NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2012

MATHEMATICS: PAPER III

EXAMINATION NUMBER												

Time: 2 hours 100 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 18 pages and an Information Sheet of 2 pages (i – ii). Please check that your paper is complete.

2. Read the questions carefully.

3. **Answer ALL the questions on the question paper and hand this in at the end of the examination.**

4. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.

5. Round off your answers to four decimal digits where necessary.

6. All the necessary working details must be clearly shown.

7. It is in your own interest to write legibly and to present your work neatly.

8. The last pages can be used for additional working, if necessary. If this space is used, make sure that you indicate clearly which question is being answered.

Question	1	2	3	4	5	6	7	8	9	10	11	12	**Total**
Marks													

SECTION A

QUESTION 1

The recursive formula for a sequence is defined by $T_{n+1} = -T_n\left(\frac{n+2}{n+1}\right)$ with $T_1 = 2$; $n \geq 1; n \in \mathrm{N}$

(a) Show all working to prove that the fourth term (T_4) is equal to –5.

(4)

(b) Find the sum of the first 50 terms.

(3)

[7]

QUESTION 2

All answers containing factorials must be calculated e.g.: 4! = 24.

(a) Riaan packs his suitcase for his holiday with 3 caps, 5 shirts, 3 pairs of jeans and 2 pairs of takkies.

(1) How many different outfits can he put together, if, when he dresses, he must wear a shirt, a pair of jeans, a pair of takkies and a cap?

_____

_____ (3)

(2) Riaan hangs all 5 shirts and the 3 pairs of jeans (each item separately) on a different hanger, on the rail in the cupboard. How many different arrangements are there of his clothing?

_____

_____ (1)

(3) What is the probability that the shirts are all hanging together next to each other in the cupboard?

_____

_____

_____

_____

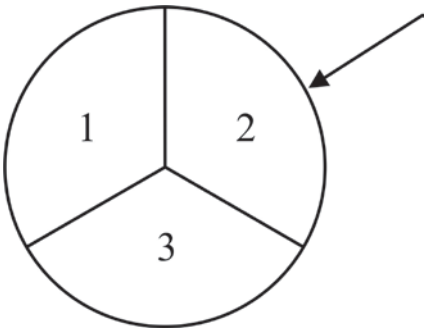
_____ (4)

(b) Find the number of different arrangements of the letters DDD EE F G, if all 7 letters must be used and there are no restrictions.

(3)
[11]

QUESTION 3

A circular wheel is divided into three equal sectors, numbered 1, 2 or 3 as shown in the figure. The wheel is spun twice. Each time, the score is the number to which the black arrow points. If the arrow points to a boundary line, the wheel is spun again. The sample space for the two spins, is therefore:



{(1 , 1); (1 , 2); (1 , 3); (2 , 1); (2 , 2); (2 , 3); (3 , 1); (3 , 2); (3 , 3)}

Calculate the probability of the following events:

(a) Both scores are the same.

_____ (1)

(b) Neither of the scores is a 1.

_____ (1)

(c) Neither of the scores is a 1 and both scores are the same.

_____ (1)

(d) Neither of the scores is a 1 or both scores are the same.

_____ (1)
[4]

QUESTION 4

The probability that any one person develops a disease is 0,12.
People develop the disease independently of other people.

(a) What is the probability that a specific person will not develop a disease?

_____ (1)

(b) What is the probability that out of a group of three people, at least one develops the disease?

_____

_____

_____

_____

_____

_____ (4)
[5]

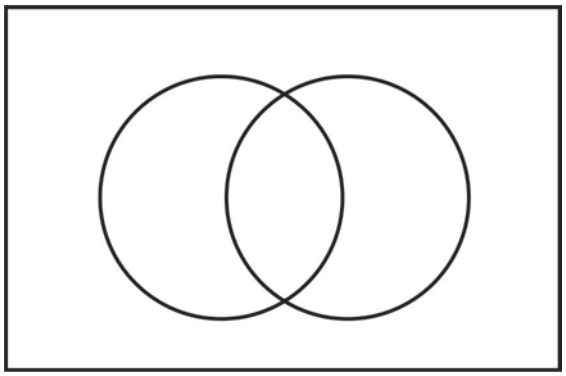
QUESTION 5

A local club has facilities that include tennis courts and a golf course.
A survey of the club members indicated that 72% regularly use the golf course and 48% regularly use the tennis courts.
Some members regularly use both while 8% use neither of the facilities.
The club has 700 members.

(a) Determine the number of members that regularly use at least one of the facilities. You may find that the given Venn-diagram is a useful aid.

_____

_____ (2)



(b) Suppose we randomly select a member of the club. What is the probability that this person uses exactly (only) one facility?

_____

_____

_____

_____ (3)

(c) P(using the golf course) $\times$ P(using the tennis courts) $= 0,72 \times 0,48 = 0,3456$.
Validate statistically whether these events are independent or not.

_____

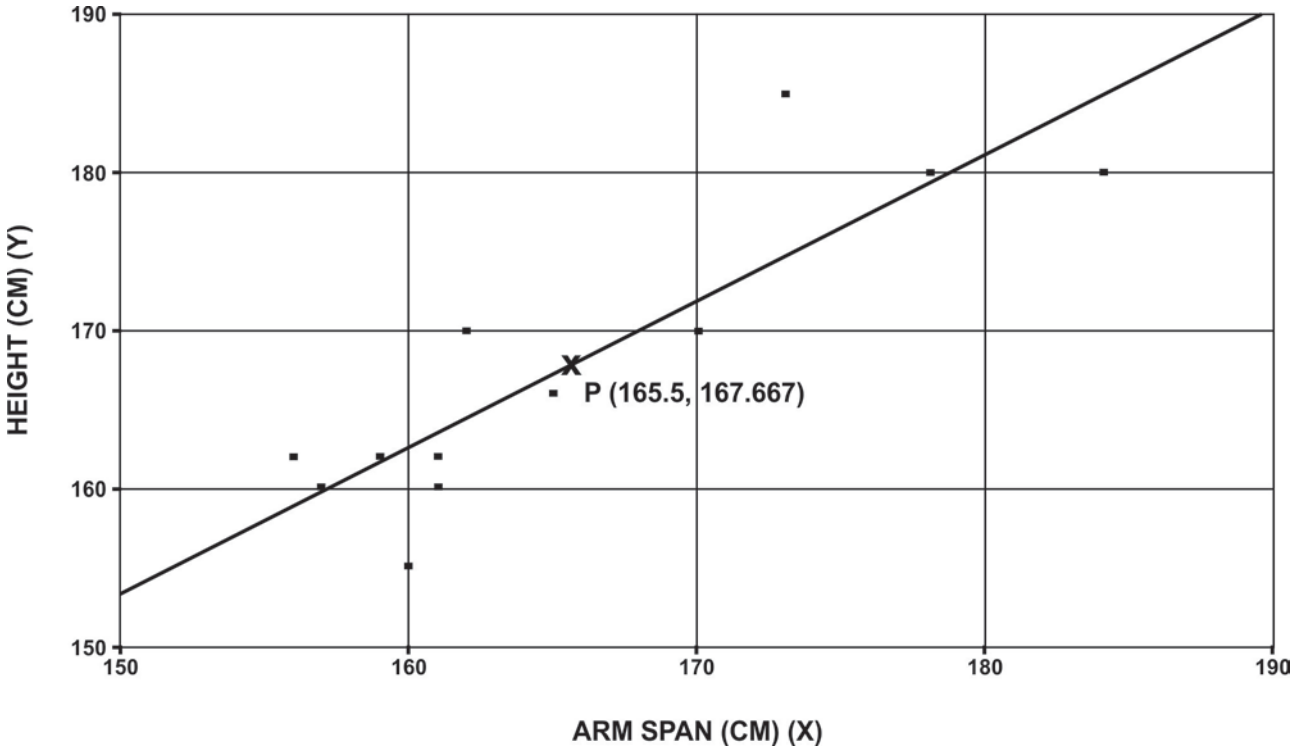
_____ (2)
[7]

QUESTION 6

Measurements were collected from 12 people to determine whether people with longer arms are taller than people with shorter arms.

Candidate	Arm span (cm) (x)	Height (cm) (y)
A	156	162
B	157	160
C	159	162
D	160	155
E	161	160
F	161	162
G	162	170
H	165	166
I	170	170
J	173	185
K	178	180
L	184	180

The regression line is drawn below.



(a) Use your calculator to determine the equation of the least squares regression line. Give your answers correct to 4 decimal digits.

_____

_____ (5)

(b) Marie's arm span is measured to be 160 cm. What would her estimated height be?

_____

_____ (2)

(c) The point $(\overline{x};\overline{y})$ with co-ordinates (165,5; 167,667) is labelled P in the scatterplot. Use this information and the scatterplot to identify and write down, (using a letter A, B, C, … L), a candidate having below average arm-span with above average height. (The letter must be selected from the table at the beginning of the question.)

_____ (1)

(d) Suppose the correlation (r) between arm span and height is calculated for two cases:

Case 1: Candidate J is included.

Case 2: Candidate J is excluded.

How would the correlations compare? Choose the correct answer from (i) to (iv) below. Write down only your choice.

(i) The r values would be the same.

(ii) The r value would be closer to 1 for case 1 than for case 2.

(iii) The r value would be closer to 1 for case 2 than for case 1.

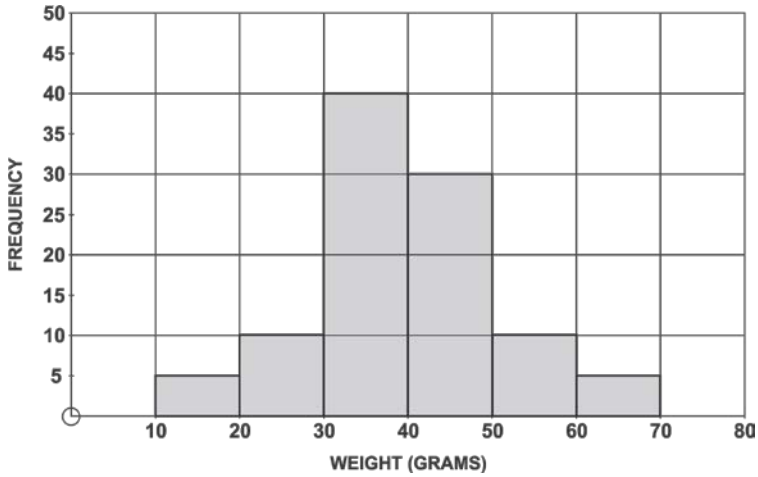
(iv) The r value would be closer to –1 for case 1 than for case 2.

Answer: _____ (1)

[9]

QUESTION 7

The following histogram is of 100 weights in grams.



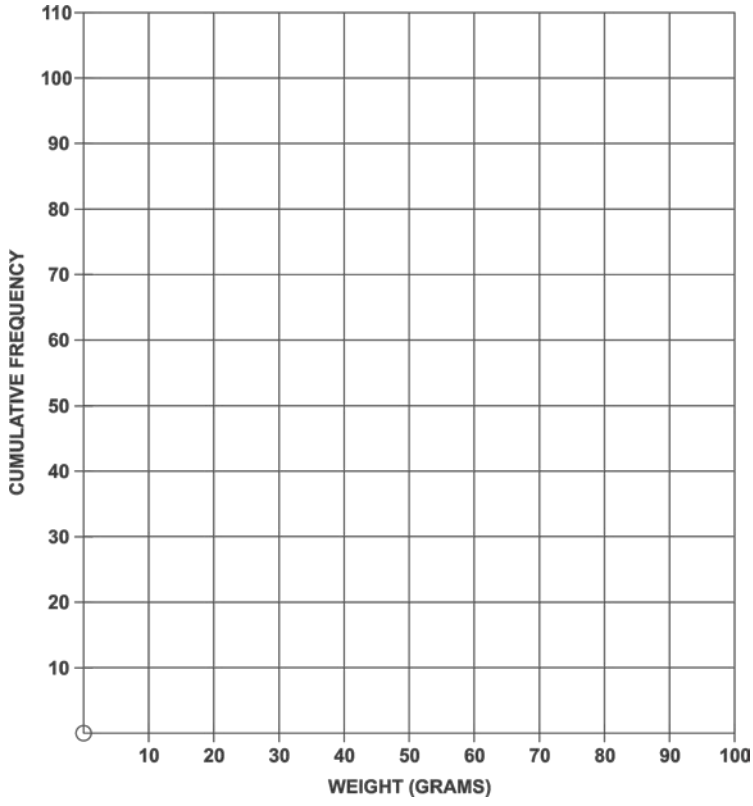
(a) Find the

(1) estimated mean _____ (3)

(2) (i) estimated standard deviation _____ (2)

(ii) estimated variance _____ (1)

(b) Draw an ogive (cumulative frequency diagram) and then estimate the median.



(2)

Estimated median _____ (1)

(c) What percentage of the weights lie between 25 grams and 40 grams? Show where you took your readings.

_____ (2)

(d) It was discovered that 5 of the scores in the class 50 – 60 were actually 61, 63, 64, 64 and 69. State, without more calculations, what effect (if any) there would be if this information was taken into account on the:

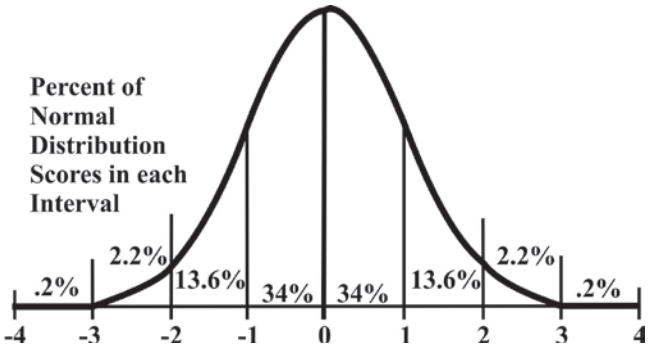
Mean _____ (1)

Median _____ (1)

Standard Deviation _____ (1)

[14]

QUESTION 8



A company manufactures ball bearings. The optimum diameter of the spherical bearing is 42 mm. Records show that the diameter follows a normal distribution with a mean of 42 mm and a standard deviation of 0,06 mm. A diameter of 41,94 mm and 42,06 mm is acceptable.

(a) What percentage of output is unacceptable?

_____

_____ (1)

(b) If updated machinery cuts the standard deviation in half, what percentage would now be unacceptable?

_____

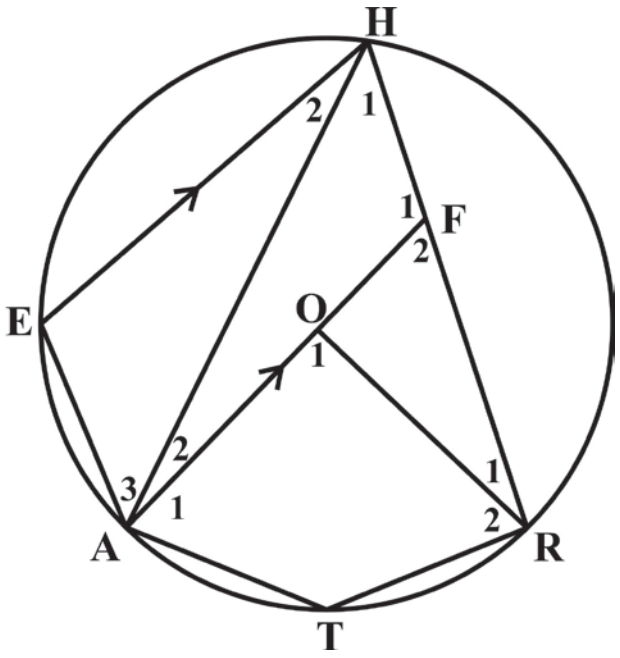
_____ (2)

[3]

60 marks

SECTION B

QUESTION 9



In the diagram O is the centre of the circle HEATR. AOF is parallel to EH.
$\hat{F}_2 = 78°$ and $\hat{R}_1 = 22°$.

Calculate, with reasons, the size of:

(a) $\hat{O}_1$

_____ (2)

(b) $\hat{H}_1$

_____ (2)

(c) $\hat{T}$

_____ (2)

(d) $\hat{H}_2$

_____

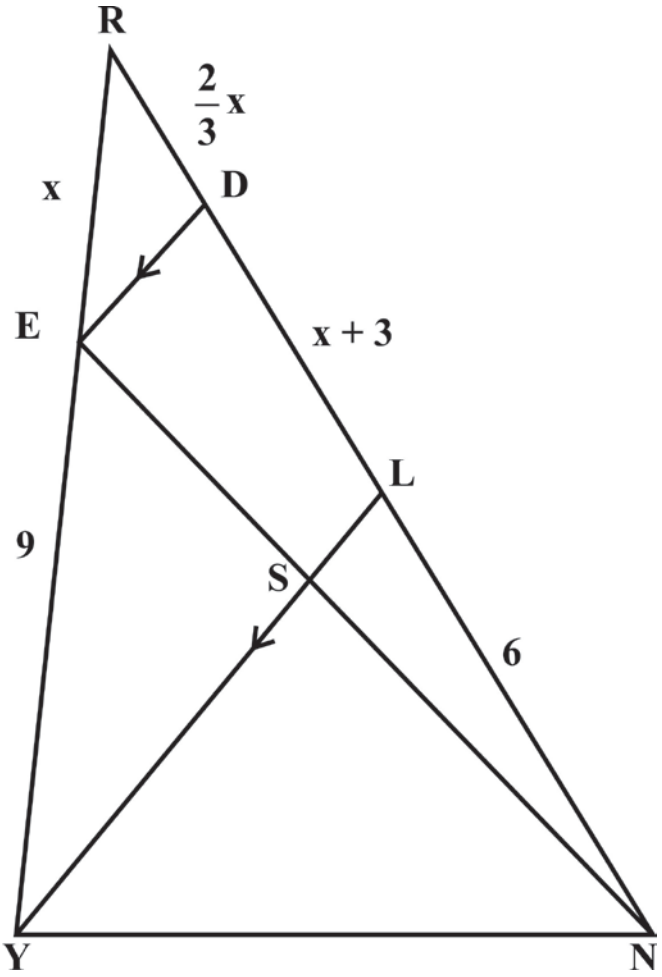
_____

_____ (3)

[9]

QUESTION 10

NL = 6 units. RE = x units; $RD = \frac{2}{3}x$ units; EY = 9 units and DL = x + 3 units.
S is a point on YL and ED // YL.



(a) Show that L is the midpoint of DN by first solving for x.

(4)

(b) If SL = 1,4 units write down the length of DE.

_____ (1)

(c) If Area ΔRED = 2,7 units^2 find the area of ΔREN.

_____

_____

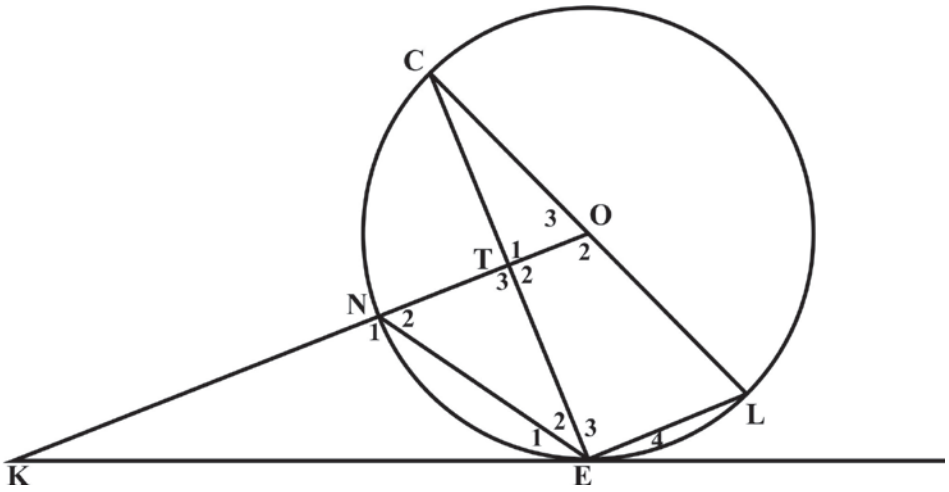
_____

_____

_____ (3)

[8]

QUESTION 11



In the diagram O is the centre of circle CNEL. KE is a tangent at E.
N and T are points on KO. KO // EL.

Complete the following tables:

Statement	**Reason**
(a) $\hat{E}_3 = 90°$	
$\therefore \hat{T}_2 = 90°$	
$\therefore CT = TE$	

(3)

(b) $\hat{E}_1 + \hat{E}_2 = \hat{L}$	
But $\hat{L} = \hat{O}_3$	
$\therefore \hat{O}_3 = \hat{E}_1 + \hat{E}_2$	
$\therefore$ COEK is a cyclic quadrilateral	

(3)

(c) Prove that ΔCOT /// ΔKET.

(3)

(d) Find CO if OT = 1 unit and KT = 9 units.

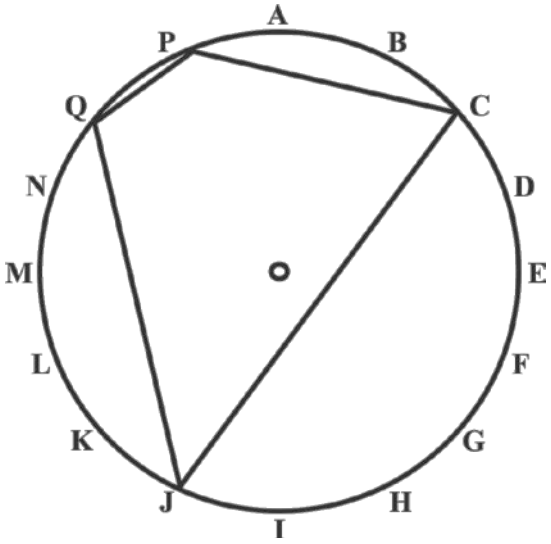
(4)

(e) Find the size of $\hat{K}$ correct to 2 decimal digits.

(3)
[16]

QUESTION 12

A cyclic quadrilateral is formed by joining P, C, J and Q on a circle centre O, which has 16 equal arcs marked off around its circumference.



(a) Find $P\hat{C}J$, giving reasons.

(5)

(b) Find $P\hat{Q}J$, giving reasons.

(2)

[7]

40 marks

Total: 100 marks