NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2019

TECHNICAL MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

QUESTION 1

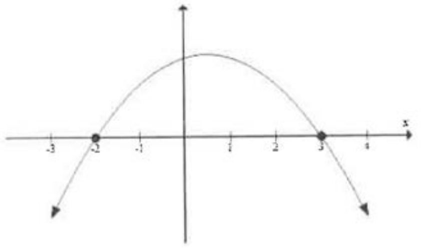
1.1 1.1.1 $6 - x^2 + x = 0$

$x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0$

$x = 3$ or $x = -2$

1.1.2

$f(x) > 0$ for $x \in (-2; 3)$ $x \in \mathbb{R}$ OR $-2 < x < 3$ 	OR using number line 

1.2 $3^y = 3^{4x}$ OR $y = x^2 - 6x + 9$ ……… (2)

$\therefore y = 4x$ ……… (1)

Subst (1) → (2): $4x = x^2 - 6x + 9$

$0 = x^2 - 10x + 9$

$0 = (x - 9)(x - 1)$

$x = 9$ or $x = 1$

Subst in (1) $y = 36$ or $y = 4$

1.3 1.3.1 $-(x-2)^2 + 3 = 5$

$-x^2 + 4x - 4 - 2 = 0$

$x^2 - 4x + 6 = 0$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(6)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{-8}}{2} = \frac{4 \pm 2\sqrt{-2}}{2}$$

$x = 2 \pm \sqrt{-2}$ OR $x = 2 \pm \sqrt{2}i$

1.3.2 $-x^2 + 4x - 1 = 5 + k$

$0 = x^2 - 4x + 6 + k$

$\Delta = (-4)^2 - 4(1)(6 + k)$

$= 16 - 24 - 4k$

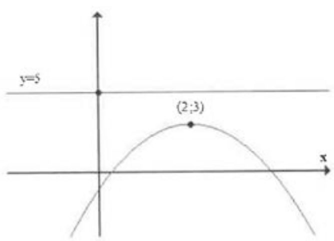
$= -4k - 8$

Fw 2 real, diff roots $-4k - 8 > 0$

$-4k > 8$

$k < -2$

OR



by insp from graph
$g(x) + k$ will meet f twice if
$5 + k < 3$
$k < -2$

1.4 $\dfrac{1\times 2^4 + 1\times 2^3 + 0\times 2^2 + 1\times 2^1 + 1\times 2^0}{1\times 2^5 + 1\times 2^4 + 0\times 2^3 + 0\times 2^2 + 1\times 2^1 + 1\times 2^0}$

$= \dfrac{16+8+2+1}{32+16+2+1}$

$= \dfrac{27}{51}$

$= \dfrac{9}{17}$

1.5 $\varepsilon = \dfrac{\Delta L}{L}$

$0{,}77 = \dfrac{182 - L}{L}$

$0{,}77L + L = 182$

$L = \dfrac{182}{1{,}77}$

$= 102{,}824858 \ldots\ldots$

$\approx 1{,}02825 \times 10^2$

QUESTION 2

2.1 $\left((x+2)^{\frac{3}{4}}\right)^{\frac{4}{3}} = (27)^{\frac{4}{3}}$

$x + 2 > 0$
$x > -2$

OR

$(x+2)^{\frac{3}{4}} = 27$

$\left((x+2)^{\frac{3}{4}}\right)^{\frac{4}{3}} = (27)^{\frac{4}{3}}$

$(x+2)^{\frac{1}{4}} = 3$

$x + 2 = 3^4$

$x + 2 = 81$

$x = 79$ Valid

2.2 2.2.1 $\left(2\sqrt{3}-\sqrt{3}-2\sqrt{2}\right)\left(2\sqrt{3}-\sqrt{3}+2\sqrt{2}\right)$

$= \left(\sqrt{3}-2\sqrt{2}\right)\left(\sqrt{3}+2\sqrt{2}\right)$

$= 3 - 8$

$= -5$

2.2.2 $\dfrac{3\cdot 2^{2x+1} - 2^{2x-2} + 4^x}{4\cdot 2^{2x-3}}$

$= \dfrac{3\cdot 2^{2x}\cdot 2^1 - 2^{2x}\cdot 2^{-2} + 2^{2x}}{4\cdot 2^{2x}\cdot 2^{-3}}$

$= \dfrac{\cancel{2^{2x}}\left(3\cdot 2^1 - 2^{-2} + 1\right)}{4.\cancel{2^{2x}}\cdot 2^{-3}}$

$= \dfrac{6-\frac{1}{4}+1}{4\cdot\frac{1}{8}}$

$= \dfrac{27}{4}\times\dfrac{2}{1}$

$= \dfrac{27}{2}$

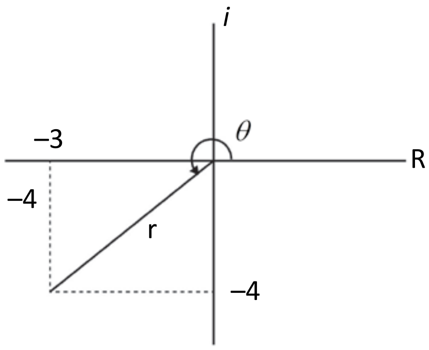
2.3 $2(5-2i) - i(6i-1)$

$= 10 - 4i - 6i^2 + i$

$= 10 - 4i + 6 + i$

$= 16 - 3i$

2.4 2.4.1



point
ray

2.4.2 $r = |p|$

$r^2 = (-3)^2 + (-4)^2$

$= 9 + 16 = 25$

$r = 5$

$\tan\theta = \frac{-4}{-3} = \frac{4}{3}$

$\theta = 180° + 53,13°$

$= 233,13°$

P = (5; 233,13°) OR (5 cos 233,13° ; 5 sin 233,13°)

OR p = +5 cos 233,13° + 5 sin 233,13°

OR p = 5 cis 233,13°

OR p = $5\underline{|233,13°}$

QUESTION 3

3.1 3.1.1 At B: $x + 2 = 0$
$x = -2$
B is (–2; 0)

3.1.2 Roots are – 12 and – 2

Eqn is $y = a(x+12)(x+2)$

Subst (–13; –11): $-11 = a(-1)(-11)$

$-11 = 11a$

$-1 = a$

$y = -1(x+12)(x+2)$

$y = -x^2 - 14x - 24$

3.1.3 $x_E = -7$ (by symmetry) OR $f'(x) = 0$
$-2x + 14 = 0$
$x = 7$

$y_E = -(-7)^2 - 14(-7) - 24$

$= -49 + 98 - 24$

$= 25$

$x_F = x_E = -7$: Subst. in g: $y_F = -7 + 2 = -5$

EF = $y_E - y_F$

$= 25 - (-5) = 30$

3.1.4 $x_E = x_G = -7$

G is (–7; 3)

At K, $y = 3$: $3 = x + 2$

$1 = x$

K is (1; 3)

$GF = 3 - (-5) = 8$ $GK = 1 - (-7) = 8$

Area $\Delta GFK = \frac{1}{2} GF.GK$

$= \frac{1}{2} \times 8 \times 8$

$= 32$

3.2 3.2.1 Let $y = 0$

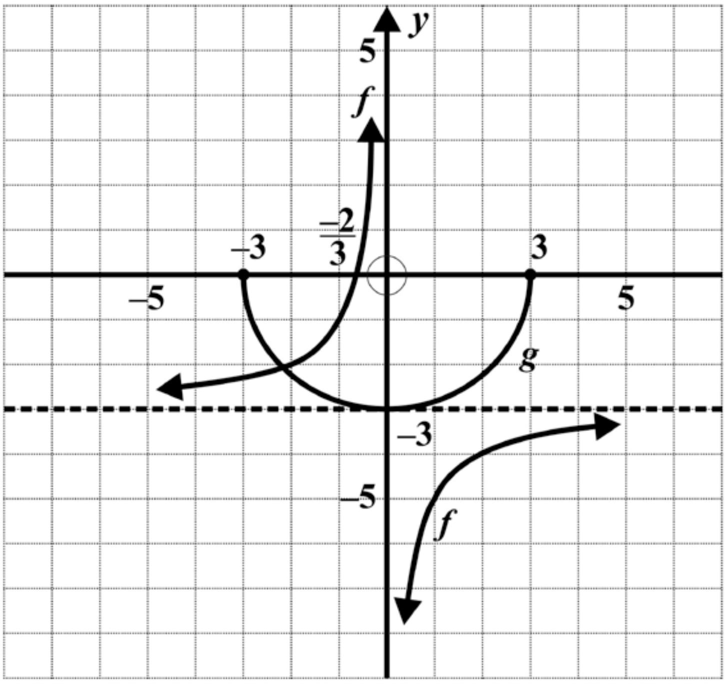
$$0 = \frac{-2}{x} - 3$$

$$3 = \frac{-2}{x}$$

$$3x = -2$$

$$x = -\frac{2}{3} \quad \text{OR} \quad \left(-\frac{2}{3}; 0\right)$$

3.2.2



For g: $r = 3$

3.2.3 $x \in \left[-\frac{2}{3}; 0\right)$ OR $-\frac{2}{3} \le x < 0$

3.3 Asymptote y = –3

i.e.: q = –3

$y = a.b^x - 3$

Subst (0; –2): $-2 = a.b^0 - 3$

$1 = a$

Subst (1; –1): $-1 = b^1 - 3$

$2 = b$

i.e.: $y = 2^x - 3$

QUESTION 4

4.1 4.1.1 $1+i\ eff=\left(1+\frac{0{,}072}{12}\right)^{12}$

i eff = 0,074424 ……

i.e.: effective rate $\approx$ 7,44%

4.1.2 $150\ 000=120\ 000\left(1+\frac{0{,}072}{12}\right)^{n}$

$\frac{15}{12}=\left(1+\frac{0{,}072}{12}\right)^{n}$

$\log_{\left(1+\frac{0{,}072}{12}\right)}\left(\frac{5}{4}\right)=n$

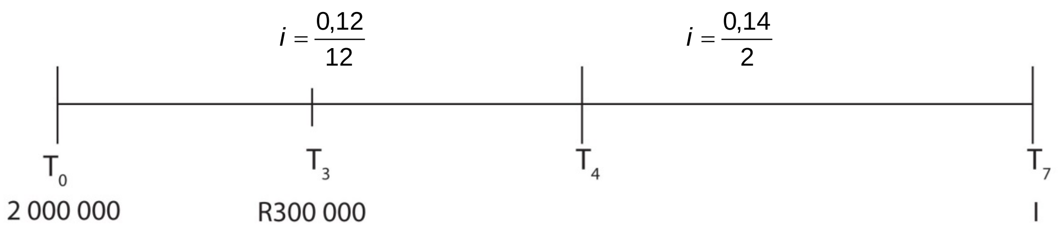
n $\approx$ 37,3 … months i.e. 38 months

4.2 $I=2\times10^{6}\left(1+\frac{0{,}12}{12}\right)^{48}\left(1+\frac{0{,}14}{2}\right)^{6}-300\ 000\left(1+\frac{0{,}12}{12}\right)^{12}\left(1+\frac{0{,}14}{2}\right)^{6}$

$\approx$ R4 331 715,06

OR $\left[2\times10^{6}\left(1+\frac{0{,}12}{12}\right)^{36}-300\ 000\right]\left(1+\frac{0{,}12}{12}\right)^{12}\left(1+\frac{0{,}14}{2}\right)^{6}$

$\approx$ R4 331 715,06



4.3 4.3.1 Reducing balance OR diminishing balance

4.3.2 110 940 = 150 000 $(1-i)^2$

$\sqrt{\frac{110\ 940}{150\ 000}}=1-i$

$i\approx 0{,}14$
Rate is 14%

4.3.3 A = 150 000 $(1-0{,}14)^7$
$\approx$ R52 189,17
The book value of car after 7 years.

QUESTION 5

5.1 $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{-3(x+h)+1-(-3x+1)}{h}$

$= \lim_{h\to 0} \frac{-3x-3h+1+3x-1}{h}$

$= \lim_{h\to 0} \frac{3h}{h}$

$= 3$

5.2 5.2.1 $f(x) = \frac{2\sqrt{x}}{\sqrt{x}} - \frac{5}{\sqrt{x}}$

$= 2 - 5x^{-\frac{1}{2}}$

$f'(x) = \frac{5}{2}x^{-\frac{3}{2}}$

5.2.2 $y = 2x(1-x)^2$

$= 2x - 4x^2 + 2x^3$

$\therefore \frac{dy}{dx} = 2 - 8x + 6x^2$

5.3 $f(x) = 3x^2 + 13x$

$f'(x) = 6x + 13$

$m_{tan} = \tan 45° = 1$

i.e.: $f'(x) = 1$

$6x + 13 = 1$

$6x = -12$

$x = -2$

5.4 5.4.1 Eqn is $y=(x+2)(x-1)(x-6)$

$=(x+2)(x^2-7x+6)$

$=x^3-7x^2+6x+2x^2-14x+12$

$=x^3-5x^2-8x+12$

b = –5; c = –8; d = 12

5.4.2 $f'(x)=3x^2-10x-8$

At D and E, $3x^2-10x-8=0$

$(3x+2)(x-4)=0$

$x_D=-\frac{2}{3}$ $\quad$ $x_E=4$

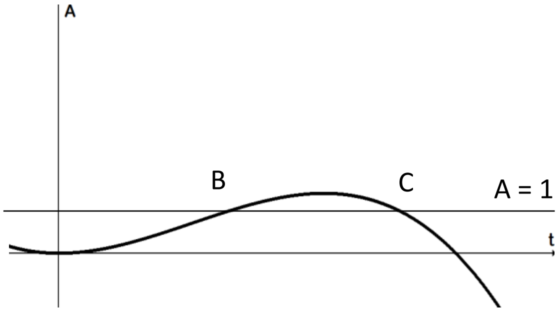
$y_D=\frac{400}{27}$ $\quad$ $y_E=-36$

(Use calculator)

D is $\left(-\frac{2}{3};\frac{400}{27}\right)$ E is $(4;-36)$

QUESTION 6

6.1 Draw A = 1



time is between t_B and t_C

At B and C, $1 = -t^3 + 2t^2$

$t^3 - 2t^2 + 1 = 0$ ✓

Let $f(t) = t^3 - 2t^2 + 1$

$f(1) = 1 - 2 + 1 = 0$

$\therefore$ $t - 1$ is a factor

$(t - 1)(t^2 - t - 1) = 0$

$t = 1$ or $t = \dfrac{1 \pm \sqrt{1 + 4}}{2}$

$$t = \frac{1 + \sqrt{5}}{2} \approx 1{,}6 \ (t > 0)$$

$\therefore$ time is 1,6 – 1 = 0,6 hours = 36 minutes

6.2 Perimeter $= 2\pi r + 2L$

$400 = 2\pi r + 2L$

$200 = \pi r + L$

$200 - \pi r = L$

$S = \pi r^2 + 2r \cdot L$

$= \pi r^2 + 2r(200 - \pi r)$

$S = 400r - \pi r^2$

$\dfrac{ds}{dr} = 400 - 2\pi r$

At max, $400 - 2\pi r = 0$

$r = \dfrac{200}{\pi}$ m

6.3 $\int \left(2x^{-1} + 3x^2 - 1\right) dx$

$= 2 \cdot \ln x + \dfrac{3x^{2+1}}{2+1} - x + c$

$= 2\ln x + x^3 - x + c$ ✓

6.4 At A & B, $2x^2 - 8x + 6 = 0$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

x = 3 or x = 1

$$\therefore \text{Area} = \int_A^B f(x)\,dx$$

$$= \int_1^3 \left(2x^2 - 8x + 6\right) dx$$

$$= \left[\frac{2x^3}{3} - \frac{8x^2}{2} + 6x\right]_1^3$$

$$= \left[\frac{2(27)}{3} - 4(9) + 6(3)\right] - \left[\frac{2}{3} - 4 + 6\right]$$

$$= \left|-2\frac{2}{3}\right|$$

$$\text{Area} = 2\frac{2}{3}$$

Total: 150 marks