TOTAL MARKS

NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2019

TECHNICAL MATHEMATICS: PAPER I

EXAMINATION NUMBER

Time: 3 hours 150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 20 pages and an Information Sheet of 2 pages (i–ii). Please check that your question paper is complete.
2. Read the questions carefully.
3. **Answer ALL the questions on the question paper and hand this in at the end of the examination. Remember to write your examination number in the space provided.**
4. Diagrams are not necessarily drawn to scale.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Round off your answers to one decimal digit where necessary, unless otherwise stated.
7. All the necessary working details must be clearly shown.
8. It is in your own interest to write legibly and to present your work neatly.
9. Two blank pages (pages 19 and 20) are included at the end of the paper. If you run out of space for a question, use these pages. Clearly indicate the question number of your answer should you use this extra space.

FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

Q1	Q2	Q3	Q4	Q5	Q6	TOTAL
30	**23**	**28**	**18**	**26**	**25**	**150**

QUESTION 1

1.1 Consider $f(x)=6-x^2+x$

1.1.1 For which value(s) of x is $f(x)=0$?

(3)

1.1.2 Hence, find the value(s) of x for which $f(x)>0$.

(2)

1.2 Solve the following simultaneous equations:
$3^y-81^x=0$ and $y=x^2-6x+9$

(7)

1.3 Given: $f(x) = -(x-2)^2 + 3$ and $g(x) = 5$

1.3.1 Solve $f(x) = g(x)$ by using the quadratic formula, where $x \in$ {complex numbers}.

(5)

1.3.2 Determine the value(s) of k for which $f(x) = g(x) + k$ has two unequal and real roots.

(4)

1.4 Convert the binary number $\frac{11011_2}{110011_2}$ to a rational number in simplest form (show all workings).

(4)

1.5 Strain measure ε of a metal rod is defined by the equation $\varepsilon = \frac{\Delta L}{L}$, where

L = original length
ΔL = change in length = new length – original length

Determine the original length (in cm) of a metal rod if $\varepsilon = 0{,}77$ and the resultant new length is 182 cm.

Write the answer in scientific notation, rounded to five decimal places.

(5)
[30]

QUESTION 2

2.1 Solve for x if $log_{x+2} 27 = \frac{3}{4}$

(4)

2.2 Simplify each of the following without the use of a calculator.

2.2.1 $\left(\sqrt{12} - \sqrt{3} - \sqrt{8}\right)\left(\sqrt{12} - \sqrt{3} + \sqrt{8}\right)$

(4)

2.2.2 $\frac{3 \cdot 2^{2x+1} - 2^{2x-2} + 4^x}{4 \cdot 2^{2x-3}}$

(6)

2.3 The complex numbers $z = 5 - 2i$ and $w = 6i - 1$ are given.
Determine $2z - iw$.

(3)

2.4 The complex number $P = -3 - 4i$ is given.

2.4.1 Draw an Argand diagram to represent $P = -3 - 4i$.

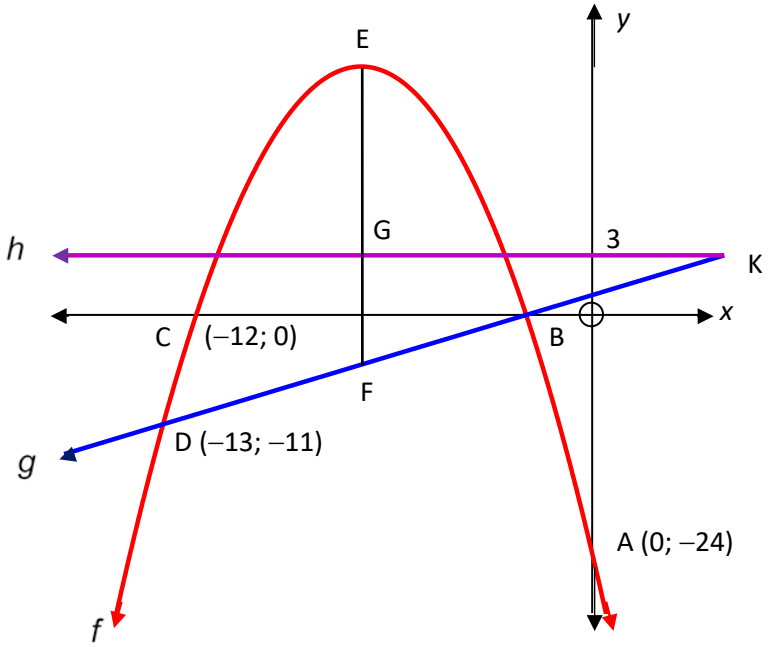
(3)

2.4.2 Hence, express P in polar form.

(3)
[23]

QUESTION 3

3.1 In the diagram, the straight line defined by $g(x) = x + 2$, and the parabola defined by $f(x)$, intersect at points B and D (–13; –11). The parabola cuts the x-axis at points B and C (–12; 0) and the y-axis at point A (0; –24). The line defined by $h(x) = 3$ is also sketched.
Functions h and g meet at K and EGF is the axis of symmetry of f with F on g and G on h.



3.1.1 Calculate the coordinates of point B.

(2)

3.1.2 Determine, clearly showing all your working details, that the equation of the parabola is given by: $f(x) = -x^2 - 14x - 24$

(4)

3.1.3 Calculate the length of EF, where E is the turning point of the parabola, and EF is parallel to the *y*-axis.

(4)

3.1.4 Calculate the area of ΔGFK.

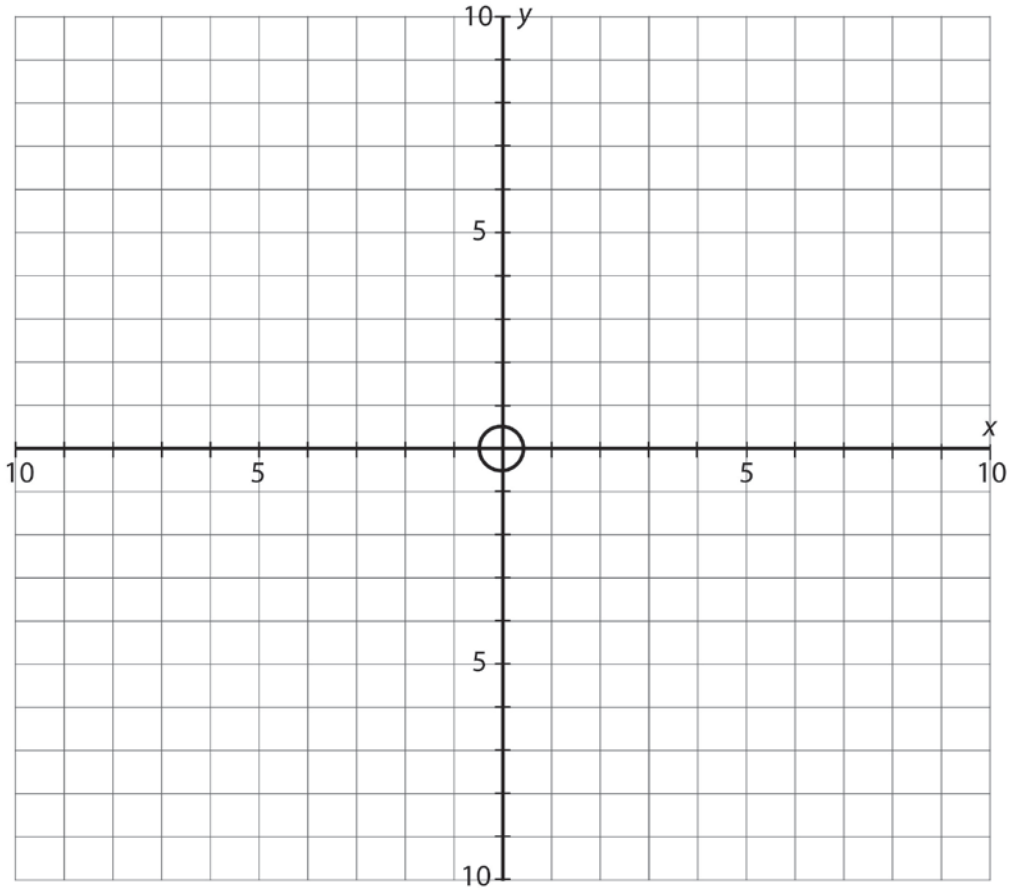
(4)

3.2 Given: $f(x) = -\frac{2}{x} - 3$ and $g(x) = -\sqrt{9 - x^2}$

3.2.1 Calculate the coordinates of the *x*-intercept of *f*.

(2)

3.2.2 Sketch the graphs of *f* and *g* clearly showing all asymptotes and intercepts with the axes.



(5)

3.2.3 For which values of *x* is *f(x)* ≥ 0?

(2)

3.3 The graph of an increasing exponential function with defining equation $f(x) = a.b^x + q$ has the following properties.

- Range: $y > -3$
- The points (0; –2) and (1; –1) lie on the graph of f.

Determine the equation that defines f.

(5)
[28]

QUESTION 4

4.1 Monica invests R120 000. She is quoted a nominal interest rate of 7,2% p.a. compounded monthly.

4.1.1 Calculate the equivalent effective annual interest rate that this investment yields. Round off your answer to two decimal digits.

(2)

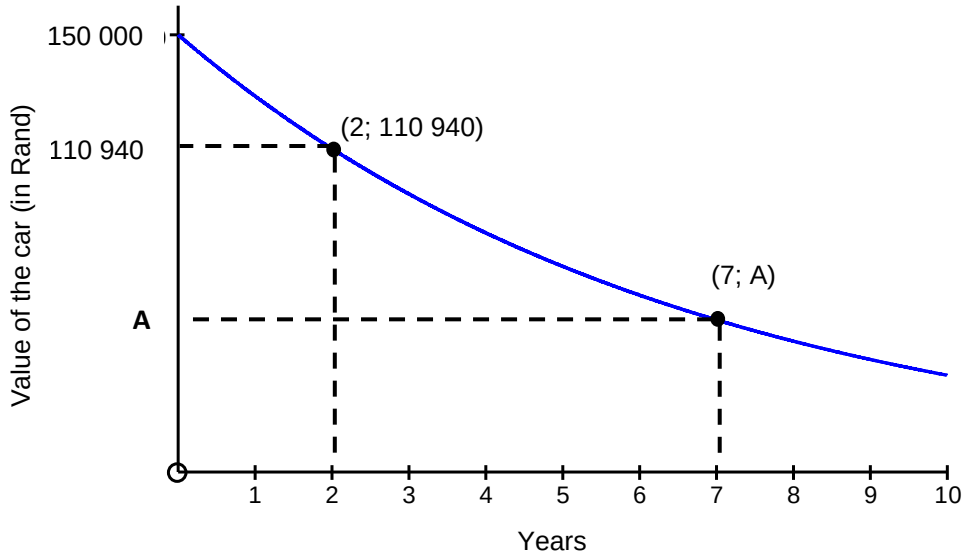
4.1.2 Calculate how long (in full months) it takes Monica's investment to be worth at least R150 000.

(5)

4.2 Amy won R2 000 000 on the Lotto and invested her winnings for a period of seven years. After three years she withdrew R300 000 to purchase a new car. During the first four years of the investment, the interest rate was 12% p.a. compounded monthly. Thereafter, interest changed to 14% p.a. compounded half yearly. Calculate the amount available in her account at the end of seven years.

(6)

4.3 The exponential graph below shows the depreciating value of a new car over a period of time (in years):



4.3.1 What type of depreciation is illustrated above?

(1)

4.3.2 Use the information on the graph to find the annual rate of depreciation.

(2)

4.3.3 Calculate the value of **A**, and interpret this value.

(2)

[18]

QUESTION 5

5.1 Find $f'(x)$ using first principles if $f(x) = -3x + 1$

(5)

5.2 5.2.1 Find $f'(x)$ if $f(x) = \dfrac{2\sqrt{x} - 5}{\sqrt{x}}$

(3)

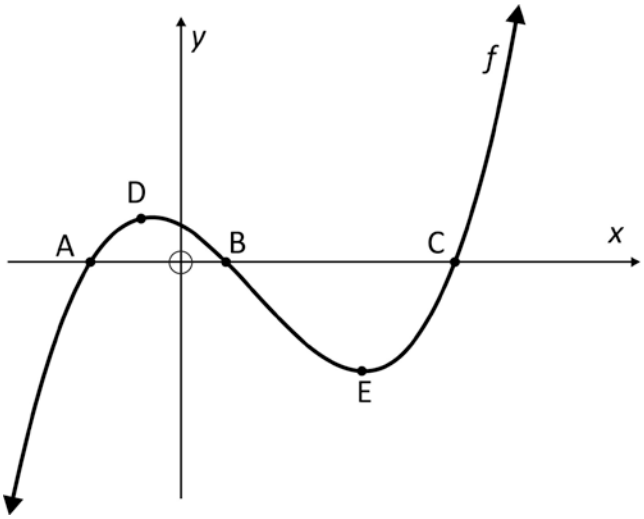
5.2.2 Determine $\dfrac{dy}{dx}$ where $\dfrac{y}{2x} = (1 - x)^2$

(4)

5.3 Find the x-value of the point on the curve defined by $f(x)=x(3x+13)$ where, a tangent to the curve will have an angle of inclination of 45°.

(4)

5.4 Sketched below is the graph of f defined by $f(x)=x^3+bx^2+cx+d$, where D and E are the turning points and A (–2; 0), B (1; 0) and C (6; 0) are *x-intercepts*.



5.4.1 Find the numerical values of b, c and d.

(4)

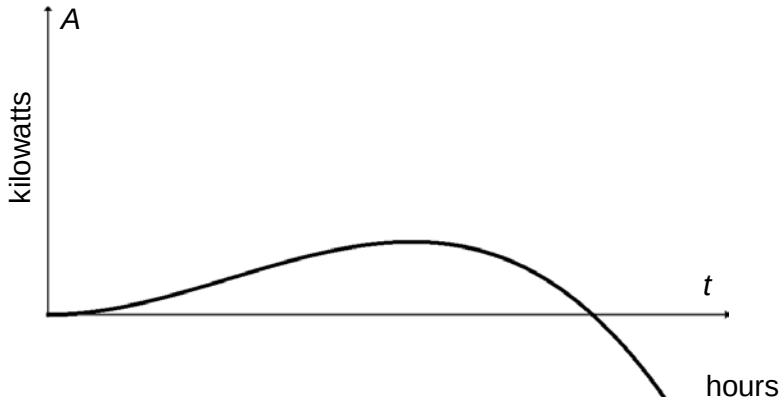
5.4.2 Hence, determine the coordinates of D and E.

(6)
[26]

QUESTION 6

6.1 Consider $A = -t^3 + 2t^2$ where A (in kilowatts) represents the amount of power available during a power outage, *t* hours after the power has been switched off.

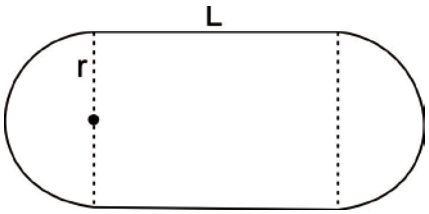
The graph of A is drawn below.



A kettle of water can only be boiled if there is at least 1 KW of power available to the household. For how long during the power outage will it be possible to boil the kettle?

(5)

6.2 A swimming pool has a perimeter of 400 metres. The shape of the pool is a rectangle with a semi-circle on each of the shorter sides of the rectangle as shown below. The length of the longer side of the rectangle is **L** metres and the radius of the semi-circle is **r** metres.



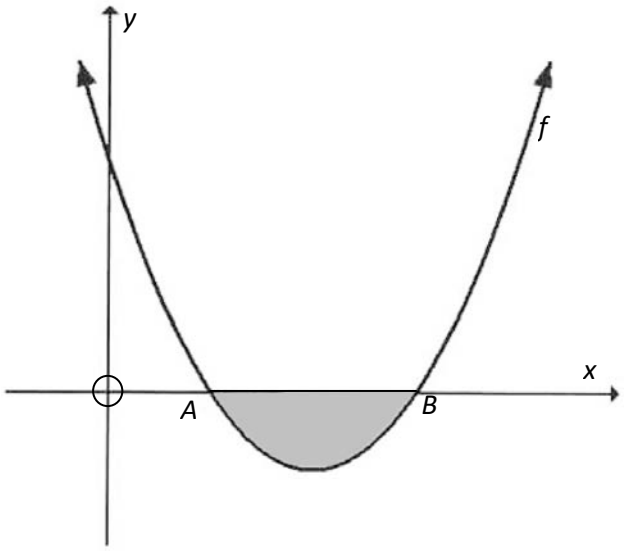
Find, in terms of π, the value of r such that the surface area of the pool is a maximum.

(7)

6.3 Evaluate $\int (2x^{-1} + 3x^2 - 1)dx$.

(6)

6.4 The sketch below shows the curve of the function defined by $f(x) = 2x^2 - 8x + 6$ with A and B representing the x-intercepts:



Calculate the shaded area bounded by the curve and the *x*-axis between points A and B. Show all workings.

(7)
[25]

Total: 150 marks

ADDITIONAL SPACE (ALL questions)

REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.

