdot \sin x=\sec x-\cos x$
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$\qquad$
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$\qquad$
3.5 Solve for $x$ :
3.5.1 $\operatorname{cosec} 2 x=2,114 ; 2 x \in\left[0^{\circ} ; 180^{\circ}\right]$
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$\qquad$
$\qquad$
$\qquad$
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$$
\text { 3.5.2 Simplify: } \frac{\sin \left(360^{\circ}-x\right) \cdot \cos \left(180^{\circ}-x\right) \cdot \tan \left(180^{\circ}+x\right)}{\cos ^{2} x \cdot \sin \left(\frac{5}{6} \pi\right)}
$$

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## QUESTION 4

Given: $f(x)=2 \cos x$ and $g(x)=\sin \left(x-30^{\circ}\right)$ for $x \in\left[0^{\circ} ; 360^{\circ}\right]$
4.1 Draw a sketch graph of $f$ and $g$ on the same set of axes on the grid below. Clearly indicate ALL turning points and intercepts with the axes.

4.2 Write down the amplitude of $f$.
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$\qquad$
4.3 Give the period of $g$.
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$\qquad$
4.4 Determine the values of $x$ for which $f(x)>g(x)$
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## QUESTION 5

The diagram below shows a vertical pole AD with points $C$ and $B$ on the same horizontal plane as point $A$, the base of the pole. If $C \hat{D} A=58^{\circ}, C \hat{A} B=108^{\circ}, C \hat{B} A=30^{\circ}, C D=2 \mathrm{~m}$ and $A B=2,3 \mathrm{~m}$.


Calculate:
5.1 the length of $A C$
$\qquad$
$\qquad$
5.2 the area of $\triangle A B C$
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$\qquad$
$\qquad$
5.3 the length of $B C$
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$\qquad$
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$\qquad$
(3)
5.4 the size of $C \hat{D} B$, if $B D=2,5 \mathrm{~m}$
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## QUESTION 6

In the figure below, O is the centre of circle SPR with $\mathrm{PT}=\mathrm{PR}$ and $\hat{O}_{1}=120^{\circ}$

6.1 Calculate, with reasons, the size of the following angles:
6.1.1 $\hat{P}_{2}$
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$\qquad$
$\qquad$
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(4)
6.1.2 $\hat{R}_{1}$
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(3)
6.2 ABCD is a cyclic quadrilateral. MK is a tangent of the circle at $C$.

CA bisects BĈD.


If $A C$ and $B D$ intersect at $E$ and $B \hat{C} M=50^{\circ}$ and $B \hat{E} A=110^{\circ}$, calculate, with reasons, the size of each of the following:
6.2.1 $\hat{D}_{2}$
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$\qquad$
6.2.2 $\hat{B}_{1}$
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$\qquad$
$\qquad$
(3)
6.2.3 $\hat{D}_{1}$
$\qquad$
$\qquad$
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$\qquad$
(5)
6.3 In the diagram, points $\mathrm{P}, \mathrm{Q}, \mathrm{R}$ and T lie on the circle.

MW is a tangent to the circle at $P$.
PT is produced to meet $R U$ at $U$.
$\mathrm{MPR}=78^{\circ}$
$\mathrm{PQ} T=49^{\circ}$
QT̂R $=32^{\circ}$


Calculate, with reasons, the size of the following:
6.3.1 a
$\qquad$
$\qquad$
6.3.2 b
$\qquad$
$\qquad$
$\qquad$

### 6.3.3 c

## QUESTION 7

A picture of a shelf stand is shown below, which is modelled by the accompanying diagram.
In $\Delta K L M, C$ and $Q$ are points on $K M$
$B$ and $P$ are points on $K L$
$B C\|P Q\| L M$ and $B Q \| P M$
$K M=20$ units, $P L=4$ units and $K Q: Q M=3: 1$

7.1 Determine, stating reasons, the lengths of each of the following:

### 7.1.1 QM

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(3)
7.1.2 KP
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### 7.1.3 KB

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7.2 7.2.1 Prove that $\Delta \mathrm{KPM}||\mid \Delta \mathrm{KBQ}$.
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7.2.2 Hence, or otherwise, determine the length of $B Q$ if $P M=10$ units.
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## QUESTION 8

8.1 A wheel has a radius of 25 cm . The circumferencial velocity of the rotating wheel is given as $8,75 \mathrm{~cm}$ per second. Calculate the angular velocity of the rotating wheel in radians per second.
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(5)
8.2 The picture and diagram below show one of the mechanisms of a machine in a textile factory where two pulleys with centres $A$ and $E$ are connected with a crossbelt. CP, CQ, CR and CT are tangents to the circles at points $\mathrm{P}, \mathrm{Q}, \mathrm{R}$ and T .


The reflex angles on both pulleys are $240^{\circ}$
The radius of the large pulley is 28 cm and the radius of the small pulley is 12 cm $A B=B C$ and $C D=D E$
8.2.1 Calculate the length of the section of the belt that is in contact with the large pulley.
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8.2.2 Calculate the total length of the belt, if the length of the section of the belt that is in contact with the small pulley, is $50,3 \mathrm{~cm}$.
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## QUESTION 9

9.1 A regular octahedron is given with lengths of all edges $3 \sqrt{2}$ units. $\operatorname{ABCD}$ is a square.

9.1.1 Calculate the height of $\triangle A B E$, the left face of the octahedron.
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9.1.2 Calculate the total outer surface area of the octahedron.
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(4)
9.2 The picture and diagram below show a solid shape that consists of a right cylindrical section with a hemispherical section at one end, with a right cylindrical section removed.

Calculate the volume of this hollowed solid shape as illustrated in this diagram.


The following formulae may be used:
Volume of sphere $=\frac{4}{3} \pi r^{3}$
Volume of cylinder $=\pi r^{2} h$
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9.3 The irregular figure as shown in the picture below has one straight side of $6,5 \mathrm{~m}$ long. It is divided into 5 equal parts with ordinates $0,8 \mathrm{~m}, 1,3 \mathrm{~m}, 1,1 \mathrm{~m}$ and $0,5 \mathrm{~m}$ given. Calculate the area of the irregular figure using the mid-ordinate rule.

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## ADDITIONAL SPACE (ALL questions)

REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.
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