## MATHEMATICS: PAPER II



## PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 24 pages and an Information Sheet of 2 pages ( $\mathrm{i}-\mathrm{i}$ ). Please check that your question paper is complete.
2. Read the questions carefully.
3. Answer ALL the questions on the question paper and hand this in at the end of the examination. Remember to write your examination number on the paper.
4. Diagrams are not necessarily drawn to scale.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. All necessary working details must be clearly shown.
7. Round off your answers to one decimal digit where necessary, unless otherwise stated.
8. Ensure that your calculator is in DEGREE mode.
9. It is in your own interest to write legibly and to present your work neatly.

FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

| Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |
| 22 | 22 | 10 | 21 | 25 | 19 | 12 | 19 |


| TOTAL | $/ 150$ |
| :--- | :--- |

## SECTION A

## QUESTION 1

(a) (1) $\quad \mathrm{P}(2 ; 8), \mathrm{Q}(4 ; \mathrm{y})$ and $\mathrm{R}(\mathrm{x} ; 0)$.

Determine x and y if Q is the midpoint of PR .
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(2) The points $(1 ; 2),(\mathrm{p}-1 ; 3)$ and $(-3 ;-6)$ all lie on the same straight line. Determine the value of p .
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(b) In the diagram, triangle APT is drawn.
$\mathrm{T}(3 ; q), \mathrm{A}(-3 ;-4)$ and $\mathrm{P}(5 ;-2)$.
TA passes through the origin.
$R$ is the midpoint of AP.
$K$ is the $x$ intercept of TR.

(1) Show that $\mathrm{q}=4$.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(2) Show that $K\left(\frac{13}{7} ; 0\right)$.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(3) Determine, correct to one decimal digit, the size of
(i) $\hat{\mathrm{K}}_{1}$
(ii) $\hat{\mathrm{K}}_{2}$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## QUESTION 2

## PLEASE ENSURE YOUR CALCULATOR IS IN DEGREE MODE.

(a) On the axes the graph of $f(x)=a \operatorname{cosbx}$ for $-180^{\circ} \leq x \leq 180^{\circ}$ is sketched.

(1) Write down the values of $a$ and $b$.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(2) Write down the period of $f$.
$\qquad$
$\qquad$
(3) Show on the $x$-axis where you would read off the solution to $a \cos b x=2$, for $-180^{\circ} \leq x \leq 180^{\circ}$. (Use letters A and B on the x -axis.)
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(4) On the same set of axes, draw a second graph which would allow you to read off the solution of the equation.
$a \cos b x=1+\sin \left(x-45^{\circ}\right) ;-180^{\circ} \leq x \leq 180^{\circ}$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(b) If $\cos \beta+\sin \beta=\mathrm{T}$, express $\frac{\cos 2 \beta}{\sin \left(\beta-45^{\circ}\right)}$ in terms of T .
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(c) Solve for A if $\tan \mathrm{A}=\tan 135^{\circ}$ and
(1) $180^{\circ}<\mathrm{A}<360^{\circ}$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(2) $360^{\circ}<\mathrm{A}<720^{\circ}$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(d) In the diagram below, similar triangles $\triangle \mathrm{OPR}$ and $\triangle \mathrm{OQT}$ are drawn.

O is the origin.
R and T are points on the x -axis.


Determine, leaving answers in surd form if necessary:
(1) $\cos \left(90^{\circ}+\theta\right)$.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(2) The value of a .
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## QUESTION 3

A spelling test was given to 1000 learners, in which the maximum possible mark that could be obtained was 90 .

The cumulative frequency graph is drawn below and contains the points $\mathrm{A}(10 ; 50)$, $\mathrm{B}(20 ; 170), \mathrm{C}(30 ; 240), \mathrm{D}(40 ; 300), \mathrm{E}(50 ; 450), \mathrm{F}(60 ; 590), \mathrm{G}(70 ; 785), \mathrm{H}(80 ; 920), \mathrm{I}(90 ; 1000)$.

(a) Determine approximate values for:
(1) Lower quartile
$\qquad$
$\qquad$
(2) Median
$\qquad$
$\qquad$
(3) Upper quartile
$\qquad$
$\qquad$
(b) How many learners obtained more than 40 marks on the spelling test?
$\qquad$
$\qquad$
(c) How many learners obtained marks in the interval $40 \leq \mathrm{x}<50$ ?
$\qquad$
$\qquad$
(d) A data value is said to be an 'isolated' value if it is less than $\mathrm{Q}_{1}-1,5 \times \mathrm{IQR}$ or it is greater than $\mathrm{Q}_{3}+1,5 \times \mathrm{IQR}$ where:
$\mathrm{Q}_{1}=$ Lower Quartile
$\mathrm{Q}_{3}=$ Upper Quartile
$\mathrm{IQR}=$ Inter-Quartile range
Show clearly that the set of data has no 'isolated' values.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## QUESTION 4

(a) In the diagram below, a circle is drawn passing through $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$ and E . The tangents at B and C meet at T .
$B \hat{E} C=x$ and $\hat{D}=y$.


Express each of the following in terms of x and/or y , giving reasons.
(1) $\quad \hat{\mathrm{B}}_{2}$ $\qquad$
$\qquad$
(2) $\hat{\mathrm{C}}_{4}$ $\qquad$
$\qquad$
(3) $\hat{T}$ $\qquad$
$\qquad$
$\qquad$
$\qquad$
(4) $\hat{\mathrm{A}}$ $\qquad$
$\qquad$
(5) $\quad \hat{\mathrm{B}}_{1}$ $\qquad$
$\qquad$
(b) In the diagram below, $\mathrm{P}, \mathrm{Q}, \mathrm{R}$ and S are points on a circle. $\mathrm{PQ} \mathrm{R}=90^{\circ}$.

PS and QR produced meet at E.
RS produced meets the line through E parallel to QP at D.

(1) Why is $\mathrm{DER}=90^{\circ}$ ?
$\qquad$
$\qquad$
(2) Why is $\hat{\mathrm{S}}_{2}=90^{\circ}$ ?
$\qquad$
$\qquad$
(3) Prove that $\triangle \mathrm{DER}\|\| \mathrm{ESR}$.
$\qquad$
$\qquad$
$\qquad$
$\qquad$ (2)
(4) If $\mathrm{ER}=6$ and $\mathrm{DR}=10$, calculate DS .
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## SECTION B

## QUESTION 5

(a) (1) Prove the identity $\frac{2 \tan \theta}{1+\tan ^{2} \theta}=\sin 2 \theta$.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(2) Hence, or otherwise, determine the maximum value of $\frac{(1+\tan \theta)^{2}}{1+\tan ^{2} \theta}$.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(b) Determine the general solution to
$3 \sin \theta \cdot \sin 22^{\circ}=3 \cos \theta \cdot \cos 22^{\circ}+1$.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(c) $\quad \triangle \mathrm{XYZ}$ has lengths 4,5 and 6 as shown in the diagram.


Using the Cosine rule, show that $\cos \hat{Y}+\cos \hat{Z}=\frac{7}{8}$.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(d) In the diagram below, $\triangle \mathrm{ECQ}$ is equilateral with sides 20 cm . Vertices $\mathrm{E}, \mathrm{C}$ and Q lie on the circumference of the circle at the top of a cylinder.

M is the centre of the base of the cylinder.
CEQM forms a triangular pyramid and is cut out of the wooden cylinder.
The volume of the triangular pyramid is $3000 \mathrm{~cm}^{3}$.


Calculate the volume of the remaining wood correct to the nearest cubic centimetre.
Useful Formulae: $\mathrm{V}=\pi \mathrm{r}^{2} \mathrm{~h}$ and $\mathrm{V}=\frac{1}{3} \times$ Area of base $\times \perp$ height
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## QUESTION 6

(a) In the Cartesian plane below, a circle passing through $\mathrm{A}(-1 ; 6)$ and $\mathrm{C}(-1 ;-1)$ is drawn. The tangent to the circle at A is drawn and passes through the point Q(-3;2).

(1) Determine the co-ordinates of $M$, the midpoint of $A C$.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(2) Determine the co-ordinates of the centre of the circle.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(b) $\quad \mathrm{G}(1 ; 5), \mathrm{H}(3 ; 7), \mathrm{I}(8 ; 2)$ and $\mathrm{K}(\mathrm{x} ; 0)$ are points in the Cartesian plane.
(1) Show that GHI $=90^{\circ}$.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(2) Show that the area of $\Delta \mathrm{GHI}=10$.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(3) Determine x if area $\Delta \mathrm{GHI}=$ area $\Delta \mathrm{GHK}$.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## QUESTION 7

(a) In the diagram below, a scatter plot is drawn. The line of best fit for the entire data set is drawn.


State whether the following statements are True or False, giving reasons.
(1) The point marked A is an outlier.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(2) Removing the point marked A would result in a higher value for the correlation coefficient.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(3) Removing the point marked A would result in a smaller value for the slope of the least squares regression line.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(b) Study the 3 histograms given below and then answer the questions that follow:

(1) Which set of data (A, B or C) has the smallest standard deviation? Explain.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(2) For data set A, the mean is 3,5 and the standard deviation is 1,2 .

The values in the data set are all doubled.
Determine the mean and the standard deviation for the new data set.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(3) The table below represents the data set C .

| CLASS INTERVAL | FREQUENCY |
| :---: | :---: |
| $0 \leq \mathrm{x}<2$ | a |
| $2 \leq \mathrm{x}<4$ | b |
| $4 \leq \mathrm{x}<6$ | c |
| $6 \leq \mathrm{x}<8$ | d |
| $8 \leq \mathrm{x}<10$ | e |

It is given that the mean is p and the standard deviation is q .
If it is given that the frequency of each class interval is doubled, write down the mean and the standard deviation for the new data set. Give your answer in terms of p and q .
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## QUESTION 8

(a) In the diagram below, triangle BEC is drawn. D and A are points on BC and BE respectively so that $\mathrm{AD} \| \mathrm{EC}$ and DA bisects CÂB.

(1) Complete the reasons for each of the given statements.

| STATEMENT | REASON |
| :--- | :--- |
| $\hat{\mathrm{A}}_{2}=\hat{\mathrm{A}}_{3}$ |  |
| $\hat{\mathrm{~A}}_{2}=\hat{\mathrm{C}}_{1}$ |  |
| $\hat{\mathrm{E}}=\hat{\mathrm{A}}_{3}$ |  |

(2) If the statements in (1) are true, what can be deduced about $\triangle$ CEA? Explain.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(3) Prove that $\frac{\mathrm{BD}}{\mathrm{DC}}=\frac{\mathrm{AB}}{\mathrm{AC}}$.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(b) In the diagram below, a circle centre O is drawn.

- AB is a diameter of the circle and C is a point on the circle.
- AB produced meets the tangent at C at D .
- $\mathrm{AC}=\mathrm{DC}$.


Determine, giving reasons, the size of $\hat{A}$.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(c) In the diagram below, AD is a chord of the circle with radius 3 units. $A B$ and $B C$ are equal chords of length 2 units each.

(1) Determine the size of $\hat{\mathrm{D}}_{2}$.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$ (4)
(2) Determine the size of $A \hat{B} C$.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Total: 150 marks

