# MATHEMATICS: PAPER II

Time: 3 hours                                                                                            150 marks

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## PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1.      This question paper consists of 14 pages, an Answer/Diagram Sheet of 4 pages (i to iv) and an Information Sheet of 2 pages (i – ii). Please check that your paper is complete.

2.      Please detach the Answer/Diagram Sheet from the middle of your question paper. Write your examination number in the space provided in your Answer Book and the Answer/Diagram Sheet.

3.      Any changes made to a diagram must be shown on the Answer/Diagram Sheet, and not on the question paper. Please hand in the Answer/Diagram Sheet with your Answer Book.

4.      Answer ALL the questions.

5.      Please note that diagrams are not necessarily drawn to scale.

6.      All necessary working details must be shown.

7.      Approved non-programmable and non-graphical calculators may be used, unless otherwise stated.

8.      Ensure that your calculator is in **DEGREE** mode.

9.      It is in your own interest to write legibly and to present your work neatly.

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**SECTION A**

**QUESTION 1**

**PLEASE ENSURE THAT YOUR CALCULATOR IS IN DEGREE MODE**
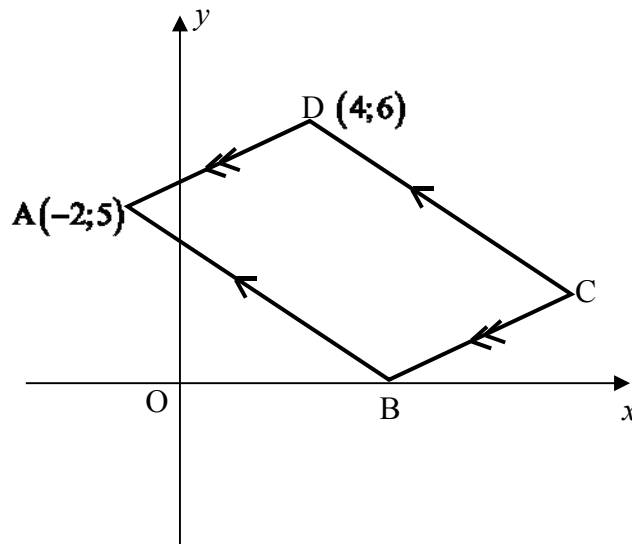
(a)     Given: $E(4;3)$, $F(0;-1)$ and $G(t;1)$.

Determine the value of $t$ for which

    (1)     E, F and G all lie on the same straight line.                    (4)

    (2)     $\Delta$FEG is right angled at F.                    (2)

(b)     In the diagram below, ABCD is a parallelogram.
A is the point $(-2;5)$, D is the point $(4;6)$ and B is on the $x$-axis.
The equation of line CD is given by $2y = -x + 16$.



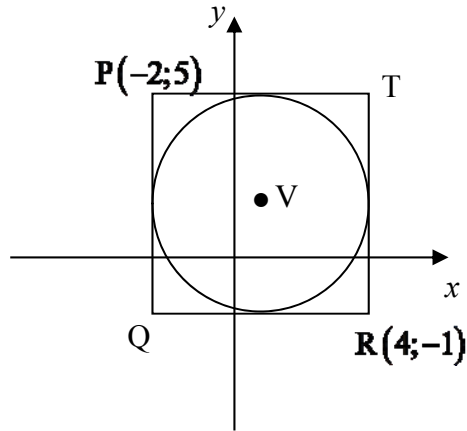    (1)     Determine $A\hat{B}O$ correct to one decimal digit.                    (3)

    (2)     Determine the equation of AB in the form $y = mx + c$.                    (3)

    (3)     Determine the co-ordinates of B.                    (2)

(c)     In the diagram below, the four sides of square PQRT are tangents to a circle with centre V. PT is parallel to the *x*-axis.

$P(-2;5)$ and $R(4;-1)$



(1)     Write down the co-ordinates of T and Q.                                  (2)

(2)     Write down the equation of the circle.                                    (4)

(3)     Given S (3;4), determine the length of VS. Hence decide whether S (3;4) lies inside the circle, outside the circle or on the circle.              (4)

**[24]**

## QUESTION 2

**PLEASE ENSURE THAT YOUR CALCULATOR IS IN DEGREE MODE**

(a)    In the diagram below, ABCD is a quadrilateral. AC is drawn.
       AB = 70 mm, BC = 60 mm, AD = 50 mm, DC = 40 mm and $A\hat{B}C = 34°$.
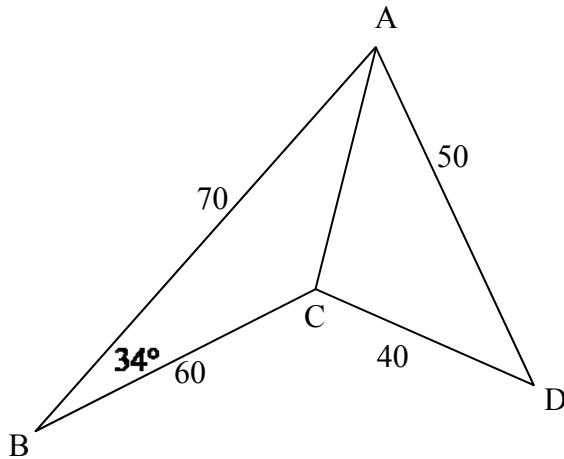


(1)    Show that AC = 39,2 mm, correct to one decimal digit.                          (3)

(2)    Show that $A\hat{D}C = 50,1°$, correct to one decimal digit.                    (3)

(b)    Solve for θ if $-180° < \theta < 180°$ and $2\sin(90° - \theta) = \dfrac{1}{8}$.

Give your answer correct to one decimal digit.                                          (3)
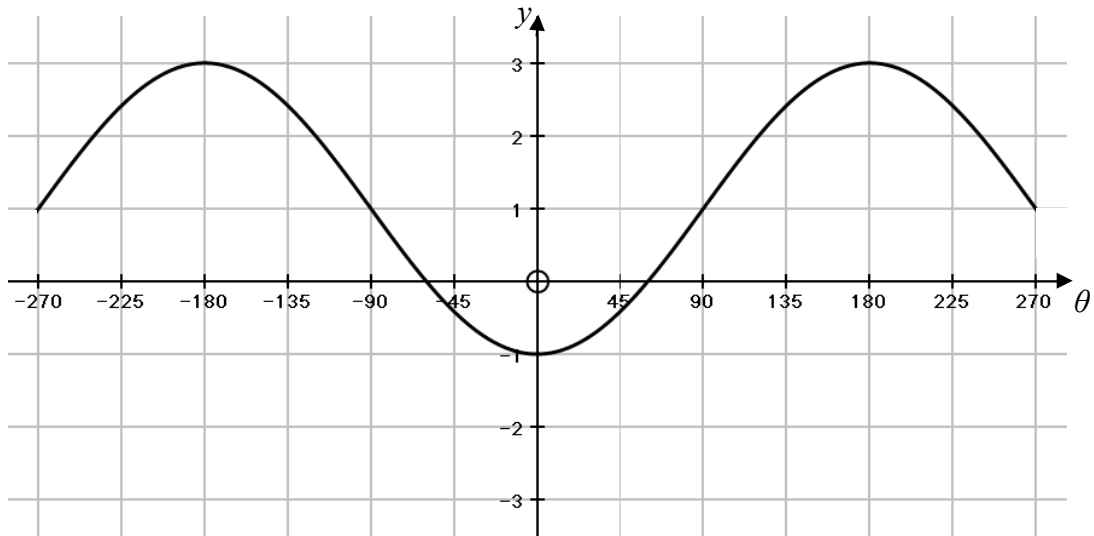                                                                                      **[9]**

**QUESTION 3**

(a)     Simplify without using a calculator. Show all working details.

$$\sin 20°.\cos 320° + \cos(-20°).\sin 400°. \tag{5}$$

(b)     The graph of

$$f(\theta) = p\cos\theta + q \text{ is sketched for } -270° \le \theta \le 270°.$$



(1)     Write down the amplitude of $f$.                                          (1)

(2)     Write down the range of $f$.                                              (1)

(3)     Write down the values of $p$ and $q$.                                     (2)

(4)     On the same set of axes, sketch the graph of $g(\theta) = -2\tan\theta$ for
        $\theta \in (-270°; 270°)$.                                              (3)

(5)     Indicate on the graph, using thickened lines, the intervals on the horizontal
        ($\theta$) axis where: $p\cos\theta + q \ge -2\tan\theta$ for $\theta \in (-270°; 270°)$.   (3)

        **[15]**

## QUESTION 4

(a)     In the diagram below, quadrilateral OABC is transformed to quadrilateral $OA'B'C'$ by undergoing one single transformation.
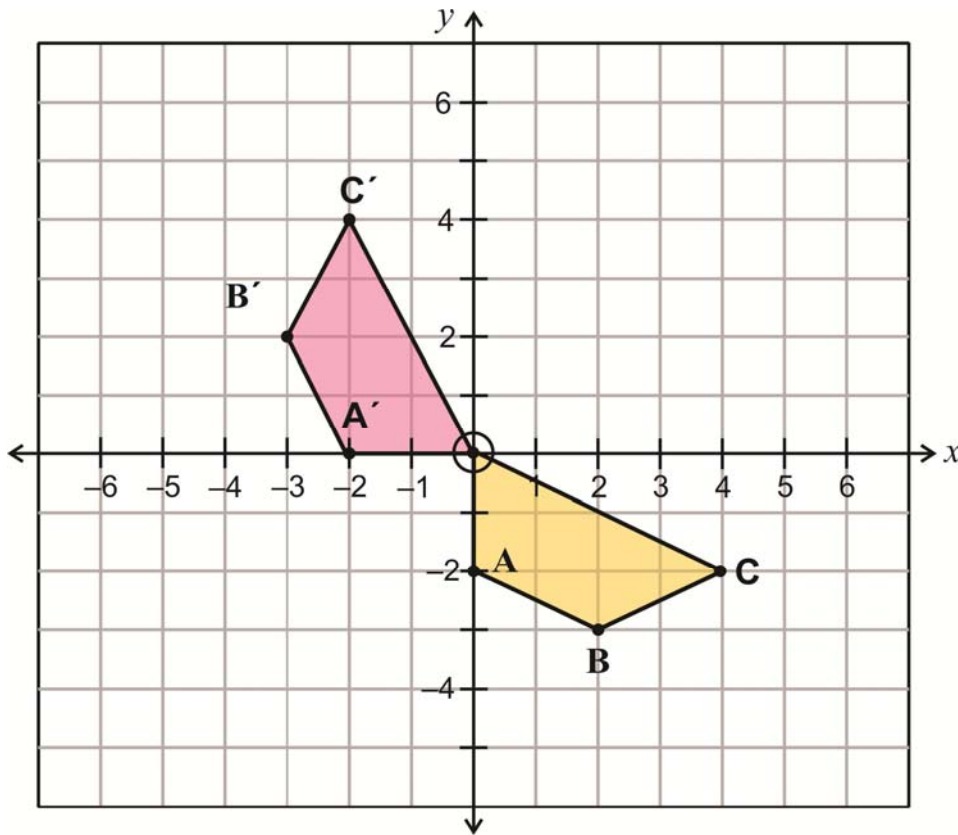


(1)     Describe fully, in words, the single transformation applied.                    (2)

(2)     Write down two different transformations that will transform OABC to $OA'B'C'$ if applied one after the other. Write the transformations down in the form $(x; y) \to ...$                    (2)

(3)     On the grid provided in the Answer/Diagram Sheet, draw the final image of OABC if it undergoes the following transformations one after the other in the order that they are given:

   1:     $(x; y) \to (y; -x)$
   2:     $(x; y) \to (x-1; y+1)$

Label your final image $OA''B''C''$.                    (4)

(b)     In the diagram below, $\Delta P'OR'$ is an enlargement through the origin of $\Delta POR$ by a factor of $k$.
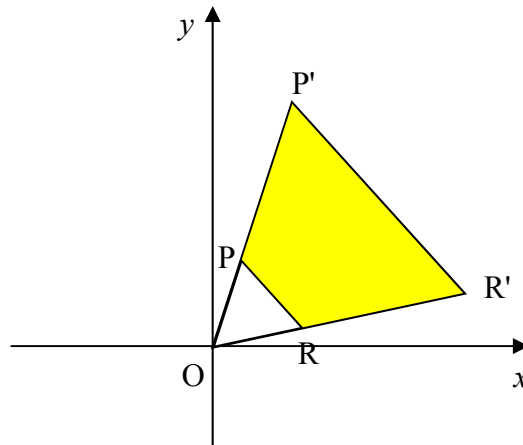


(1)     Determine the value of $k$ for each of the given scenarios:

      (i)     $OP = PP'$                                                    (1)

      (ii)    $P'R' = 3PR$                                                (1)

      (iii)   $\dfrac{\text{Perimeter of } \Delta POR}{\text{Perimeter of } \Delta P'OR'} = \dfrac{1}{4}$                    (1)

(2)     If $k = 3$ and the area of $P'OR' = 72$ square units, determine

      (i)     $\dfrac{\text{Area of } \Delta P'OR'}{\text{Area of } \Delta POR}$                                       (1)

      (ii)    the area of $PRR'P'$                                         (2)

**[14]**

**QUESTION 5**

(a)     The number of children per family in 500 families is summarised in the table below.

(1)

| Number of Children $(x_i)$ | Frequency $(f_i)$ | $(x_i)(f_i)$ |
|:---:|:---:|:---:|
| 0 | 58 | |
| 1 | 84 | |
| 2 | 166 | |
| 3 | 102 | |
| 4 | 72 | |
| 5 | 18 | |
| | **Total** | |

On the Answer/Diagram Sheet provided complete the table and show that
the mean number of children per family is 2,2.                                    (3)

(2)     Study the table below.

| Number of Children $(x_i)$ | Frequency $(f_i)$ | $(\bar{x} - x_i)^2$ | $(f_i)(\bar{x} - x_i)^2$ |
|:---:|:---:|:---:|:---:|
| 0 | 58 | $a$ | 280,72 |
| 1 | 84 | 1,44 | 120,96 |
| 2 | 166 | 0,04 | $b$ |
| 3 | 102 | $c$ | $d$ |
| 4 | 72 | 3,24 | 233,28 |
| 5 | 18 | 7,84 | 141,12 |

Determine the values of *a, b, c* and *d*.                                        (4)

(3)     Determine the standard deviation for the data in the table if
$$\sum f_i (\bar{x} - x_i)^2 = 848.$$
Give your answer correct to one decimal digit.                                    (1)

(b)     A keen photographer managed to obtain the following cumulative frequency curves about the battery lifespan of two types of batteries. The graphs indicate the percentage of batteries that die after $t$ minutes of usage.

**Type A**



**Time ($t$)**

**Type B**



**Time ($t$)**
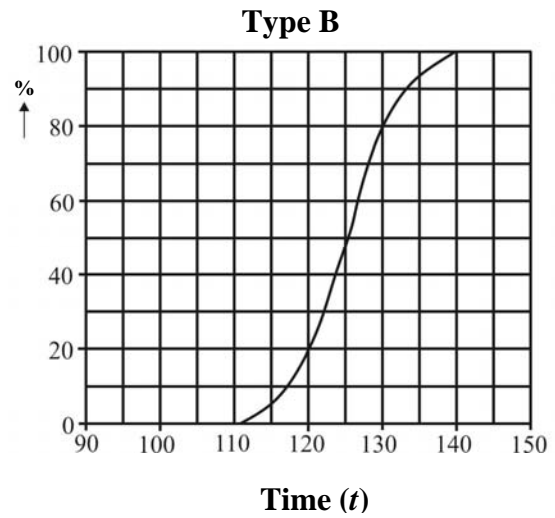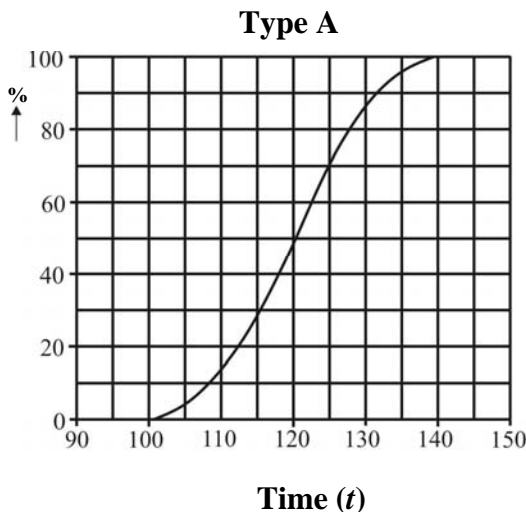
(1)     Give the median battery lifespan for each type of battery.                          (2)

(2)     Which of the two types has a range which is more than 30?                      (1)

(3)     Which of the two types has an inter quartile range which is less than 10?     (1)

(4)     The photographer uses the above information to make a decision about which battery to purchase. Give TWO reasons why he would choose type B batteries rather than type A batteries.                                                              (2)
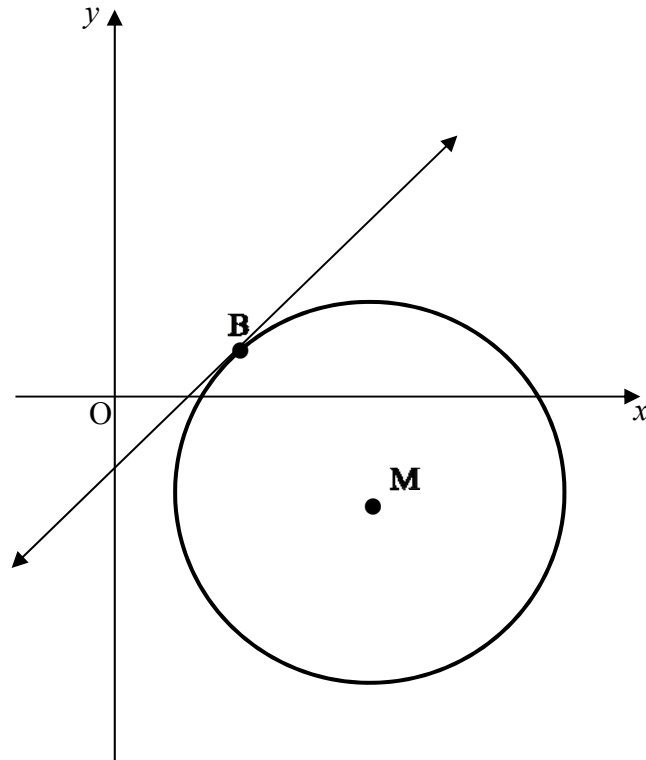
**[14]**

**76 marks**

**SECTION B**

**QUESTION 6**

(a)     In the diagram below, the circle centre  M  with equation
        $x^2 + y^2 - 12x + 4y + 27 = 0$ is drawn.
        The tangent to the circle at  B  has equation $3y - 2x + 5 = 0$.



        (1)     Determine the equation of BM.                                                        (6)

        (2)     Determine the co-ordinates of B.                                                     (3)

(b)    In the diagram below, two circles with equations $(x-4)^2 + (y-6)^2 = 25$ and
       $(x+10)^2 + y^2 = 1$ are drawn. CD is a common tangent to the circles with centres A and B at
       C and D respectively. C is a point on the $y$-axis.
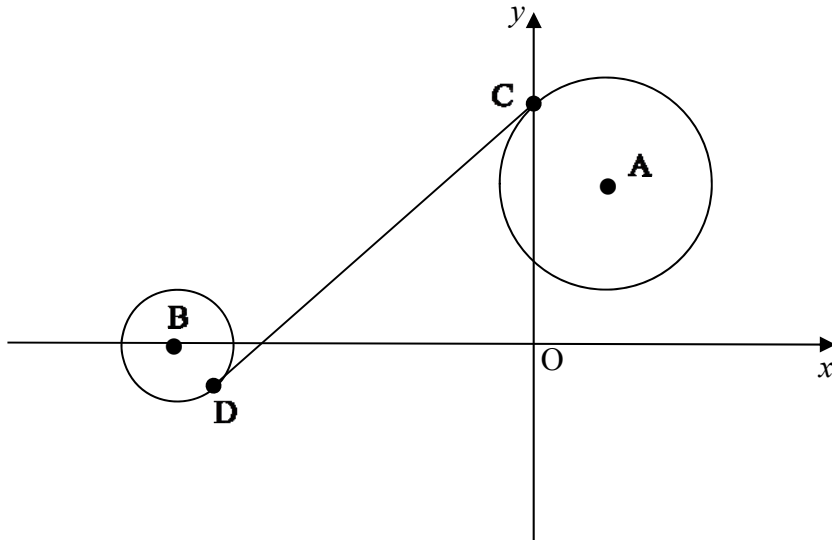


(1)    Determine the co-ordinates of C.                                                    (4)

(2)    Determine the length of the common tangent CD. Leave your answer in
       surd form.                                                                          (5)
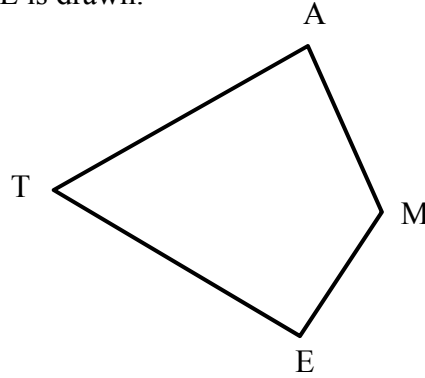
                                                                                          **[18]**

## QUESTION 7

(a)    Given: $\sin\theta = \dfrac{3}{5}$ with $0° \leq \theta \leq 360°$.

       Calculate, without using a calculator, the value of:

       (1)    $\cos(90° + \theta)$                                                         (2)

       (2)    $\sin(-180° - \theta)$                                                       (2)

       (3)    $\cos(\theta - 60°)$. Leave your answer in surd form.                        (3)

(b)    Determine the complete general solution to $4\cos^2\theta - \sin\theta\cos\theta = 4$.
       Give answers correct to one decimal digit.                                          (8)

(c)     In the diagram below, quadrilateral TAME is drawn.

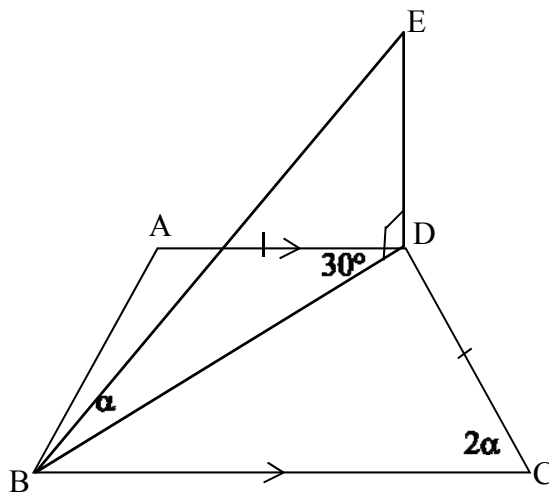$\hat{T} + \hat{M} = 180°$ and $\cos \hat{E} = -\dfrac{1}{3}$.



(1)     Explain why $T\hat{E}M$ is an obtuse angle.                                          (2)

(2)     Determine the value of $\tan \hat{A}$, without using a calculator.                    (4)

**[21]**


## QUESTION 8

In the diagram below, trapezium ABCD is in a horizontal plane and triangle BDE is in a vertical plane.

$A\hat{D}B = 30°$, $B\hat{C}D = 2\alpha$, $D\hat{B}E = \alpha$ and AD = DC.


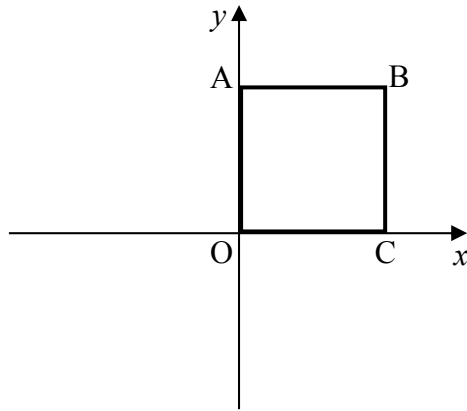
(a)     Prove that $\dfrac{\text{Area of } \Delta BDE}{\text{Area of } \Delta ADB} = \dfrac{2DE}{AD}$.                                 (3)

(b)     Hence, or otherwise prove that $\dfrac{\text{Area of } \Delta BDE}{\text{Area of } \Delta ADB} = 8\sin^2 \alpha$.                    (7)
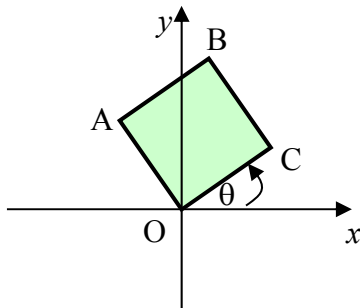
**[10]**

## QUESTION 9

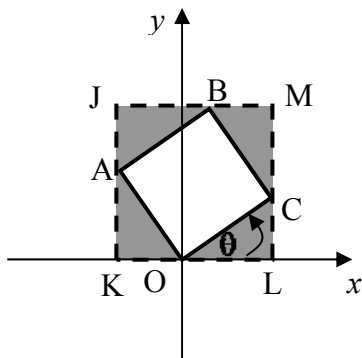In the diagram below, OABC is a square with $O(0;0)$, $A(0;1)$, $B(1;1)$ and $C(1;0)$.



OABC is rotated about the origin in an anticlockwise direction through an angle of $\theta$ for $0° \le \theta \le 90°$.



(a)     Determine, in terms of $\theta$, the co-ordinates of

   (1)     C                                                                          (2)

   (2)     A                                                                          (2)

   (3)     B                                                                          (2)

(b)     Consider JKLM, the rectangle surrounding square OABC as it is rotated about the origin.
   JKLM passes through O, A, B and C.
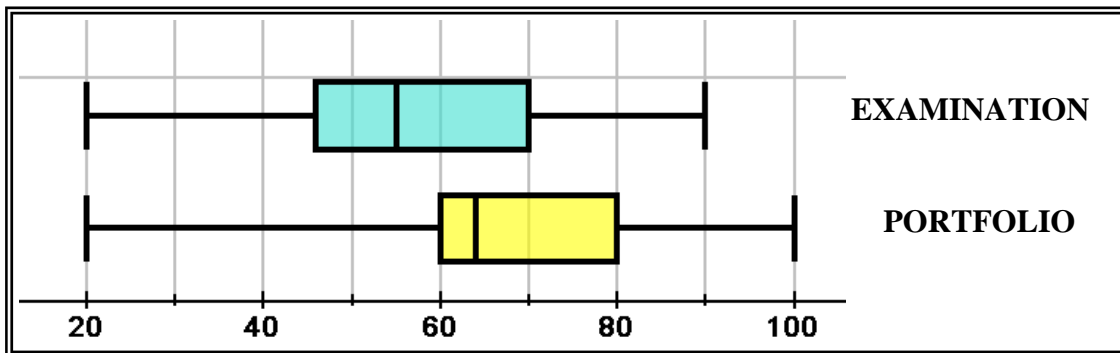   JK and ML are perpendicular to the $x$-axis.



   Prove that the area JKLM is given by $A = 1 + \sin 2\theta$ for $0° \le \theta \le 90°$.          (3)

                                                                                      **[9]**

## QUESTION 10

(a)     A certain school has 80 learners doing Mathematics. The learners' end of year mark
        is calculated as follows:

        Final Mark = 75% of (Examination) + 25% of (Portfolio)

        The box and whisker plots below summarise the performance of the 80 learners in
        the separate assessments.



(1)     Calculate the highest possible end of year mark that any learner could have
        obtained. Give your answer correct to the nearest whole number.                    (3)

(2)     Consider the results obtained on the portfolio.
        Which of the following is greater? Explain why.

        P:      *The range for the marks of the lowest scoring 20 learners*
                            OR
        Q:      *The range for the marks of the highest scoring 20 learners*               (2)

(3)     Can one conclude that the lowest **final** mark by a learner was 20%? Explain.     (2)

(4)     The mean end of year mark obtained by the 80 learners was 60%. One of
        the learners got 46% for the examination and 78% for the portfolio. The
        learner was allowed to retake the examination. His new mark for the
        examination was 86%. Calculate the new overall mean mark for the group.
        Give your answer correct to 1 decimal digit.                                        (5)

(b)     A learner claims that in general, the higher the range of a set of data, the higher the
        standard deviation. Determine whether the following pair of data sets prove or
        disprove the learner's claim. Justify fully.

        Data set 1: 60; 80; 80; 80; 80; 100
        Data set 2: 62; 62; 62; 98; 98; 98                                                 (4)

                                                                                           **[16]**

                                                                                  **74 marks**

                                                                          **Total: 150 marks**