



NASIONALE SENIOR SERTIFIKAAT-EKSAMEN  
NOVEMBER 2019

**TEGNIESE WISKUNDE: VRAESTEL II**

**NASIENRIGLYNE**

Tyd: 3 uur

150 punte

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Hierdie nasienriglyne is opgestel vir gebruik deur eksaminators en hulp-eksaminators van wie verwag word om almal 'n standaardiseringsvergadering by te woon om te verseker dat die riglyne konsekwent vertolk en toegepas word by die nasien van kandidate se skrifte.

Die IEB sal geen bespreking of korrespondensie oor enige nasienriglyne voer nie. Ons erken dat daar verskillende standpunte oor sommige aangeleenthede van beklemtoning of detail in die riglyne kan wees. Ons erken ook dat daar sonder die voordeel van die bywoning van 'n standaardiseringsvergadering verskillende vertolkings van die toepassing van die nasienriglyne kan wees.

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**VRAAG 1**

$$\begin{aligned}
 1.1 \quad AB &= \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} \quad \text{korrekte formule} \\
 &= \sqrt{(200 - 100)^2 + (0 - 250)^2} \quad \text{vervanging} \\
 &= \sqrt{(100)^2 + (-250)^2} \\
 &= 50\sqrt{29} \quad \text{eenhede vereenvoudigde wortelvorm}
 \end{aligned}$$

$$\begin{aligned}
 1.2 \quad m_{AB} &= \frac{y_B - y_A}{x_B - x_A} \\
 &= \frac{0 - 250}{200 - 100} \quad \text{vervanging} \\
 &= -2,5 \quad \text{vereenvoudiging}
 \end{aligned}$$

$$\begin{aligned}
 1.3 \quad M &= \left( \frac{x_A + x_O}{2} ; \frac{y_A + y_O}{2} \right) \\
 M &= \left( \frac{100 + 0}{2} ; \frac{250 + 0}{2} \right) \\
 M &= (50; 125) \quad \text{x-koördinaat y-koördinaat}
 \end{aligned}$$

$$\begin{aligned}
 1.4 \quad \tan B &= m \\
 \tan B &= -2,5 \quad \text{vervanging} \\
 \therefore \hat{O}BA &\approx 68,2^\circ \quad \hat{O}BA \\
 \therefore \hat{A}OB &= 68,2^\circ \quad \hat{A}OB \\
 \therefore \theta &= 180^\circ - 68,2^\circ - 68,2^\circ = 43,6^\circ \quad \text{vereenvoudiging}
 \end{aligned}$$

**VRAAG 2**

$$2.1 \quad 2.1.1 \quad m_{LP} \times m_{LN} = -1$$

$$-1 \times m_{LN} = -1$$

$$\therefore m_{LN} = 1 \quad \text{LN gradiënt}$$

$$\therefore y = x + c \quad \text{met } M(-4; 4)$$

$$\therefore 4 = -4 + c$$

$$\therefore 8 = c$$

$$\therefore y = x + 8 \quad \text{vergelyking}$$

$$\text{OF} \quad y - y_1 = m(x - x_1)$$

$$y - 4 = 1(x + 4) \quad \checkmark$$

$$y = x + 8 \quad \text{vergelyking}$$

$$2.1.2 \quad \therefore y = x + 8 = -x + 2 \quad \text{volg uit Vraag 2.1.1}$$

$$\therefore 2x = -6$$

$$\therefore x = -3$$

$$\therefore y = x + 8$$

$$\therefore y = -3 + 8 = 5$$

$$\therefore L(-3; 5) \quad \text{koördinate}$$

$$m\left(\frac{x_L + x_N}{2}; \frac{y_L + y_N}{2}\right) = (-4; 4)$$

$$\frac{-3 + x_N}{2} = -4 \quad \frac{5 + y_N}{2} = 4 \quad \text{vergelykings}$$

$$x_N = -5 \quad y_N = 3$$

$$\therefore N(-5; 3) \quad \text{koördinate}$$

$$2.1.3 \quad y = -x + 2$$

$$\text{x-afsnit: } 0 = -x + 2$$

$$x = 2 \quad \therefore P(2; 0) \quad \text{koördinate}$$

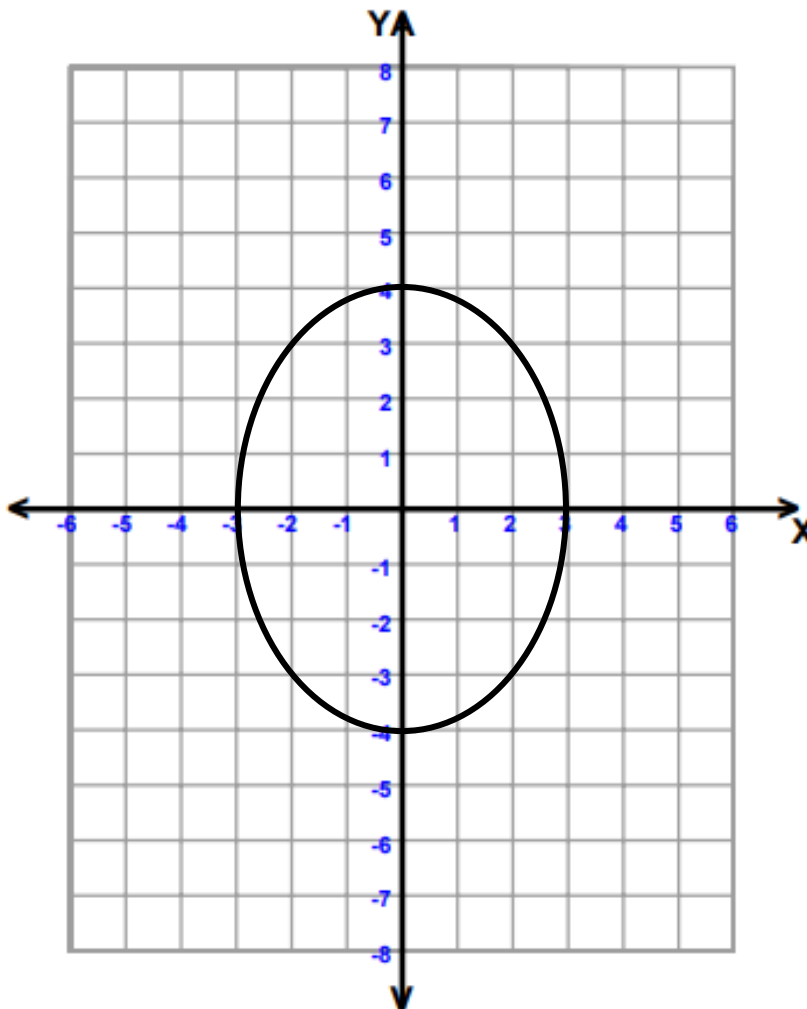
$$2.1.4 \quad x^2 + y^2 = r^2 \quad p(2; 0)$$

$$\therefore (2)^2 + (0)^2 = r^2 \quad \text{vervanging}$$

$$4 = r^2$$

$$\therefore x^2 + y^2 = 4 \quad \text{vergelyking}$$

2.2



x-afsnitte y-afsnitte vorm

**VRAAG 3**

3.1 3.1.1  $x^2 + y^2 = r^2$   
 $(2\sqrt{3})^2 + (-2)^2 = r^2$       vervanging  
 $16 = r^2$   
 4 eenhede =  $r$  vereenvoudiging

3.1.2  $\tan\theta = \frac{-2}{2\sqrt{3}}$  funksie  
 (verwysingshoek) =  $30^\circ$  metode  
 $\theta = 360^\circ - 30^\circ = 330^\circ$  vereenvoudiging

3.2  $\sec(a - b)$   
 $= \sec(2,695 - 1,112)$       vervanging  
 $= \sec(1,583)$   
 $= \frac{1}{\cos(1,583)}$   
 $\approx -81,9$  vereenvoudiging

3.3  $\frac{\sin 210^\circ \tan 45^\circ \cos 315^\circ}{\sin 45^\circ \cos 60^\circ}$   
 $= \frac{-\sin 30^\circ \tan 45^\circ \cos 45^\circ}{\sin 45^\circ \cos 60^\circ}$   
 $= \frac{\left(\frac{-1}{2}\right) \cdot (1) \cdot \left(\frac{\sqrt{2}}{2}\right)}{\left(\frac{\sqrt{2}}{2}\right) \cdot \left(\frac{1}{2}\right)}$   
 $= -1$

OF  
 $= \frac{-\sin 30^\circ \times \frac{\sin 45^\circ}{\cos 45^\circ} \times \cos 45^\circ}{\sin 45^\circ \times \sin 30^\circ}$   
 $= -1$

3.4 LK:  $\tan x \cdot \sin x$   
 $= \frac{\sin x}{\cos x} \cdot \sin x$   
 $= \frac{\sin^2 x}{\cos x}$

RK:  $\sec x - \cos x$   
 $= \frac{1}{\cos x} - \cos x$   
 $= \frac{1 - \cos^2 x}{\cos x}$   
 $= \frac{\sin^2 x}{\cos x}$

$\therefore$  LK = RK

OF LK:  $\tan x \cdot \sin x$   
 $= \tan x (\cos x \tan x)$   
 $= \cos x \cdot \tan^2 x$   
 $= \cos x (\sec^2 x - 1)$   
 $= \cos x \left( \frac{1}{\cos^2 x} - 1 \right)$   
 $= \frac{1}{\cos x} - \cos x$   
 $= \sec x - \cos x$   
 $\therefore$  LK = RK

OF LK:  $\tan x \cdot \sin x$

$$= \frac{\sin x}{\cos x} \cdot \frac{\sin x}{1}$$

$$= \frac{\sin^2 x}{\cos x}$$

$$= \frac{1 - \cos^2 x}{\cos x}$$

$$= \frac{1}{\cos x} - \frac{\cos^2 x}{\cos x}$$

$$= \sec x - \cos x$$

$\therefore$  LK = RK

OF RK:  $\sec x - \cos x$

$$= \frac{1}{\cos x} - \cos x$$

$$= \frac{1 - \cos^2 x}{\cos x}$$

$$= \frac{\sin^2 x}{\cos x}$$

$$= \sin x \times \frac{\sin x}{\cos x}$$

$$= \sin x \cdot \tan x$$

$\therefore$  LK = RK

3.5 3.5.1  $\operatorname{cosec} 2x = 2,114$  vir  $2x \in [0^\circ; 180^\circ]$

$$\frac{1}{\sin 2x} = 2,114$$

$$\frac{1}{2,114} = \sin 2x$$

$$0,473\dots = \sin 2x$$

Verwysingshoek  $\approx 28,2316^\circ$

$2x = 28,23^\circ$  of  $2x = 180^\circ - 28,23^\circ$  korrekte kwadrante

$x = 14,12^\circ$  of  $x = 75,88^\circ$  albei antwoorde

3.5.2 
$$\frac{\sin(360^\circ - x) \cdot \cos(180^\circ - x) \cdot \tan(180^\circ + x)}{\cos^2 x \cdot \sin \frac{5}{6} \pi}$$

$$= \frac{(-\sin x)(-\cos x)(\tan x)}{(\cos^2 x) \left(\frac{1}{2}\right)}$$

$$= \frac{\sin x \cdot \tan x}{\cos x \cdot \frac{1}{2}}$$

$$= 2 \tan^2 x$$

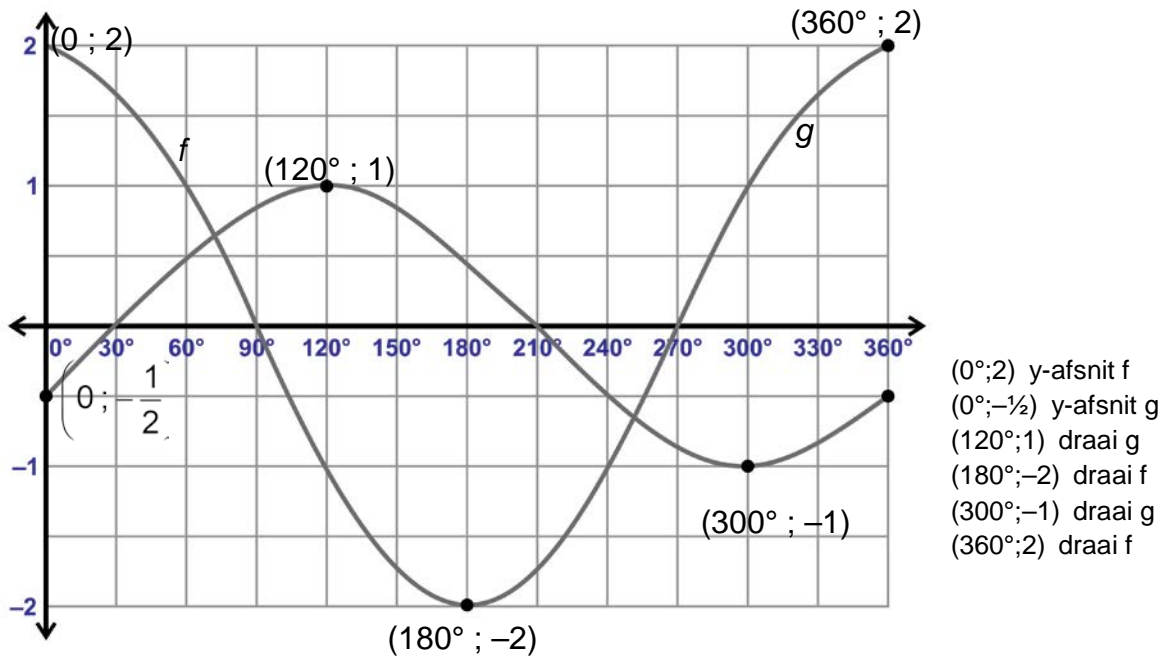
OF 
$$\frac{(-\sin x)(-\cos x)(\tan x)}{(\cos^2 x) \left(\sin \frac{5}{6} \pi\right)}$$

$$= \frac{(-\sin x)(-\cos x) \left(\frac{\sin x}{\cos x}\right)}{(\cos^2 x) \left(\frac{1}{2}\right)}$$

$$= 2 \tan^2 x$$

**VRAAG 4**

4.1



4.2 2

4.3 360°

4.4  $x \in (0^\circ; 70^\circ) \cup (250^\circ; 360^\circ)$       gee speling met 70° ( $\pm 5^\circ$ ) en 250° ( $\pm 5^\circ$ )  
 OF  $0^\circ < x < 70^\circ \cup 250^\circ < x < 360^\circ$

**VRAAG 5**

$$5.1 \quad \sin 58^\circ = \frac{AC}{2} \text{ verhouding} \quad \text{OF} \quad \frac{AC}{\sin 58^\circ} = \frac{2}{\sin 90^\circ}$$

$$\therefore AC \approx 1,7 \text{ m antwoord} \quad \therefore AC \approx 1,7 \text{ m}$$

$$5.2 \quad \text{Oppervlakte van } \triangle ABC = \frac{1}{2}(1,7 \text{ m})(2,3 \text{ m})\sin 108^\circ \text{ formule vervanging}$$

$$\approx 1,9 \text{ m}^2$$

$$5.3 \quad \frac{BC}{\sin 108^\circ} = \frac{2,3 \text{ m}}{\sin 42^\circ} \quad \text{OF} \quad BC^2 = 2,3^2 + 1,7^2 - 2(2,3)(1,7)\cos 108^\circ$$

$$BC \approx 3,3 \text{ m} \quad BC \approx 3,3 \text{ m}$$

$$5.4 \quad DB^2 = (1,1)^2 + (2,3)^2$$

$$\therefore DB \approx 2,5 \text{ m}$$

$$BC^2 = DC^2 + DB^2 - 2DC \cdot DB \cdot \cos \hat{D}$$

$$(3,3)^2 = (2)^2 + (2,5)^2 - 2(2)(2,5) \cdot \cos \hat{D}$$

$$\therefore \hat{D} \approx 93,7^\circ$$

**VRAAG 6**

$$6.1 \quad 6.1.1 \quad \hat{P}_2$$

$$\hat{P}_1 + \hat{P}_2 = 180^\circ \text{ (Hoeke op 'n reguitlyn)}$$

$$\hat{P}_1 = 60^\circ \text{ (Hoek by middelpunt} = 2 \times \text{hoek by omtrek van sirkel)}$$

$$60^\circ + \hat{P}_2 = 180^\circ$$

$$\hat{P}_2 = 120^\circ$$

$$6.1.2 \quad \hat{R}_1 = \hat{T} \text{ (Hoeke by gelyke sye)}$$

$$\text{en } \hat{P}_2 = \hat{R}_1 + \hat{T} = 180^\circ \text{ (Binnehoeke van driehoek)}$$

$$\therefore 120^\circ + 2\hat{R}_1 = 180^\circ$$

$$2\hat{R}_1 = 60^\circ$$

$$\hat{R}_1 = 30^\circ$$



6.2 6.2.1  $\hat{D}_2 = 50^\circ$  (Raaklyn-koord-stelling)

6.2.2  $\hat{B}_1$

$$\hat{A}_1 = 50^\circ \text{ (Raaklyn-koord-stelling)}$$

$$\hat{A}_1 + 110^\circ + \hat{B}_1 = 180^\circ \text{ (Binnehoeke van driehoek)}$$

$$50^\circ + 110^\circ + \hat{B}_1 = 180^\circ$$

$$\hat{B}_1 = 20^\circ$$

6.2.3  $\hat{D}_1$

$$\hat{B}_1 = \hat{C}_3 = 20^\circ \text{ (Hoeke in dieselfde segment)}$$

$$\text{en } \hat{C}_2 = 20^\circ \text{ (Gegee)}$$

$$\therefore \hat{D}_1 = 20^\circ \text{ (Hoeke in dieselfde segment)}$$

6.3 6.3.1  $a = 49^\circ$  (Raaklyn-koord-stelling)

6.3.2  $\hat{P}TR = 78^\circ$  (Raaklyn-koord-stelling)

$$32^\circ + \hat{T}_1 = 78^\circ$$

$$\hat{T}_1 = 46^\circ$$

$$\therefore b = 46^\circ \text{ (Raaklyn-koord-stelling)}$$

OF  $Q\hat{P}R = 32^\circ$  (hoeke in dieselfde segment)

$$\therefore C = 78^\circ - 32^\circ$$

$$= 46^\circ$$

6.3.3  $c + 78^\circ = 180^\circ$  (Hoeke op dieselfde lyn)

$$\therefore c = 102^\circ$$

**VRAAG 7**

7.1 7.1.1 QM

$$KQ : QM = 3 : 1 \quad (\text{Gegee})$$

$$\therefore KM : QM = 4 : 1 \quad (\text{Eweredigheidstelling})$$

$$\therefore \frac{KM}{QM} = \frac{4}{1}$$

$$\therefore \frac{20 \text{ eenhede}}{QM} = \frac{4}{1} \quad \text{OF} \quad QM = \frac{1}{4} \times 20 \text{ eenhede}$$

$$\therefore 20 = 4QM \quad = 5 \text{ eenhede}$$

$$\therefore 5 \text{ eenhede} = QM$$

7.1.2 KP

$$\frac{KQ}{QM} = \frac{KP}{PL} \quad (\text{Eweredigheidstelling } PQ \parallel LM)$$

$$\frac{3}{1} = \frac{KP}{4 \text{ eenhede}}$$

$$KP = 12 \text{ eenhede}$$

7.1.3 KB

$$\frac{KM}{QM} = \frac{KP}{BP} \quad (\text{Eweredigheidstelling } BQ \parallel PM)$$

$$\frac{20 \text{ eenhede}}{5 \text{ eenhede}} = \frac{12 \text{ eenhede}}{BP}$$

$$20BP = 60$$

$$BP = 3 \text{ eenhede}$$

$$KB + BP = KP$$

$$KB + 3 = 12$$

$$KB = 9 \text{ eenhede}$$

7.2 7.2.1 In  $\triangle KPM$  en  $\triangle KBQ$ :

$$\hat{M}KP = \hat{Q}KB \quad (\text{Gemeenskaplike hoek})$$

$$\hat{K}MP = \hat{K}QB \quad (\text{Ooreenkomstige hoeke } BQ \parallel PM)$$

$$\hat{K}PM = \hat{K}BQ \quad (\text{Ooreenkomstige hoeke } BQ \parallel PM)$$

$$\therefore \triangle KPM \parallel \triangle KBQ \quad (\text{Hoek, hoek, hoek})$$

$$7.2.2 \quad \frac{KQ}{KM} = \frac{BQ}{PM} \quad (\triangle KPM \parallel \triangle KBQ)$$

$$\frac{3}{4} = \frac{BQ}{10 \text{ eenhede}}$$

$$4BQ = 30$$

$$BQ = 7,5 \text{ eenhede}$$

**VRAAG 8**

8.1  $v = \pi Dn$   
 $8,75 = \pi(50)n$

$$0,0557 = n \qquad \text{OF} \qquad \frac{7}{40\pi}$$

$w = 2\pi n$

$$= 2\pi(0,0557\dots) \qquad \text{OF} \qquad 2\pi\left(\frac{7}{40}\pi\right)$$

$$\approx 0,35 \text{ rad/sek} \qquad \frac{7}{20} \text{ rad/sek}$$

8.2 8.2.1  $s = r\theta$

$$s = 28 \text{ cm} \times \left(240^\circ \times \frac{\pi}{180^\circ}\right)$$

$$= 117,3 \text{ cm}$$

8.2.2  $\hat{A}PC = 90^\circ$  (radius  $\perp$  raaklyn)  
 $AB = BC = 28 \text{ cm}$  (gegee)  
 $\therefore AC = AB + BC$   
 $= 28 \text{ cm} + 28 \text{ cm}$   
 $= 56 \text{ cm}$   
 $AC^2 = AP^2 + PC^2$  (Pythagoras)  
 $(56 \text{ cm})^2 = (28 \text{ cm})^2 + PC^2$   
 $\therefore \mathbf{PC = 48,5 \text{ cm}}$   
 $\therefore \mathbf{QC = 48,5 \text{ cm}}$

OF  $\frac{PC}{28} = \tan 60^\circ$   
 $PC = 48,5 \text{ cm}$   
 $QC = 48,5 \text{ cm}$

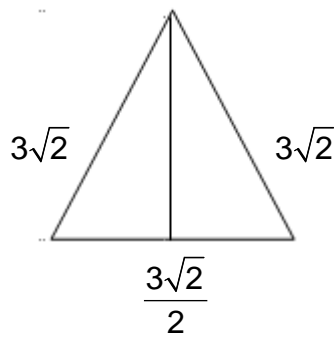
$\frac{RC}{12} = \tan 60^\circ$   
 $RC = 20,8 \text{ cm}$   
 $\therefore TC = 20,8 \text{ cm}$   
 Totaal =  $117,3 + 50,3 + 2(48,5) + 2(20,8)$   
 $\approx 306,2 \text{ cm}$

$\hat{E}RC = 90^\circ$  (radius  $\perp$  raaklyn)  
 $ED = DC = 12 \text{ cm}$  (gegee)  
 $\therefore EC = ED + DC$   
 $= 12 \text{ cm} + 12 \text{ cm}$   
 $= 24 \text{ cm}$   
 (Pythagoras)  
 $EC^2 = ER^2 + RC^2$   
 $(24 \text{ cm})^2 = (12 \text{ cm})^2 + RC^2$   
 $\therefore \mathbf{RC = 20,8 \text{ cm}}$   
 $\therefore \mathbf{TC = 20,8 \text{ cm}}$

Totale bandlengte =  $117,286 \text{ cm} + 50,3 \text{ cm} + 2(48,5 \text{ cm}) + 2(20,78 \text{ cm})$   
 $= 306,11 \text{ cm}$

**VRAAG 9**

9.1



9.1.1 Loodregte hoogte van  $\Delta ABE$ :  $(3\sqrt{2})^2 = (sh)^2 + \left(\frac{3\sqrt{2}}{2}\right)^2$

$$18 = (sh)^2 + \frac{18}{4}$$

$$sh = \frac{3\sqrt{6}}{2} \text{ eenhede of } 3,7 \text{ eenhede}$$

OF  $\frac{h}{3\sqrt{2}} = \sin 60^\circ$

$$h = \frac{3\sqrt{6}}{2}$$

9.1.2 Oppervlakte  $\Delta ABE = \frac{1}{2} \times \text{basis} \times \perp \text{ hoogte}$

$$= \frac{1}{2} \times 3\sqrt{2} \times \frac{3\sqrt{6}}{2}$$

$$= \frac{9\sqrt{3}}{2} \text{ of } 7,794$$

Totale oppervlakte van oktaëder

$$= 8 \times \frac{9\sqrt{3}}{2}$$

$$= 36\sqrt{3} \text{ of } 62,4 \text{ eenhede}^3$$

$$\begin{aligned}
 9.2 \quad \text{Volume silinder} &= \pi \times r^2 \times h \\
 &= \pi \times (30 \text{ mm})^2 \times 70 \text{ mm} \\
 &= 197\,920\,3372 \text{ mm}^3 \text{ of } 63\,000 \pi
 \end{aligned}$$

$$\begin{aligned}
 \text{Volume koepel} &= \frac{2}{3} \pi r^3 \\
 &= \frac{2}{3} \pi (30 \text{ mm})^3 \\
 &= 56\,548,668 \text{ mm}^3 \text{ of } 18\,000 \pi
 \end{aligned}$$

$$\begin{aligned}
 \text{Volume middel verwyder} &= \pi \times r^2 \times h \\
 &= \pi \times (15 \text{ mm})^2 \times 70 \text{ mm} \\
 &= 49\,480,084 \text{ mm}^3 \text{ of } 15\,750 \pi
 \end{aligned}$$

$$\begin{aligned}
 \text{Totale volume} &= 197\,920,3372 \text{ mm}^3 + 56\,548,668 \text{ mm}^3 - 49\,480,084 \text{ mm}^3 \\
 &= 204\,988,92 \text{ mm}^3 \text{ of } 29\,250 \pi
 \end{aligned}$$

$$9.3 \quad a = 6,5 \text{ m} \div 5 = 1,3 \text{ m}$$

$$\begin{aligned}
 \text{Oppervlakte} &= a(m_1 + m_2 + m_3 + m_4 + m_5) \\
 &= 1,3 \left( \frac{0+0,8}{2} + \frac{0,8+1,3}{2} + \frac{1,3+1,1}{2} + \frac{1,1+0,5}{2} + \frac{0,5+0}{2} \right) \\
 &= 1,3(0,4 + 1,05 + 1,2 + 0,8 + 0,25) \\
 &= 4,81 \text{ m}^2
 \end{aligned}$$

**TOTAAL: 150 punte**