## MATHEMATICS: PAPER I

Time: 3 hours

## PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 8 pages and an Information Sheet of 2 pages (i - ii). Please check that your paper is complete.
2. Read the questions carefully.
3. Answer all the questions.
4. Number your answers exactly as the questions are numbered.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Round off your answers to one decimal digit where necessary.
7. All the necessary working details must be clearly shown. Answers only, without relevant calculations, may incur penalties.
8. It is in your own interest to write legibly and to present your work neatly.

## SECTION A

## QUESTION 1

Solve for $x$ :
(a) $2 x^{2}+11=x+21$
(b) $3 x^{3}+x^{2}-x=0$
(c) $2 x+p=p(x+2)$, stating any restrictions.
(d) (1) $\quad-3 x+2<-2$
(2) $(2 x-3)^{2} \leq 169$
(3) $(2 x-3)^{2} \leq 169$ and $-3 x+2<-2$

## QUESTION 2

(a) Write down the next term of the number pattern:

$$
\begin{equation*}
\frac{1}{2} ; \frac{8}{9} ; \frac{27}{28} \ldots \tag{2}
\end{equation*}
$$

(b) Given: $2 ; 6 ; k$

Write down the value of $k$ if the sequence is:
(1) arithmetic
(2) geometric
(c) Evaluate the sum of the infinite series: 5,6 $+3,36+2,016+1,2096+\ldots$
(d) Given: $0 ;-1 ; 1 ; 6 ; 14$
(1) Show that this sequence has a constant second difference.
(2) Determine a simplified expression for the $n^{\text {th }}$ term of the sequence.
(3) Find the $30^{\text {th }}$ term.

## QUESTION 3

(a) Refer to the figure showing a sketch of $y=f(x)$.

(1) Give the domain and range for the graph.
(2) Use the graph to determine the values of $x$ for which $f(x)>0$.
(3) Use the graph to determine the values of $x$ for which $f^{\prime}(x)<0$.
(4) Determine $f(f(3))$.
(b) Refer to the figure showing a hyperbola.

Determine the equation of this graph.

(c) Given: $g(x)=3 x^{2}-7$.

The graph of $y=g(x)$ is shifted 3 units down and 2 units to the left, giving $y=h(x)$.

Determine an expression for $h(x)$ in the form $a x^{2}+b x+c$.
(d) Given: $f(x)=1+2^{x}$.
(1) Show that $f(x) \times f(-x)=f(x)+f(-x)$.
(2) If $g(x)=f(x)-1$, determine $g^{-1}(x)$ in the form $y=\ldots$

## QUESTION 4

(a) Given: $f(x)=-2 x^{2}$. Determine $f^{\prime}(x)$ from first principles.
(b) Find $\frac{d y}{d x}$ for $y=3 \sqrt{x^{3}}+\frac{4}{\sqrt{x}}-\sqrt{2}$.

The exponents in your answer must be positive values.
(c) Given $f(x)=x^{3}+3 x^{2}+x+1$.
(1) Show that the tangent to the curve $y=f(x)$ at the point where $x=-2$ is $y=x+5$.
(2) Determine the $x$-coordinate of the point where this tangent intersects the curve again.

## SECTION B

## QUESTION 5

(a) Sarah received a valuation of R130 000 for her car that has depreciated at a rate of $15 \%$ p.a. on a reducing balance over the last 5 years.
Determine the value of the car 5 years ago. (Round off your answer to the nearest thousand rands.)
(b) Emily saves R300 from her monthly salary in an account earning interest at $8,5 \%$ p.a. compounded monthly. At the end of 10 years of working, Emily stops making payments but leaves the money in the account to continue earning interest.

Calculate how much Emily can expect to have in the account 20 years after she started her saving plan.
(c) Yerma starts working at the same time as Emily but does not have a saving fund for the first ten years. Calculate how much Yerma needs to save each month if she is to have R130 000 available at the end of the next ten years, using the same interest rate of $8,5 \%$ p.a. compounded monthly.
(d) Emily paid R36 000 in total. Calculate how much more Yerma paid in total.

## QUESTION 6

Refer to the diagram below showing a shaded feasible region bounded by a set of constraints.

(a) Determine the inequalities involved with each of the constraints.
(b) Write down the maximum value of $y$ that satisfies the constraints.
(c) The equation $k=8 x+2 y$ has no solution in the feasible region. Determine all possible values of $k$.

## QUESTION 7

(a) In a picture on a computer, the height of a building is 50 mm .


By clicking on a button, the picture size can be enlarged by a factor of 1,2 so that the height of the building in the picture becomes 60 mm .
(1) Calculate the height of the building in the picture after the button has been clicked eight times. Give your answer to the nearest mm .
(2) The height of the building in the picture is required to be more than 400 mm . Determine the least number of times the button must be clicked to ensure this measurement.
(b) Refer to the diagram, showing the first three figures in a pattern that Anthea is investigating.


Fig. 1


Fig. 2


Fig. 3
(1) Determine the total number of squares (grey and white together) as well as the number of dots needed for Figure 4.
(2) Considering the $n^{\text {th }}$ figure, determine a simplified expression in terms of $n$ for:
(i) The number of grey squares.
(ii) The number of white squares.
(iii) The number of dots.
(3) There are 320 dots in some pattern. Determine the number of grey and white squares in the pattern.

## QUESTION 8

(a) Find the values of $x$ and $y$ if $x+y=60$ and $K=x y^{3}$ is a maximum.
(b) Refer to the figure showing the graph of $y=f(x)$.


In your answer book, draw a copy of the above graph, then on the same axes, add a possible sketch of $y=f^{\prime}(x)$.
(c) Refer to the figure showing the graph of cubic function $y=g(x)$ that has intercepts with axes $\mathrm{C}(-4 ; 0), \mathrm{D}(-2 ; 0), \mathrm{E}(-0,5 ; 0)$ and $\mathrm{F}(0 ; 8)$.


Determine the equation of the graph. You do not need to simplify your answer.
(d) Refer to the figure showing the graphs of cubic functions $f(x)=-4 x^{3}+6 x^{2}+26 x$ and $g(x)=2 x^{3}+13 x^{2}+22 x+8$ with PQ the vertical distance between the graphs.


Calculate the minimum length of PQ for $x>0$.

## QUESTION 9

Tashmira, an enthusiastic basketball player, is practising her shooting.


She throws from a point $1,7 \mathrm{~m}$ from the floor. Each throw follows the path of a parabola. On one of her throws, the ball reaches its maximum height of $3,1625 \mathrm{~m}$ when it has covered a horizontal distance of 3 m . Unfortunately, the ball does not go into the basket but hits the front of the rim which is 3 m above the floor.

Determine how far Tashmira is from the rim, that is: the horizontal distance between Tashmira's hand and the front of the rim.

