NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2019

TECHNICAL MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 3 hours | 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

QUESTION 1

1.1 $AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$ correct formula

$= \sqrt{(200-100)^2 + (0-250)^2}$ substitution

$= \sqrt{(100)^2 + (-250)^2}$

$= 50\sqrt{29}$ units simplified surd form

1.2 $m_{AB} = \dfrac{y_B - y_A}{x_B - x_A}$

$= \dfrac{0-250}{200-100}$ substitution

= –2,5 simplification

1.3 $M = \left(\dfrac{x_A - x_O}{2} \; ; \; \dfrac{y_A - y_O}{2} \right)$

$M = \left(\dfrac{100+0}{2} \; ; \; \dfrac{250+0}{2} \right)$

M = (50 ; 125) *x*-coordinate *y*-coordinate

1.4 $\tan B = m$

$\tan B = -2{,}5$ substitution

$\therefore O\hat{B}A \approx 68{,}2^\circ$ angle $O\hat{B}A$

$\therefore A\hat{O}B = 68{,}2^\circ$ angle $A\hat{O}B$

$\therefore \theta = 180^\circ - 68{,}2^\circ - 68{,}2^\circ = 43{,}6^\circ$ simplification

QUESTION 2

2.1 2.1.1 $m_{LP} \times m_{LN} = -1$

$-1 \times m_{LN} = -1$

$\therefore m_{LN} = 1$ LN gradient

$\therefore y = x + c$ with M(–4;4)

$\therefore 4 = -4 + c$

$\therefore 8 = c$

$\therefore y = x + 8$ equation

OR $y - y_1 = m(x - x_1)$

$y - 4 = 1\,(x + 4)$

$y = x + 8$ equation

2.1.2 $\therefore y = x + 8 = -x + 2$ follow up from Question 2.1.1

$\therefore 2x = -6$

$\therefore\ x = -3$

$\therefore y = x + 8$

$\therefore y = -3 + 8 = 5$

$\therefore L(-3;\,5)$ coordinates

$$m\left(\frac{x_L + x_N}{2}\,;\,\frac{y_L + y_N}{2}\right) = (-4;\,4)$$

$$\frac{-3 + x_N}{2} = -4 \qquad \frac{5 + y_N}{2} = 4$$ equations

$x_N = -5$ $\qquad y_N = 3$

$\therefore N(-5;\,3)$ coordinates

2.1.3 $y = -x + 2$

x-intercept: $0 = -x + 2$

$x = 2$ $\therefore$ P (2; 0) coordinates

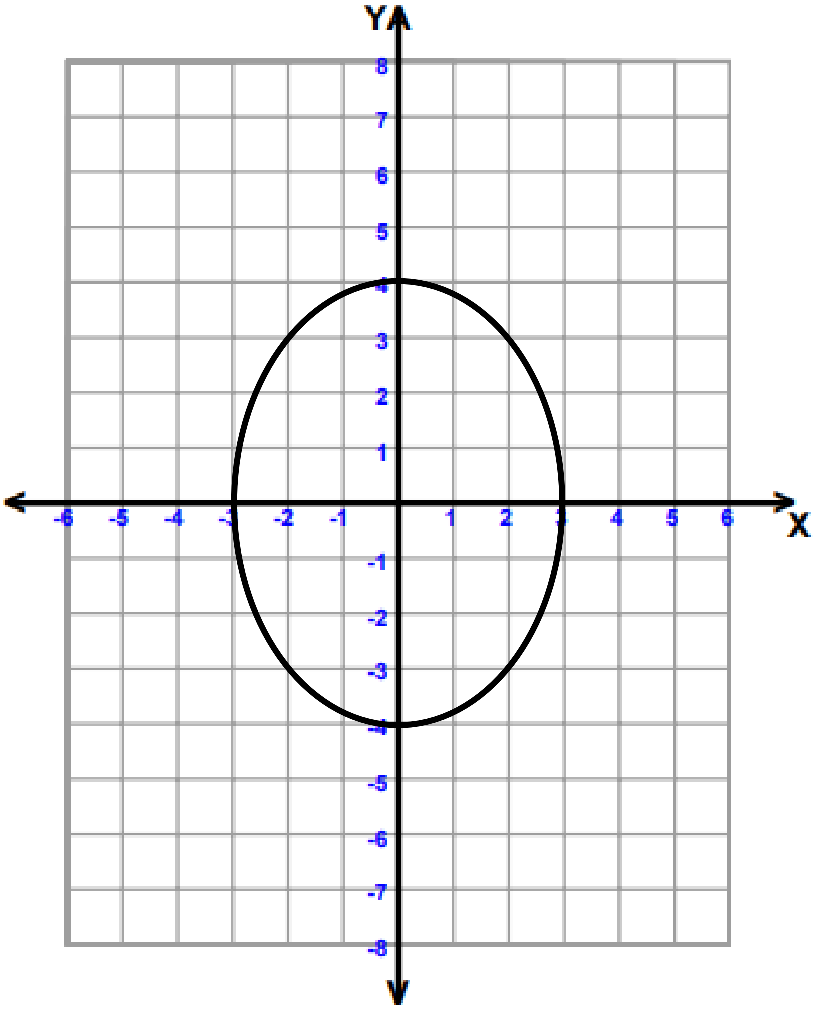
2.1.4 $x^2 + y^2 = r^2$ $\qquad p(2\,;0)$

$\therefore (2)^2 + (0)^2 = r^2$ substitution

$4 = r^2$

$\therefore x^2 + y^2 = 4$ equation

2.2



x-intercepts y-intercepts shape

QUESTION 3

3.1 3.1.1 $x^2 + y^2 = r^2$

$\left(2\sqrt{3}\right)^2 + \left(-2\right)^2 = r^2$ substitution

$16 = r^2$

4 units = r simplification

3.1.2 $tan\theta = \frac{-2}{2\sqrt{3}}$ function

(Ref angle) = 30° method

$\theta = 360° - 30° = 330°$ simplification

3.2 $\sec\left(a - b\right)$

$= \sec\left(2{,}695 - 1{,}112\right)$ substitution

$= \sec\left(1{,}583\right)$

$= \frac{1}{\cos\left(1{,}583\right)}$

$\approx -81{,}9$ simplification

3.3 $\frac{\sin 210° \tan 45° \cos 315°}{\sin 45° \cos 60°}$

$= \frac{-\sin 30° \tan 45° \cos 45°}{\sin 45° \cos 60°}$

$= \frac{\left(\frac{-1}{2}\right)\cdot\left(1\right)\cdot\left(\frac{\sqrt{2}}{2}\right)}{\left(\frac{\sqrt{2}}{2}\right)\cdot\left(\frac{1}{2}\right)}$

= –1

OR

$= \frac{-\sin 30° \times \frac{\sin 45°}{\cos 45°} \times \cos 45°}{\sin 45° \times \sin 30°}$

= –1

3.4 LHS: $\tan x \cdot \sin x$

$= \frac{\sin x}{\cos x}.\sin x$

$= \frac{\sin^2 x}{\cos x}$

RHS: $\sec x - \cos x$

$= \frac{1}{\cos x} - \cos x$

$= \frac{1 - \cos^2 x}{\cos x}$

$= \frac{\sin^2 x}{\cos x}$

$\therefore LHS = RHS$

OR

LHS: $\tan x \cdot \sin x$

$= \tan x\left(\cos x \tan x\right)$

$= \cos x \cdot \tan^2 x$

$= \cos x\left(\sec^2 x - 1\right)$

$= \cos x\left(\frac{1}{\cos^2 x} - 1\right)$

$= \frac{1}{\cos x} - \cos x$

$= \sec x - \cos x$

$\therefore LHS = RHS$

OR LHS : $\tan x \cdot \sin x$

$$= \frac{\sin x}{\cos x} \cdot \frac{\sin x}{1}$$
$$= \frac{\sin^2 x}{\cos x}$$
$$= \frac{1 - \cos^2 x}{\cos x}$$
$$= \frac{1}{\cos x} - \frac{\cos^2 x}{\cos x}$$
$$= \sec x - \cos x$$
$$\therefore LHS = RHS$$

OR RHS : $\sec x - \cos x$

$$= \frac{1}{\cos x} - \cos x$$
$$= \frac{1 - \cos^2 x}{\cos x}$$
$$= \frac{\sin^2 x}{\cos x}$$
$$= \sin x \times \frac{\sin x}{\cos x}$$
$$= \sin x \cdot \tan x$$
$$\therefore LHS = RHS$$

3.5 3.5.1 cosec $2x = 2{,}114$ for $2x \in [0°;180°]$

$$\frac{1}{\sin 2x} = 2{,}114$$
$$\frac{1}{2{,}114} = \sin 2x$$
$$0{,}473\ldots = \sin 2x$$

Ref angle $\approx 28{,}2316°$

$2x = 28{,}23°$ or $2x = 180° - 28{,}23°$ correct quadrants

$x = 14{,}12°$ or $x = 75{,}88°$ both answers

3.5.2 $$\frac{\sin(360° - x) \cdot \cos(180° - x) \cdot \tan(180° + x)}{\cos^2 x \cdot \sin \frac{5}{6}\pi}$$

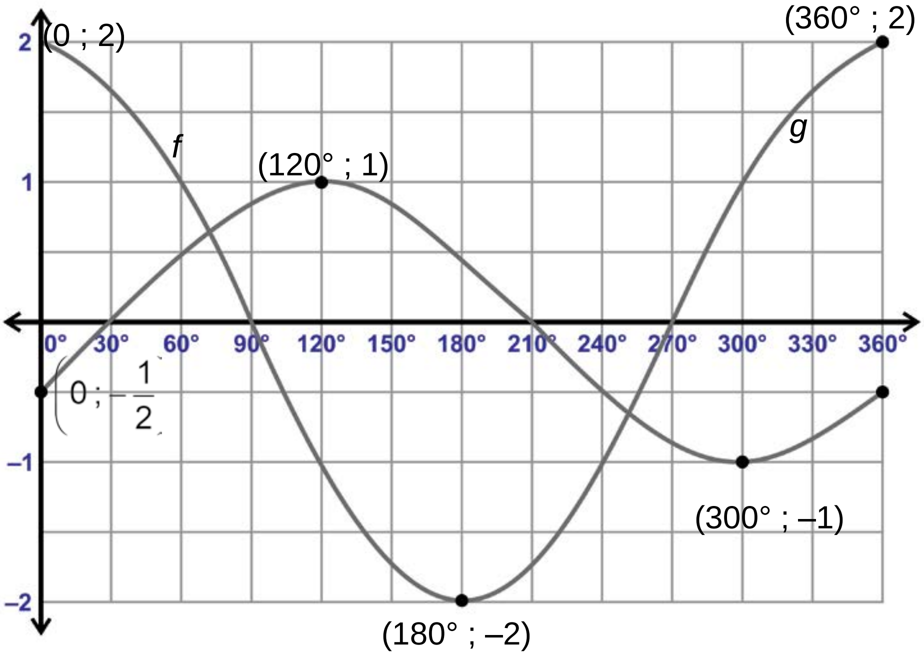
$$= \frac{(-\sin x)(-\cos x)(\tan x)}{(cos^2 x)\left(\frac{1}{2}\right)} \checkmark$$
$$= \frac{\sin x \cdot \tan x}{\cos x \cdot \frac{1}{2}}$$
$$= 2\tan^2 x$$

OR

$$\frac{(-\sin x)(-\cos x)(\tan x)}{(\cos^2 x)\left(\sin \frac{5}{6}\pi\right)}$$
$$= \frac{(-\sin x)(-\cos x)\left(\frac{\sin x}{\cos x}\right)}{(\cos^2 x)\left(\frac{1}{2}\right)}$$
$$= 2\tan^2 x$$

QUESTION 4

4.1



(0°;2) y-intercept f
(0°;–½) y-intercept g
(120°;1) turning g
(180°;–2) turning f
(300°;–1) turning g
(360°;2) turning f

4.2 2

4.3 360°

4.4 $x \in (0°;70°) \cup (250°;360°)$

OR $0° < x < 70° \quad \cup \quad 250° < x < 360°$

give leeway with 70° (±5°) and 250° (±5°)

QUESTION 5

5.1 $\sin 58^\circ = \dfrac{AC}{2}$ ratio OR $\dfrac{AC}{\sin 58^\circ} = \dfrac{2}{\sin 90^\circ}$

$\therefore AC \approx 1,7 \text{ m}$ answer $\therefore AC \approx 1,7 \text{ m}$

5.2 Area of Δ ABC $= \dfrac{1}{2}(1,7 \text{ m})(2,3 \text{ m})\sin 108^\circ$ formula substitution

$\approx 1,9 \text{ m}^2$

5.3 $\dfrac{BC}{\sin 108^\circ} = \dfrac{2,3 \text{ m}}{\sin 42^\circ}$ OR $BC^2 = 2,3^2 + 1,7^2 - 2(2,3)(1,7)\cos 108^\circ$

$BC \approx 3,3 \text{ m}$

$BC \approx 3,3 \text{ m}$

5.4 $DB^2 = (1,1)^2 + (2,3)^2$

$\therefore DB \approx 2,5 \text{ m}$

$BC^2 = DC^2 + DB^2 - 2DC \cdot DB \cdot \cos\hat{D}$

$(3,3)^2 = (2)^2 + (2,5)^2 - 2(2)(2,5)\cdot\cos\hat{D}$

$\therefore \hat{D} \approx 93,7^\circ$

QUESTION 6

6.1 6.1.1 $\hat{P}_2$

$\hat{P}_1 + \hat{P}_2 = 180^\circ$ (Angles on a straight line)

$\hat{P}_1 = 60^\circ$ (Angle at centre = 2 × angle on circumference of circle)

$60^\circ + \hat{P}_2 = 180^\circ$

$\hat{P}_2 = 120^\circ$

6.1.2 $\hat{R}_1 = \hat{T}$ (Angles at equal sides)

and $\hat{P}_2 = \hat{R}_1 + \hat{T} = 180^\circ$ (int. angles of triangle)

$\therefore 120^\circ + 2\hat{R}_1 = 180^\circ$

$2\hat{R}_1 = 60^\circ$

$\hat{R}_1 = 30^\circ$

6.2 6.2.1 $\hat{D}_2 = 50°$ (Tan-chord theorem)

6.2.2 $\hat{B}_1$

$\hat{A}_1 = 50°$ (Tan-chord theorem)

$\hat{A}_1 + 110° + \hat{B}_1 = 180°$ (Interior angles of triangle)

$50° + 110° + \hat{B}_1 = 180°$

$\hat{B}_1 = 20°$

6.2.3 $\hat{D}_1$

$\hat{B}_1 = \hat{C}_3 = 20°$ (Angles in same segment)

and $\hat{C}_2 = 20°$ (Given)

$\therefore \hat{D}_1 = 20°$ (Angles in same segment)

6.3 6.3.1 $a = 49°$ (Tan-chord theorem)

6.3.2 $P\hat{T}R = 78°$ (Tan-chord theorem)

$32° + \hat{T}_1 = 78°$

$\hat{T}_1 = 46°$

$\therefore b = 46°$ (Tan-chord theorem)

OR $Q\hat{P}R = 32°$ (angles in same segment)

$\therefore C = 78° - 32°$

$= 46°$

6.3.3 $c + 78° = 180°$ (Angles on same line)

$\therefore c = 102°$

QUESTION 7

7.1 7.1.1 QM

$KQ : QM = 3:1$ (Given)

$\therefore KM : QM = 4:1$ (Proportionality theorem)

$\therefore \frac{KM}{QM} = \frac{4}{1}$

$\therefore \frac{20 \text{ units}}{QM} = \frac{4}{1}$ OR $QM = \frac{1}{4} \times 20 \text{ units}$

$\therefore 20 = 4QM$ $= 5$ units

$\therefore 5 \text{ units} = QM$

7.1.2 KP

$\frac{KQ}{QM} = \frac{KP}{PL}$ (Proportionality theorem PQ || LM)

$\frac{3}{1} = \frac{KP}{4 \text{ units}}$

$KP = 12 \text{ units}$

7.1.3 KB

$\frac{KM}{QM} = \frac{KP}{BP}$ (Proportionality theorem BQ || PM)

$\frac{20 \text{ units}}{5 \text{ units}} = \frac{12 \text{ units}}{BP}$

20BP = 60

BP = 3 units

KB + BP = KP

KB + 3 = 12

KB = 9 units

7.2 7.2.1 *In* ΔKPM *and* ΔKBQ

$M\hat{K}P = Q\hat{K}B$ (Common angle)

$K\hat{M}P = K\hat{Q}B$ (Corresponding angles BQ||PM)

$K\hat{P}M = K\hat{B}Q$ (Corresponding angles BQ||PM)

$\therefore \Delta KPM \,|||\, \Delta KBQ$ (Angle, angle, angle)

7.2.2 $\frac{KQ}{KM} = \frac{BQ}{PM}$ ($\Delta KPM \,|||\, \Delta KBQ$)

$\frac{3}{4} = \frac{BQ}{10 \text{ units}}$

$4BQ = 30$

$BQ = 7{,}5 \text{ units}$

QUESTION 8

8.1 $v = \pi D n$

$8{,}75 = \pi(50)n$

$0{,}0557 = n$ OR $\frac{7}{40\pi}$

$w = 2\pi n$

$= 2\pi(0{,}0557\ldots)$ OR $2\pi\left(\frac{7}{40}\pi\right)$

$\approx 0{,}35$ rad/sec $\frac{7}{20}$ rad/sec

8.2 8.2.1 $s = r\theta$

$s = 28\text{ cm} \times \left(240° \times \frac{\pi}{180°}\right)$

= 117,3 cm

8.2.2 $A\hat{P}C = 90°$ (radius⊥ tangent)

AB = BC = 28 cm (Given)

∴ AC = AB + BC

$= 28\text{ cm} + 28\text{ cm}$

$= 56\text{ cm}$

$AC^2 = AP^2 + PC^2$ (Pyth)

$(56\text{ cm})^2 = (28\text{ cm})^2 + PC^2$

∴ **PC** $= 48{,}5$ **cm**

∴ **QC** $= 48{,}5$ **cm**

$E\hat{R}C = 90°$ (radius⊥ tangent)

ED = DC = 12 cm (Given)

∴ EC = ED + DC

= 12 cm + 12 cm

= 24 cm

(Pyth)

$EC^2 = ER^2 + RC^2$

$(24\text{ cm})^2 = (12\text{ cm})^2 + RC^2$

∴ **RC** $= 20{,}8$ **cm**

∴ **TC** $= 20{,}8$ **cm**

Total belt length = $117{,}286\text{ cm} + 50{,}3\text{ cm} + 2(48{,}5\text{ cm}) + 2(20{,}78\text{ cm})$

= 306,11 cm

OR

$\frac{PC}{28} = \tan 60°$

PC = 48,5 cm

QC = 48,5 cm ✓

$\frac{RC}{12} = \tan 60°$

RC = 20,8 cm

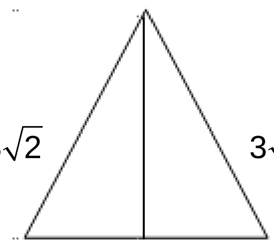
∴ TC = 20,8 cm

Total = 117,3 + 50,3 + 2 (48,5) + 2 (20,8)

≈ 306,2 cm

QUESTION 9

9.1



9.1.1 **Perpendicular height of ΔABE:** $\left(3\sqrt{2}\right)^2 = (\boldsymbol{sh})^2 + \left(\frac{3\sqrt{2}}{2}\right)^2$

$$18 = (\boldsymbol{sh})^2 + \frac{18}{4}$$

$\boldsymbol{sh} = \frac{3\sqrt{6}}{2}$ **units** or 3,7 units

OR $\frac{h}{3\sqrt{2}} = \sin 60^\circ$

$$h = \frac{3\sqrt{6}}{2}$$

9.1.2 Area ΔABE = $\frac{1}{2} \times \mathit{base} \times \perp \mathit{height}$

$$= \frac{1}{2} \times 3\sqrt{2} \times \frac{3\sqrt{6}}{2}$$

$= \frac{9\sqrt{3}}{2}$ or 7,794

Total area of octahedron

$$= 8 \times \frac{9\sqrt{3}}{2}$$

$= 36\sqrt{3}$ *or* 62,4 units^3

9.2 Volume cylinder = $\pi \times r^2 \times h$

$= \pi \times (30 \text{ mm})^2 \times 70 \text{ mm}$

= 197 920 3372 mm^3 or 63 000 π

Volume dome $= \dfrac{2}{3}\pi r^3$

$= \dfrac{2}{3}\pi (30 \text{ mm})^3$

= 56 548,668 mm^3 or 18 000 π

Volume centre removed $\pi \times r^2 \times h$

$= \pi \times (15 \text{ mm})^2 \times 70 \text{ mm}$

= 49 480,084 mm^3 or 15 750 π

Total volume = 197 920,3372 mm^3 + 56 548,668 mm^3 – 49 480,084 mm^3
= 204 988,92 mm^3 or 29 250 π

9.3 $a = 6{,}5 \text{ m} \div 5 = 1{,}3 \text{ m}$

$Area = a(m_1 + m_2 + m_3 + m_4 + m_5)$

$= 1{,}3\left(\dfrac{0 + 0{,}8}{2} + \dfrac{0{,}8 + 1{,}3}{2} + \dfrac{1{,}3 + 1{,}1}{2} + \dfrac{1{,}1 + 0{,}5}{2} + \dfrac{0{,}5 + 0}{2}\right)$

$= 1{,}3(0{,}4 + 1{,}05 + 1{,}2 + 0{,}8 + 0{,}25)$

$= 4{,}81 \text{ m}^2$

TOTAL: 150 marks