NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2012

MATHEMATICS: PAPER III

MARKING GUIDELINES

Time: 2 hours | 100 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A

QUESTION 1 [LO 1: AS 12.1.3]

(a) $T_1 = 2$

$T_{1+1} = -(2)\left(\frac{1+2}{1+1}\right) = -2\left(\frac{3}{2}\right) = -3$

$T_{2+1} = -(-3)\left(\frac{2+2}{2+1}\right) = 3\left(\frac{4}{3}\right) = 4$

$T_{3+1} = -(4)\left(\frac{3+2}{3+1}\right) = -4\left(\frac{5}{4}\right) = -5$ (4)

(b) $2-3+4-5+6-7+8\ \dots +50-51 = (-1)+(-1)+(-1)+(-1)\ \dots = -25$

OR

$2+4+6+\dots+50$ and $-3-5-7\ \dots -51$

$= \frac{25}{2}(52)$ $= \frac{25}{2}(-54)$

$= (25)(26)$ $= (25)(-27)$

$= 650$ $= -675$

add two together $= -25$ (3)

[7]

OR

$s_1 = 2; s_2 = -1; s_3 = 3; s_4 = -2; s_5 = 4 \dots\dots s_{50} = -25$

QUESTION 2 [LO 4: AS 11.4.2; 12.4.2]

(a) (1) $3 \times 5 \times 3 \times 2 = 90$ (3)

(2) $8! = 40320$ (1)

(3) Total number of arrangements $= 4! \times 5! = 2880$

5 shirts			

Probability $= \frac{4! \times 5!}{8!} = \frac{1}{14} = 0,0714$ OR $\frac{2880}{40320}$ (4)

(b) $\frac{7!}{3! \times 2!} = 420$ (3)

[11]

QUESTION 3 [LO 4: AS 11.4.2; 12.4.2]

(a) $\frac{1}{3}$ (1)

(b) $\frac{4}{9}$ (1)

(c) $\frac{2}{9}$ (1)

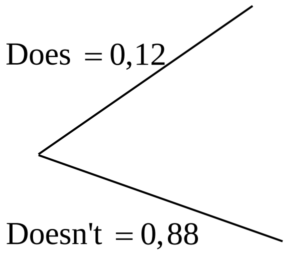
(d) $\frac{5}{9}$ (1)

[4]

QUESTION 4 [LO 4: AS 11.4.2; 12.4.2]

(a) 0,88 (1)

(b) P (does; doesn't; doesn't) $= (0,12)(0,88)^2 = \dfrac{1452}{15625} = 0,092928$



OR

(doesn't; does; doesn't) **OR** (doesn't; doesn't; does)

P (does; does; doesn't) $= (0,12)^2(0,88) = \dfrac{198}{15625} = 0,012672$

P (does; does; does) $= (0,12)^3 = \dfrac{27}{15625} = 0,001728$

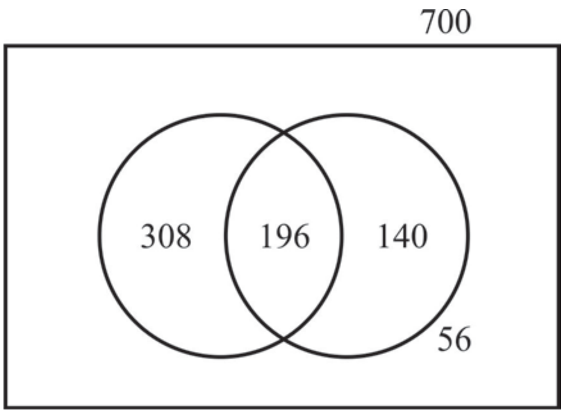
$= 3(0,092928) + 3(0,012672) + 0,001728 = 0,318528$

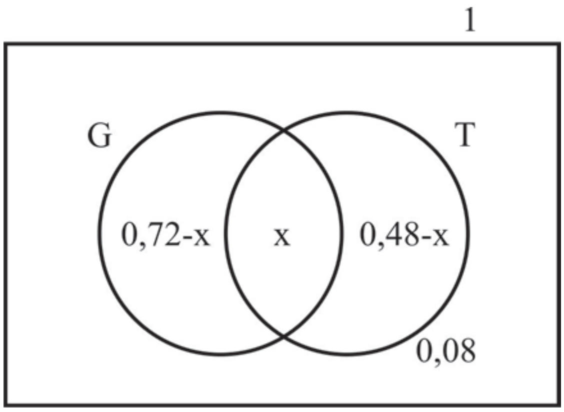
OR

1 – Prob (none) $= 1 - (0,88)^3 = \dfrac{4977}{15625} \approx 0,3185$ (4)

[5]

QUESTION 5 [LO 4: AS 11.4.2; 12.4.2]





(a) $0,92 \times 700 = 644$

OR

$0,08 \times 700 = 56$

$700 - 56 = 644$ (2)

(b) $0,72 - x + x + 0,48 - x = 0,92$

$-x = -0,28$

$x = 0,28$

So probability of using exactly one facility $= 0,92 - 0,28 = 0,64$

$504 + 336 = 840$

OR

840 – 644 = 196

so $x = 196$

$\frac{308+140}{700} = \frac{448}{700} = \frac{16}{25} = 0{,}64$ (3)

(c) $P(\text{tennis} \cap \text{golf}) = 0{,}28$

$\therefore P(\text{tennis}) \times P(\text{golf}) \neq P(\text{tennis} \cap \text{golf})$

$\therefore$ events are not independent. (2)

[7]

QUESTION 6 [LO 4: AS 12.4.1]

(a) $A = 14,0566$

$B = 0,9282$ each $\therefore y = 0{,}9282x + 14{,}0566$ (5)

(b) $\hat{y} = 0,9282(160) + 14,0566 = 162,5686$ OR $160\hat{y} = 162,5618$ (2)

(c) Candidate G (1)

(d) choice (iii) – the r value would be closer to 1 for case 2 than for case 1 (1)

[9]

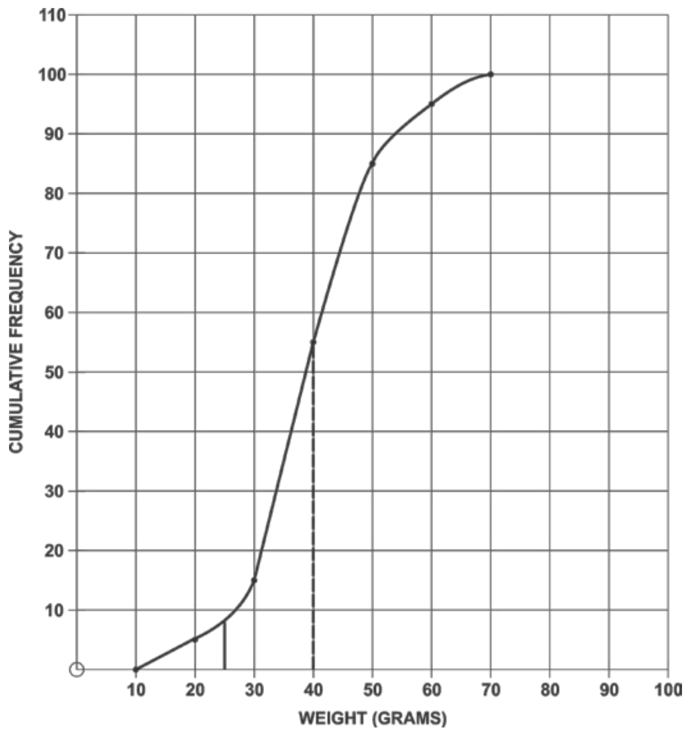
QUESTION 7 [LO 4: AS 12.4.3]

(a) (1) $\overline{x} = 39{,}5$ (3)

(2) (i) $\sigma = 11{,}1692$ (2)

(ii) $\sigma^2 = 124{,}75$ (1)

(b) for drawing with endpoints plotted together with cumulative frequency.
for anchoring at (10 ; 0) (2)



Estimated median $= 38$ or 39 (from their ogive) (1)

(c) below 25 grams is about 8% and below 40 grams is 55% therefore between them is 47% (mark according to their graph) (2)

(d) Mean would go up/increase
Median would stay same
Standard deviation would go up/increase (3)

[14]

QUESTION 8 [LO 1: AS 12.4.1 and 11.4.3]

(a) $\overline{x} - \sigma n \quad = \quad 42 - 0{,}06 = 41{,}94$
$\overline{x} + \sigma n \quad = \quad 42 + 0{,}06 = 42{,}06$
$42 - 0{,}06 = \quad 41{,}94$
$42 + 0{,}06 = \quad 42{,}06$

This is within 1 standard deviation either side of the mean.
Therefore unacceptable $100\% - 68\% = 32\%$ (1)

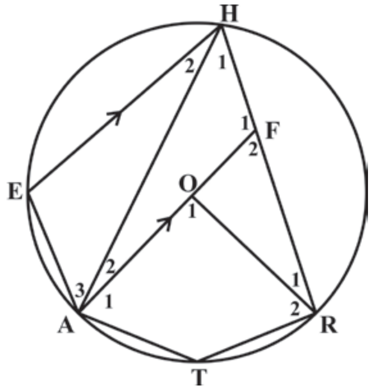
(b) standard deviation $= 0{,}03$
So 41,94 would be within 2 standard deviations and so would 42,06. Now 4,8% would be unacceptable. (2)

[3]

60 marks

SECTION B

QUESTION 9 [LO 3: AS 12.3.2]



(a) $\hat{O}_1 = 100°$ ext angle of a triangle (2)

(b) $\hat{H}_1 = 50°$ angle at centre (2)

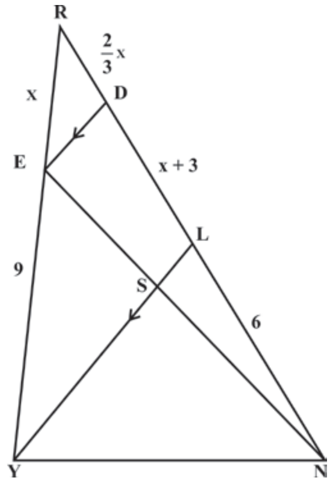
(c) $\hat{T} = 130°$ opp. angles cyclicquad (2)

(d) $\hat{A}_2 = 28°$ angles of a triangle

$\hat{H}_2 = 28°$ alt angles EH//AF (3)

[9]

QUESTION 10 [LO 3: AS 11.3.2]



(a) $\frac{x}{9} = \frac{\frac{2x}{3}}{x+3}$ prop int theorem

$x^2 - 3x = 0$

$x(x-3) = 0$

$x = 3$

$\therefore DL = x + 3 = 6$

and $DL = LN$ **and so L is now the midpoint of DN** (4)

(b) DE = 2,8 units (1)

(c) $\frac{\text{Area}\Delta\text{RED}}{\text{Area}\Delta\text{REN}} = \frac{RD}{RN} = \frac{2}{14} = \frac{1}{7}$ same height

$\text{Area}\Delta\text{REN} = 7 \times \text{Area}\Delta\text{RED} = 18,9 \text{ units}^2$ (3)

OR

$AreaRED = \frac{1}{2}.3.2.\sin R$ *and* $AreaREN = \frac{1}{2}3.14.\sin 64,15^o$

$2,7 = 3\sin R$ $= 18,9 units^2$

$R = 64,15^o$

[8]

QUESTION 11 [LO 3: AS 12.3.2]

Statement	Reason
(a) $\hat{E}_3 = 90°$	Angle in a semi-circle
$\therefore \ \hat{T}_2 = 90°$	Co-interior angles; KO//EL
$\therefore CT = TE$	Line from centre perpendicular to chord Or Midpoint theorem Or Similar triangles COT and CEL

(3)

(b) $\hat{E}_1 + \hat{E}_2 = \hat{L}$	tan chord theorem alt seg theorem
But $\hat{L} = \hat{O}_3$	Corres. angles KO//EL
$\therefore \hat{O}_3 = \hat{E}_1 + \hat{E}_2$	
$\therefore$ COEK is a cyclic quadrilateral	Converse angles same segment Line subtends equal angles

(3)

(c) InΔCOT and ΔKET

$\hat{C} = \hat{K}$ (angles same segment)

$\hat{O}_3 = \hat{E}_1 + \hat{E}_2$ (proved)

$\hat{T}_1 = \hat{T}_3$ (3rd angle of triangles/vert. opp.)

$\therefore \Delta COT /// \Delta KET$ (AAA)

OR

$\hat{T}_2 = 90°$ (proved)

$\therefore \hat{T}_1 = \hat{T}_3 = 90°$ ($\angle$s on stline)

In ΔCOT and ΔKET

$\hat{T}_1 = \hat{T}_3$ (proved)

$\hat{O}_3 = \hat{E}_1 + \hat{E}_2$ (proved)

$\hat{C} = \hat{K}$ (sum$\angle$s of triangle)

$\therefore \Delta COT /// \Delta KET$ AAA (3)

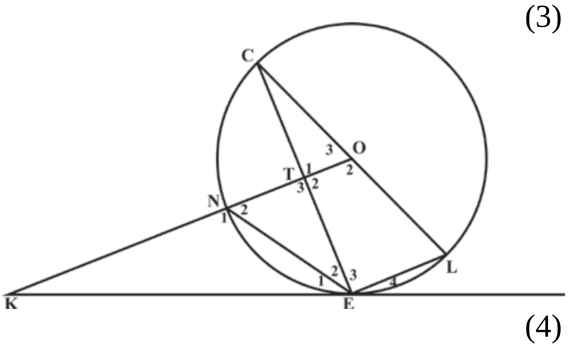
(d) ΔCOT /// ΔKET

$$\frac{OT}{ET} = \frac{CT}{KT}$$

$$\frac{1}{ET} = \frac{CT}{9}$$

But ET = CT and so $CT^2 = 9$ and CT = 3

$\therefore OC = \sqrt{10}$ (by Pythag) (4)



OR $\Delta COK : \cos \hat{O}_3 = \dfrac{CO}{10}$ *and* $\Delta COT : \cos \hat{O}_3 = \dfrac{1}{CO}$

$\therefore CO^2 = 10$ *and* $CO = \sqrt{10}$

(e) $\hat{K} = \hat{C}$ (similar Δs)

$\tan \hat{C} = \dfrac{1}{3}$ or $\sin \hat{C} / \cos \hat{C}$

$\therefore \hat{C} = \hat{K} = 18,43°$ (3)

OR

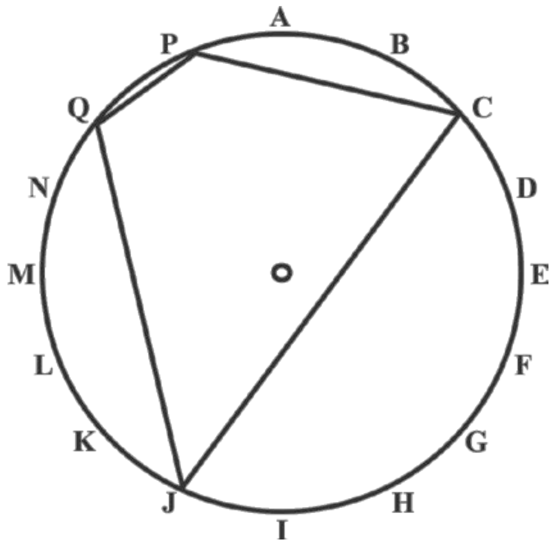
$Area \Delta COT = \dfrac{1}{2}(3)(1) = 1,5$

$\dfrac{1}{2}(3)(\sqrt{10}) \sin \hat{C} = 1,5$

$\hat{C} = 18,43^o$

[16]

QUESTION 12



(a) This circle has 16 arcs each subtending x° at the centre.

$16x = 360°$ (angles about a point) $16x = 360$

$x = 22,5°$ $x = 22,5$

$\therefore P\hat{O}J = 6 \times (22,5°) = 135°$

$\therefore P\hat{C}J = 67,5°$ (angle at centre twice angle at circum) (5)

(b) $P\hat{Q}J = 112,5°$ (opp angles cyclic quad suppl) (2)

[7]

40 marks

Total: 100 marks