MATHEMATICS: PAPER II

Time: 3 hours

150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1.   This question paper consists of 15 pages, an Answer/Diagram Booklet of 6 pages (i – vi), and an Information Sheet of 2 pages (i – ii). Please check that your question paper is complete.

2.   Please detach the Answer/Diagram Booklet from the middle of your question paper. Write your examination number in the spaces provided on your Answer Book and the Answer/ Diagram Booklet.

3.   Any changes made to a diagram must be shown on the Answer/Diagram Booklet and not on the question paper. Please hand in the Answer/Diagram Booklet with your Answer Book.

4.   Answer ALL the questions.

5.   Please note that diagrams are not necessarily drawn to scale.

6.   All necessary working details must be shown.

7.   Approved non-programmable and non-graphical calculators may be used, unless otherwise stated.

8.   Round off your answer to one decimal digit where necessary.

9.   Ensure that your calculator is in **DEGREE** mode.

10.   It is in your own interest to write legibly and to present your work neatly.

SECTION A

QUESTION 1

THIS QUESTION MUST BE ANSWERED IN THE ANSWER/DIAGRAM BOOKLET

In the table below, the area of each province in South Africa is given alongside the population estimate of each province. These figures are according to the National Census of 2011.

(a)

Province	Area of Province (km²)	Population Estimate
Eastern Cape	168 966	6 562 053
Free State	129 825	2 745 590
Gauteng	18 178	12 272 263
KwaZulu Natal	94 361	10 267 300
Limpopo	125 754	5 404 868
Mpumalanga	76 495	4 039 939
Northern Cape	372 889	1 145 861
North West	104 882	3 509 953
Western Cape	129 462	5 822 734
Total	**1 220 812**	**51 770 561**

(1)   Determine, correct to the nearest whole number, the mean population size per province.   (2)

(2)   What percentage of the total South African population is the population of Gauteng? Give your answer correct to one decimal digit.   (1)

(3)   On average, how many people live in a square kilometre in Gauteng?   (1)

(b)   The table below shows the approximate percentage of people living in Gauteng according to their annual income. People earning at least R140 000 are not included in the table.

Category	Approximate Percentage of Gauteng Population
No income	2%
$R0 \leq x < R20\,000$	43%
$R20\,000 \leq x < R40\,000$	20%
$R40\,000 \leq x < R60\,000$	10%
$R60\,000 \leq x < R80\,000$	6%
$R80\,000 \leq x < R100\,000$	4%
$R100\,000 \leq x < R120\,000$	3%
$R120\,000 \leq x < R140\,000$	2%

(1)   Use the information in the table to determine, correct to the nearest whole number, an estimate for the number of people that are unemployed in Gauteng. Assume the population of Gauteng is 12 272 263.   (1)

(2)   According to the table, what percentage of people in Gauteng earn at least R140 000?   (1)

(3)   (i)   Complete the table below, **in the Answer/Diagram Booklet**.   (2)

Category	Approximate Percentage	Cumulative Percentage Frequency
$R0 \leq x < R20\,000$	45%	
$R20\,000 \leq x < R40\,000$	20%	
$R40\,000 \leq x < R60\,000$	10%	
$R60\,000 \leq x < R80\,000$	6%	
$R80\,000 \leq x < R100\,000$	4%	
$R100\,000 \leq x < R120\,000$	3%	
$R120\,000 \leq x < R140\,000$	2%	

(ii)   On the grid provided in the **Answer/Diagram Booklet**, draw a cumulative percentage frequency curve for the annual income earned by people in Gauteng earning less than R140 000.   (4)

(4)   Show, on the curve drawn in Question (b)(3)(ii), where you would read off the:

(i)   median annual income for people in Gauteng earning less than R140 000.   (1)

(ii)   median income for all people living in Gauteng, including those who earn at least R140 000.   (1)
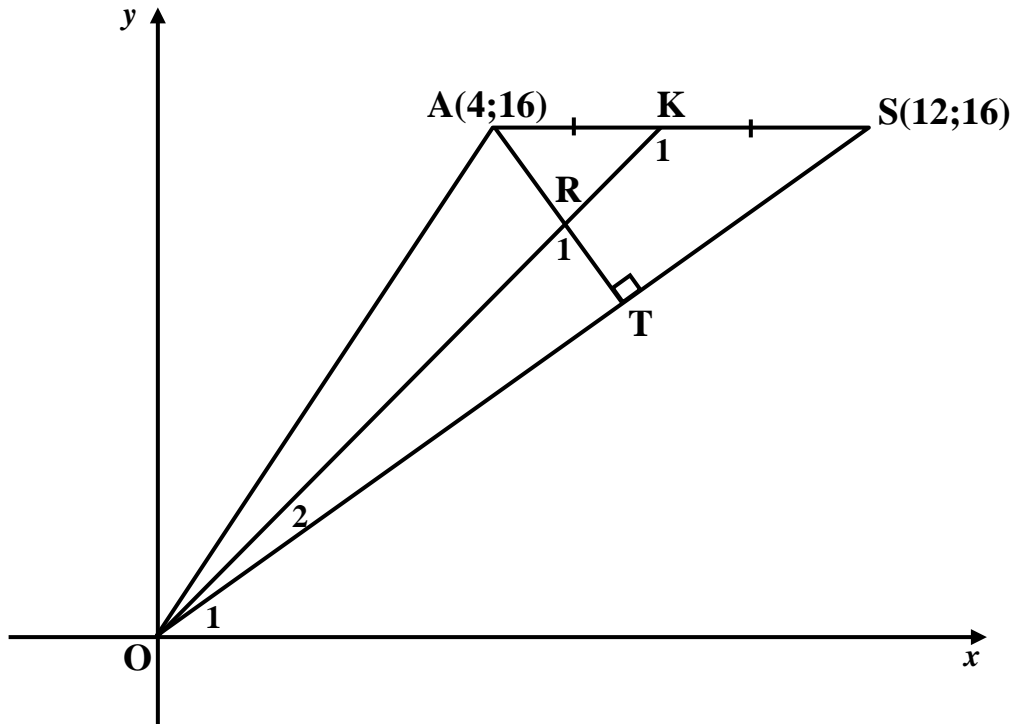
[14]

QUESTION 2

(a)   In the diagram below, ΔSAO has vertices $S(12;16)$, $A(4;16)$ and $O(0;0)$.

K is the midpoint of AS and AT is perpendicular to OS with T a point on OS.

AT and OK meet at R.

(1)   Determine the co-ordinates of K and hence the equation of line OK.   (5)

(2)   Determine the gradient of OS and hence the equation of line AT in the form
       $y = mx + c$.   (4)

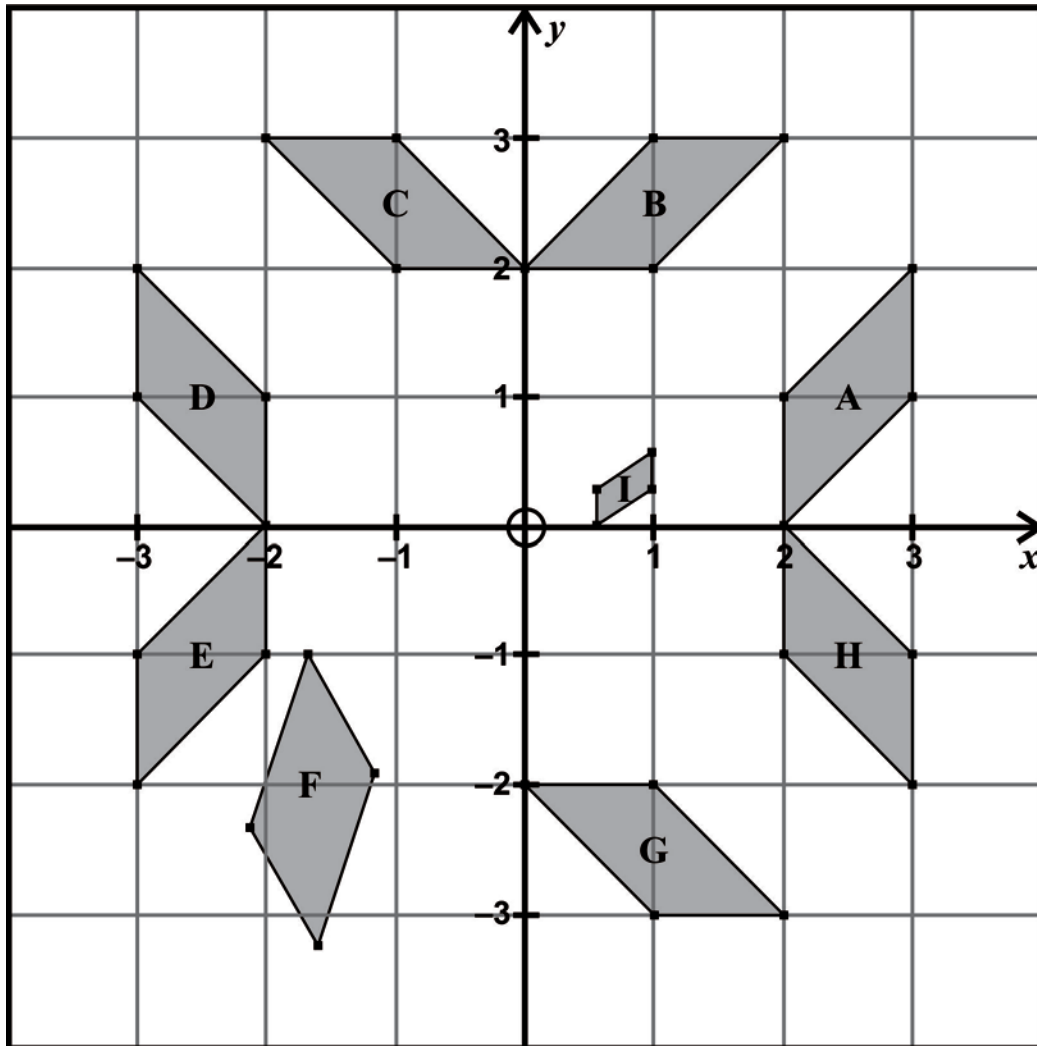(3)   Determine, correct to one decimal digit, the size of

       (i) $\hat{O}_1$ (2)

       (ii) $\hat{O}_2$ (3)

       (iii)   Hence, or otherwise, determine the size of $\hat{R}_1$ and $\hat{K}_1$.   (3)

(b)   If $B(-8;4)$ and $D(4;-8)$, determine the equation of the circle having BD as a
       diameter.   (4)

[21]

QUESTION 3

In the diagram below, shapes A to I are drawn in the Cartesian plane.

(a)   Complete the following statements:

    (1)   A and _____ are symmetrical about the line $y = x$.   (1)

    (2)   H and _____ are symmetrical about $x = 0$.   (1)

(b)   In each case, give the letter of the shape which will be the resulting image of the given transformation. Write only the letter A to I down.

    (1)   Shape E is transformed according to $(x; y) \to (x; -y)$.   (1)

    (2)   Shape E is transformed according to the rule $(x; y) \to (y; -x)$.   (1)

(c)   (1)   If shape I is transformed to shape A by the rule, $(x; y) \rightarrow (3x; 3y)$,

write down the rule that will transform shape A to shape I.   (1)

(2)   Write down the value of the following ratio:

$$\frac{\text{Area of shape I}}{\text{Area of shape A}}.$$
(1)

(d)   (1)   Give the transformation that takes shape E to shape A.

Give your answer in the form $(x; y) \rightarrow ...$ (2)

(2)   If shape E is transformed to shape F by the rule,

$$(x; y) \rightarrow (x\cos 30° - y\sin 30°; x\sin 30° + y\cos 30°),$$

write down in simplified surd form the rule that will transform shape F to
shape A.   (4)

[12]

QUESTION 4

(a)   Given: $\cos\hat{G} = 0{,}726$ and $180° < \hat{G} < 360°$.
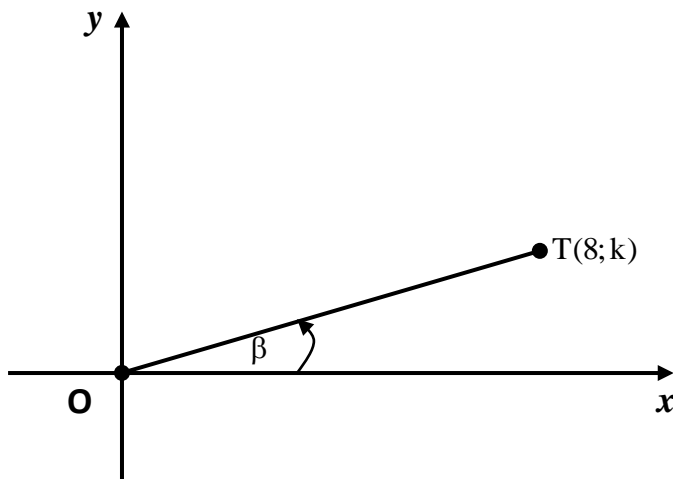
   (1)   Use a calculator to determine $\hat{G}$, correct to one decimal digit. (2)

   (2)   Hence determine the value of $\tan\left(\dfrac{2}{3}\hat{G} + 100°\right)$, correct to three decimal

          digits. (1)

(b)   Simplify as far as possible:

$$\frac{\sin(180° - A)}{\cos(90° + A) + \sin(360° - A)}.$$ (4)

(c)   In the diagram below, $T(8;k)$ is a point in the first quadrant.

   If $\tan\beta = \dfrac{1}{4}$, determine **without using a calculator**:

   (1)   the value of k. (2)

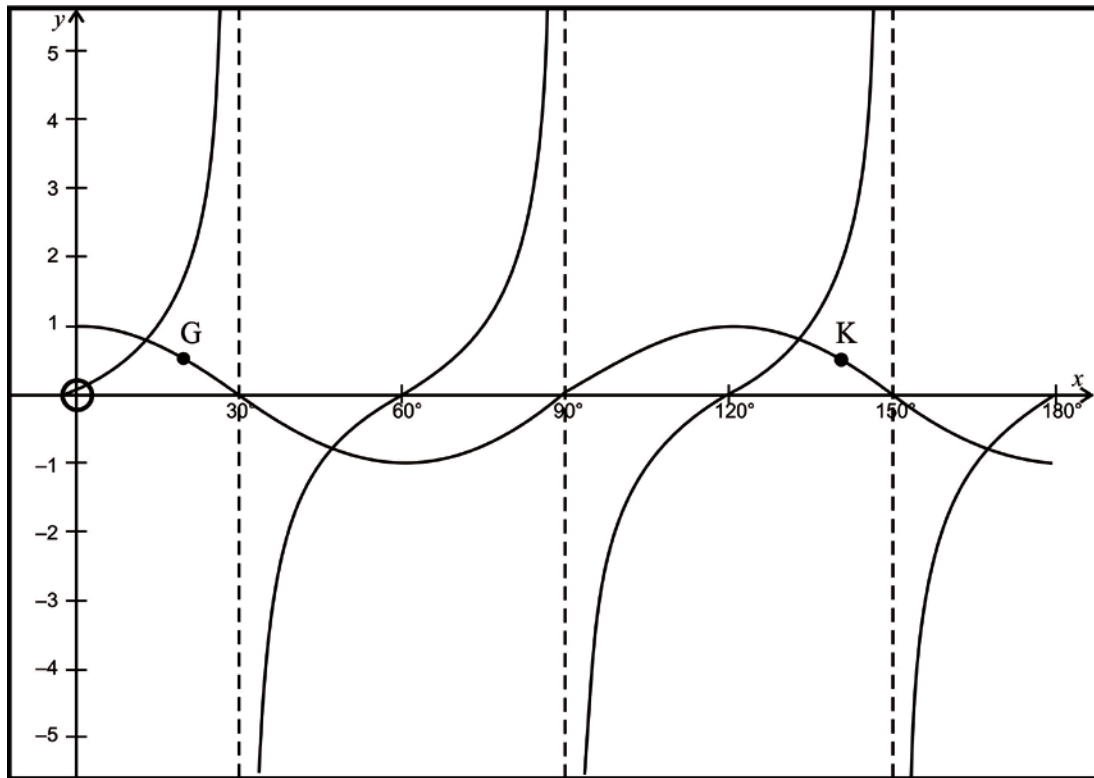   (2)   the value of $\sin\beta$. Leave your answer in surd form. (3)

(d)   Simplify without the use of a calculator:

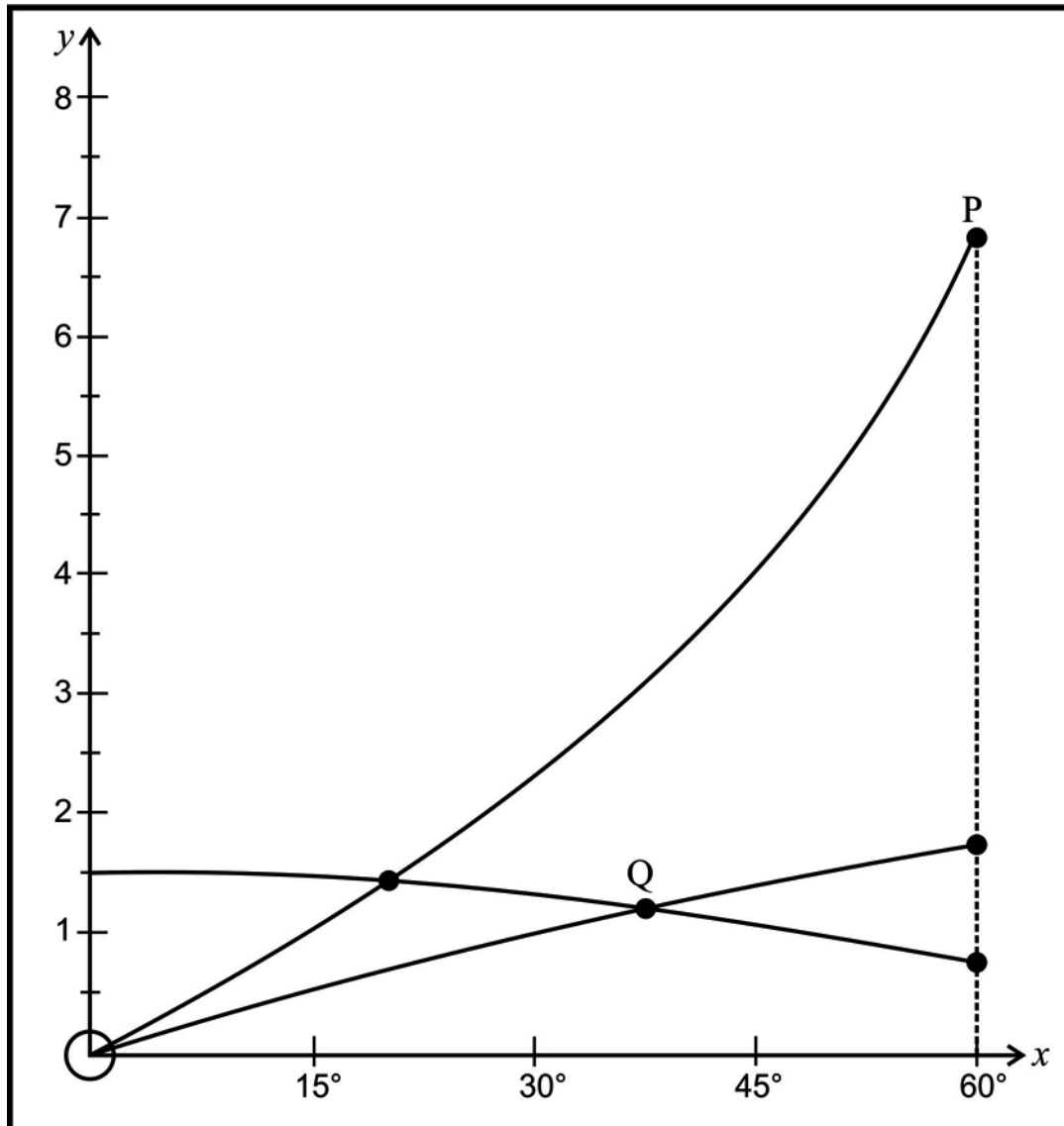$$\frac{\cos(45° - \theta)}{\cos 45°.\cos\theta} - \tan\theta.$$ (5)

[17]

QUESTION 5

(a)   The graphs of $y = \cos ax$ and $y = \tan bx$ are sketched for $x \in [0°, 180°]$.

(1)   Write down the period of $y = \cos ax$.   (1)

(2)   Write down the value of a.   (1)

(3)   Write down the period of $y = \tan bx$.   (1)

(4)   Write down the value of b.   (1)

(b)   The graphs of $f(x) = 2\sin x$, $g(x) = 1,5\cos x$ and $h(x) = 4\tan x$ are drawn for $x \in [0°,\ 60°]$.

(1)   Determine the co-ordinates of P in simplest surd form.   (2)

(2)   Determine the co-ordinates of Q correct to two decimal digits.   (4)

(3)   Give a point (x; y) which satisfies the following inequalities simultaneously:
        $y \leq 4\tan x$
        $y \geq 2\sin x$
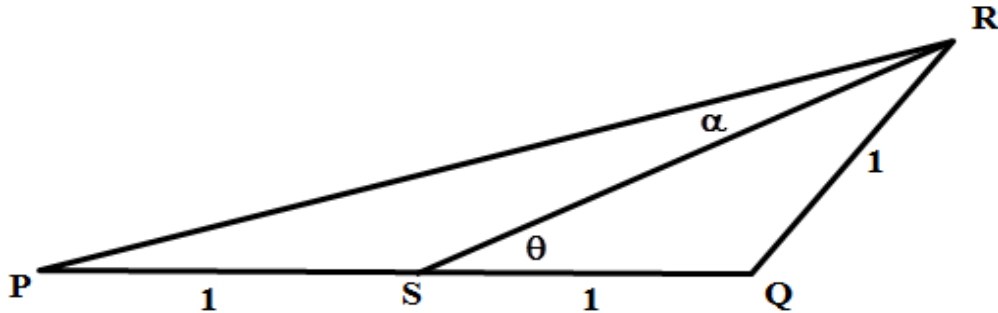 $y \leq 1,5\cos x$ (2)

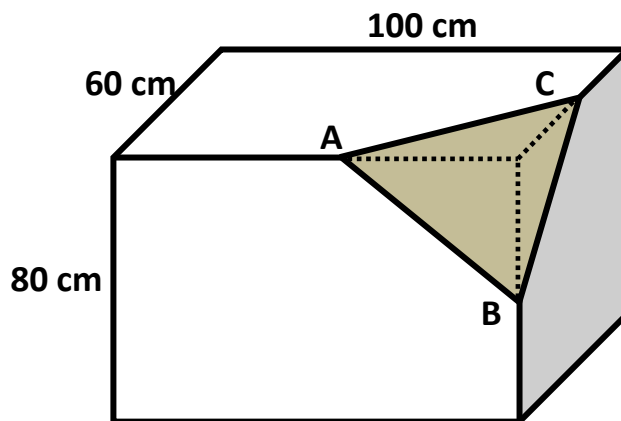   **[12]**

                                                                                    76 marks

SECTION B

QUESTION 6

(a)   In ΔPQR below, PQ = 2 and QR = 1. S is the midpoint of PQ.

$$P\hat{R}S = \alpha, R\hat{S}Q = \theta.$$

   (1)   Determine $\hat{P}$ in terms of θ and α.   (1)

   (2)   Show that $\tan\theta = 3\tan\alpha$.   (6)

(b)   A rectangular block of wood measures $80\ cm \times 100\ cm \times 60\ cm$.
        One corner is cut away from the block, in such a way that three of the edges are cut
        through their midpoints A, B and C.

        Determine the area of the triangular face ABC created by the cut. Give your answer
        correct to one decimal digit.   (7)

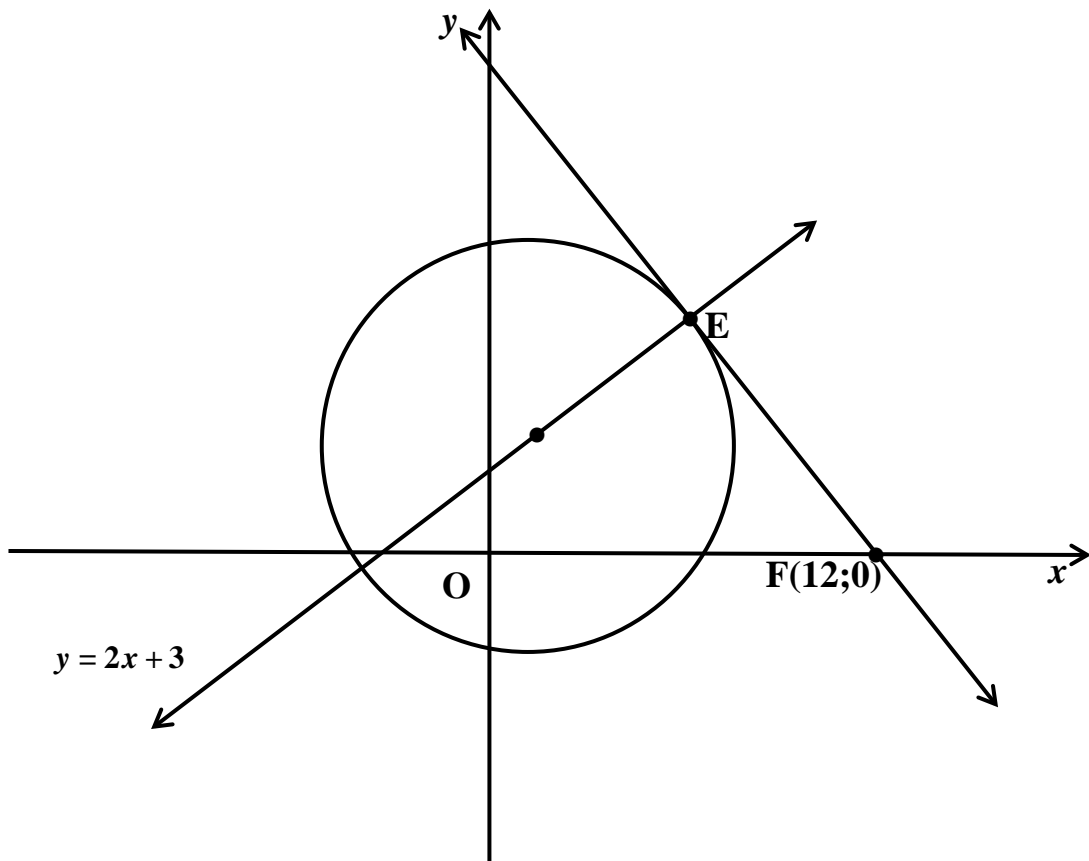   **[14]**

QUESTION 7

(a)   By replacing 2A with 3A – A and 4A with 3A + A, prove the identity:
$$\frac{\cos 2A - \cos 4A}{\sin 4A - \sin 2A} = \tan 3A.$$
(7)

(b)   Determine the general solution of $\sin 4A = \sin 2A$.   (7)

[14]

QUESTION 8

(a)   In the diagram below, a circle has a diameter with equation $y = 2x + 3$. The tangent at point E on the circle cuts the x-axis at $F(12;0)$.
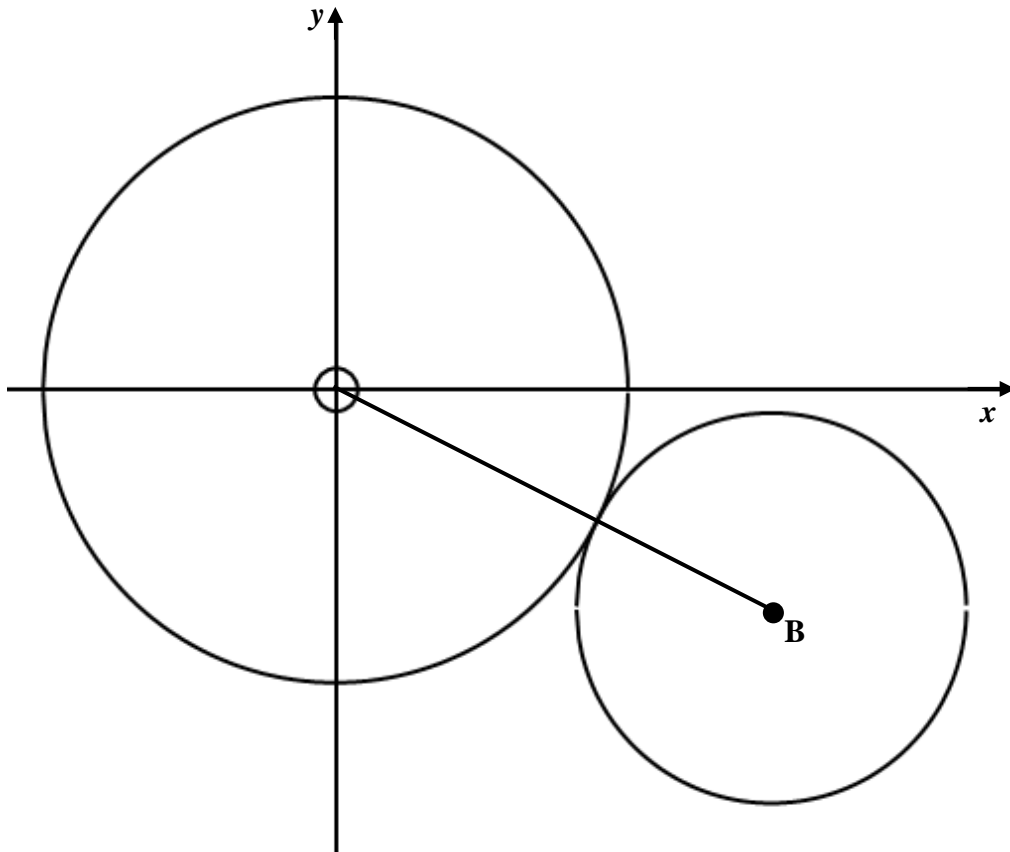

Determine the co-ordinates of E.   (6)

(b)   In the diagram below, two circles are drawn. Circle centre O touches circle centre B externally.

The equation of the circle centre the origin is given by $x^2 + y^2 = 45$.

The equation of the circle centre B is given by $(x - 2p)^2 + (y + p)^2 = 20$.

Determine the value of p.   (5)

(c)   Prove that the radius of the circle having equation $x^2 + y^2 + 4x\cos\theta + 8y\sin\theta + 3 = 0$ can never exceed $\sqrt{13}$ for any value of θ.   (5)

[16]

QUESTION 9

(a)   Consider the set of numbers: 3; x ; y ; 10 ; 12.

The mean of the set is 8 and the variance is 10. Determine the numbers x and y.   (8)

(b)   The box and whisker plots below represent the times taken in minutes by a group of students to complete a race. The times recorded have been separated into boys and girls.
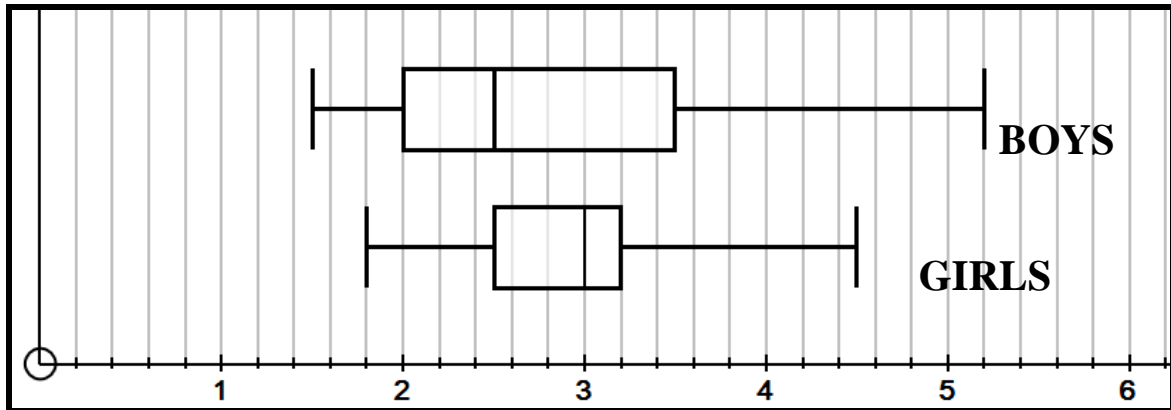

Study the box and whisker diagrams and answer the questions that follow.

(1)   Give two different reasons for why the girls did the best that you would include in a convincing argument.   (2)

(2)   Give two different reasons for why the boys did the best that you would include in a convincing argument.   (2)

[12]

QUESTION 10

Two squares with sides 12 cm overlap so that the corner of one square is at the centre of the other square, as shown in the diagram. SQUA is the lower square.
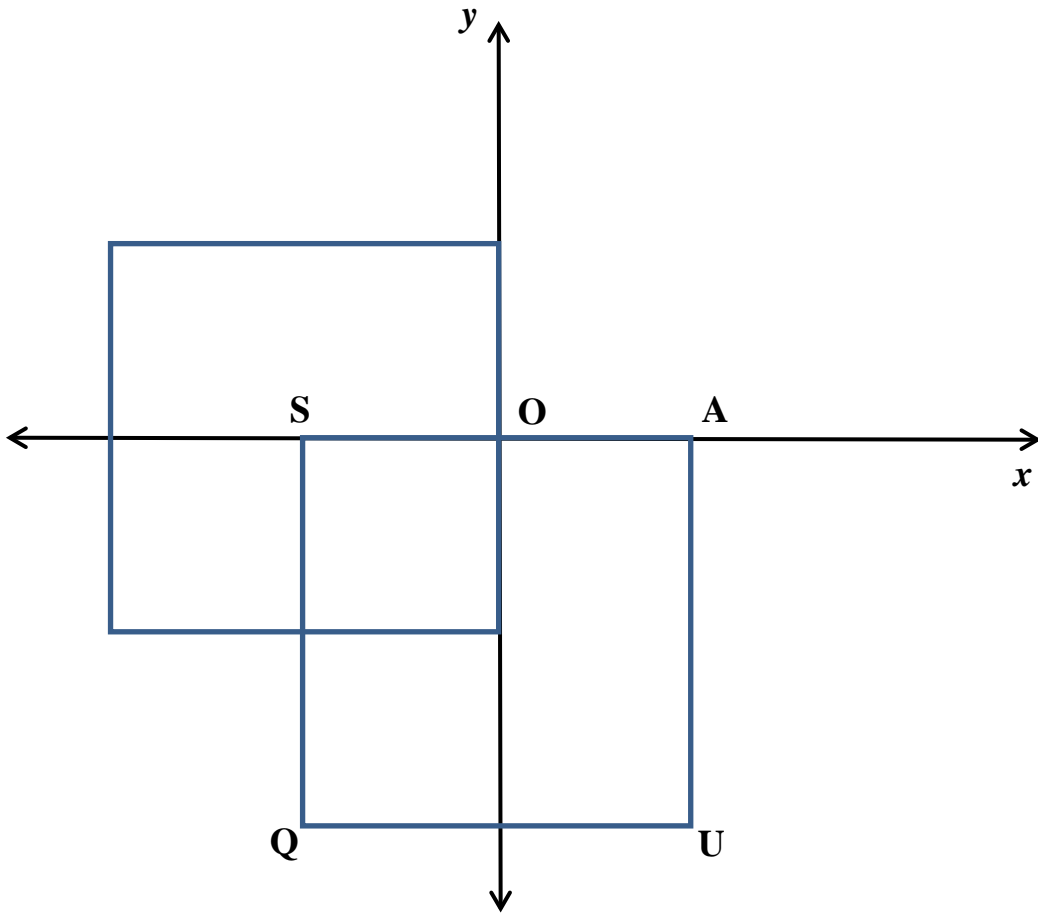
(a)   SQUA is transformed so that the two squares completely overlap.
Write down the transformation in the form $(x; y) \to \ldots$ if

    (1)   the transformation is a translation.   (2)

    (2)   the transformation is a reflection.   (2)

(b)   SQUA is rotated clockwise about the point O through θ degrees where $0° < \theta < 25°$.
        The shaded area S'OBC is a kite.
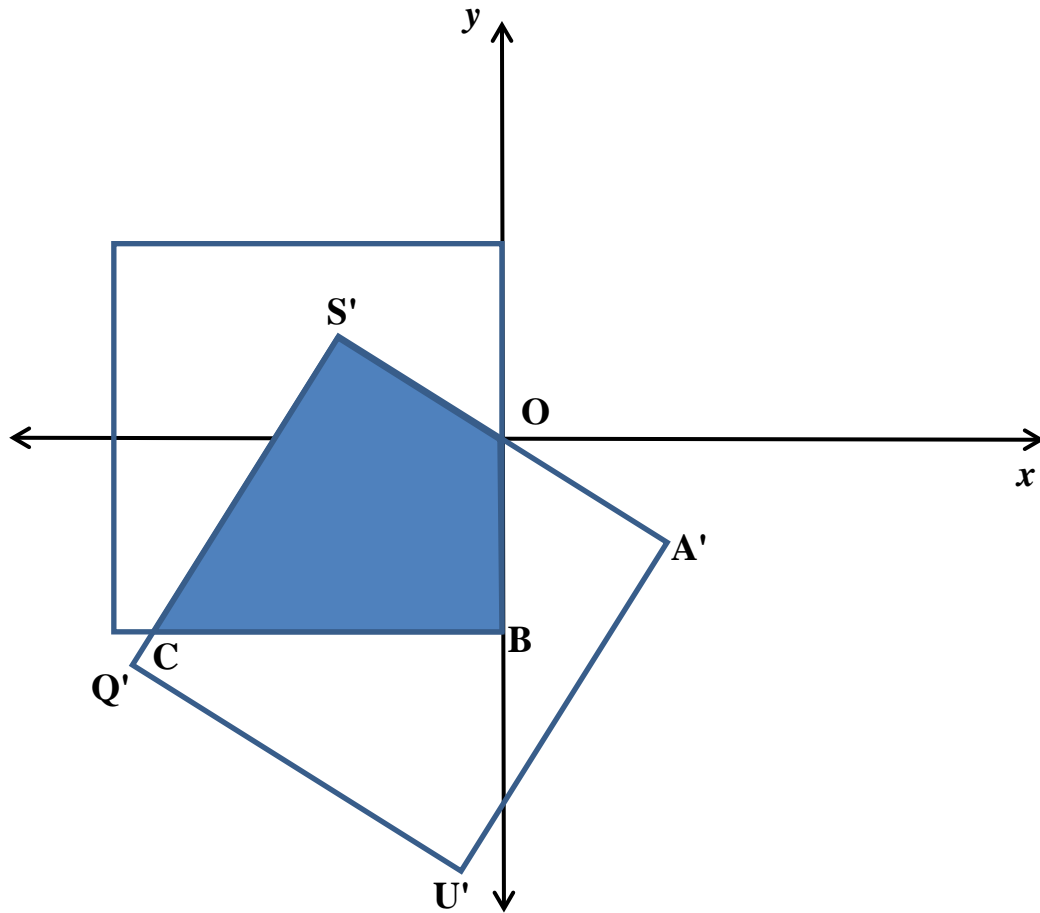
S'Q'U'A' is the image of SQUA after the rotation.

(1)   Determine the co-ordinates of S' and Q' in terms of $\cos\theta$ and $\sin\theta$.   (6)

(2)   Hence or otherwise, show that the equation of the line joining S' and Q' is
        given by $y = \dfrac{x.\cos\theta + 6}{\sin\theta}$.   (3)

(3)   Show that the shaded area is given by $36\tan\theta + \dfrac{36}{\cos\theta}$.   (5)

[18]

74 marks

Total: 150 marks