NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2012

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A

QUESTION 1

(a) $2x^2 + 11 = x + 21$
$2x^2 - x - 10 = 0$
$(2x - 5)(x + 2) = 0$
$x = \frac{5}{2}$ or $x = -2$ (3)

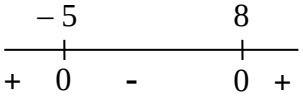
(b) $3x^3 + x^2 - x = 0$
$x(3x^2 + x - 1) = 0$
$x = 0$ or $x = \frac{-1 \pm \sqrt{1+12}}{6}$
$x = 0,4$ or $-0,8$ (5)

(c) $2x + p = p(x + 2)$
$= px + 2p$
$2x - px = 2p - p$
$x(2 - p) = p$
$x = \frac{p}{2-p}, \quad p \neq 2$ (4)

(d) (1) $-3x + 2 < -2$
$-3x < -4$
$x > \frac{4}{3}$ (2)

(2) $(2x - 3)^2 \leq 169$
$-13 \leq 2x - 3 \leq 13$
$-10 \leq 2x \leq 16$
$-5 \leq x \leq 8$ ` (4)

ALTERNATIVELY
$4x^2 - 12x + 9 \leq 169$
$4x^2 - 12x - 160 \leq 0$
$x^2 - 3x - 40 \leq 0$
$(x - 8)(x + 5) \leq 0$



$-5 \leq x \leq 8$

(3) $\frac{4}{3} < x \leq 8$ (1)

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QUESTION 2

(a) $\frac{64}{65}$ ✓ (2)

(b) (1) $k = 10$ (1)

(2) $k = 18$ (1)

(c) $5,6 + 3,36 + 2,016 + 1,2096 + ...$

$r = \frac{3,36}{5,6}$

$= 0,6$

$S_\infty = \frac{5,6}{1-0,6}$

$= 14$ (3)

(d) (1)

	0	;	– 1 ; 1 ; 6 ; 14		
1st diff.		– 1	2	5	8
2nd diff.			3	3	3

$\therefore$ Constant 2nd diff. $= 3$ (2)

(2) $T_n = T_1 + (n-1)f + \frac{(n-1)(n-2)s}{2}$

$= 0 + (n-1)(-1) + \frac{(n-1)(n-2)\times 3}{2}$

$= -n + 1 + \frac{3}{2}\left(n^2 - 3n + 2\right)$

$= -n + 1 + \frac{3n^2}{2} - \frac{9n}{2} + 3$

$= \frac{3n^2}{2} - \frac{11n}{2} + 4$ (4)

ALTERNATIVELY

$2a = 3$

$a = \frac{3}{2}$

$T_0 = 4$

$T_n = \frac{3n^2}{2} + bn + 4$

$T_1 = \frac{3}{2} + b + 4 = 0$

$b = -\frac{11}{2}$

$T_n = \frac{3n^2}{2} - \frac{11n}{2} + 4$

(3) $T_{30} = \frac{3\times 30^2}{2} - \frac{11\times 30}{2} + 4$

$= 1189$ (2)

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QUESTION 3

(a) (1) Domain: $-3 < x \leq 4$ ALT. $x \in (-3;4]$
Range: $-3 \leq y \leq 3$ $y \in [-3;3]$

1 mark for 2 endpoints of domain
1 mark for 2 endpoints of range
1 mark for correct inequalities (3)

(2) $-2 < x < 3$ (2)

(3) $1 < x < 4$ (1)

(4) $f(f(3))$
$= f(0)$
$= 2$ (2)

(b) $y = \dfrac{a}{x-p} + q$

$= \dfrac{a}{x+3} + 2$

$(0;0): \ 0 = \dfrac{a}{3} + 2$

$\dfrac{a}{3} = -2$

$a = -6$

Eqn.: $y = \dfrac{-6}{x+3} + 2$ (4)

(c) $g(x) = 3x^2 - 7$

$h(x) = 3(x+2)^2 - 7 - 3$
$= 3(x^2 + 4x + 4) - 10$
$= 3x^2 + 12x + 12 - 10$
$= 3x^2 + 12x + 2$ (4)

(d) $f(x) = 1 + 2^x$

(1) LHS $= f(x) \times f(-x)$
$= \left(1 + 2^x\right)\left(1 + 2^{-x}\right)$
$= 1 + 2^{-x} + 2^x + 2^0$
$= 1 + 2^{-x} + 2^x + 1$
$= 2 + 2^{-x} + 2^x$

RHS $= f(x) + f(-x)$
$= 1 + 2^x + 1 + 2^{-x}$
$= 2 + 2^x + 2^{-x}$ (3)

(2) $g(x) = 2^x$

$g^{-1}: y = \log_2 x$ (3)

[22]

QUESTION 4

(a) $f(x) = -2x^2$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0}\frac{-2(x+h)^2-(-2x^2)}{h}$$
$$= \lim_{h\to 0}\frac{-2(x^2+2hx+h^2)+2x^2}{h}$$
$$= \lim_{h\to 0}\frac{-2h(2x+h)}{h}$$
$$= \lim_{h\to 0} -2(2x+h)$$
$$= -4x$$ (5)

(b) $y = 3\sqrt{x^3}+\frac{4}{\sqrt{x}}-\sqrt{2}$

$$= 3x^{\frac{3}{2}}+4x^{-\frac{1}{2}}-\sqrt{2}$$
$$\frac{dy}{dx} = 3\times\frac{3}{2}.x^{\frac{1}{2}}+4\left(-\frac{1}{2}\right)x^{-\frac{3}{2}}$$
$$= \frac{9\sqrt{x}}{2}-\frac{2}{\sqrt{x^3}} \quad \text{or} \quad \frac{9x^{\frac{1}{2}}}{2}-\frac{2}{x^{\frac{3}{2}}}$$ (5)

(c) $f(x) = x^3+3x^2+x+1$

(1) $f(-2) = (-2)^3+3(-2)^2+(-2)+1$
$= 3$
$f'(x) = 3x^2+6x+1$
$f'(-2) = 3(-2)^2+6(-2)+1$
$= 1$
Eqn. of Tangent: $y-3 = 1(x+2)$
$y = x+2+3$
$y = x+5$ (6)

(2) $x^3+3x^2+x+1 = x+5$
$x^3+3x^2-4 = 0$
$(x+2)^2(x-1) = 0$
Other intersection point has $x = 1$ (5)

[21]

77 marks

SECTION B

QUESTION 5

(a) $P(1-0,15)^5 = 130\,000$

$$P = \frac{130\,000}{0,85^5}$$

$$= 292\,987,2115$$

$$= \text{R}292\,987,21$$

$$\approx \text{R}293\,000$$ (4)

(b) $$A = \frac{300\left[\left(1+\frac{0,085}{12}\right)^{120}-1\right]}{\frac{0,085}{12}} \times \left(1+\frac{0,085}{12}\right)^{120}$$

$$= 56\,441,52492 \times 2,332647116$$

$$= \text{R}131658,16$$ (6)

(c) $$\frac{x\left[\left(1+\frac{0,085}{12}\right)^{120}-1\right]}{\frac{0,085}{12}} = 130\,000$$

$$x\{188,1384164\} = 130\,000$$

$$x = \text{R}690,98$$ (4)

(d) Yerma's payments $= 690,98 \times 120$

$= 82\,917,60$

Difference $= \text{R}46\,917,60$ (2)

[16]

QUESTION 6

(a) $-2 \leq x$
$y \geq -3$
$y \leq -3x+12$
$y \leq x+4$
$y \geq -\frac{x}{2}-3$ (8)

(b) $y=6$ (1)

(c) $k = 8x+2y$
$2y = -8x+k$
$y = -4x+\frac{k}{2}$
Max. at $(5\,;-3)$
$k = 8\times 5+2\times(-3)$
$= 34$
Min. at $(-2\,;-2)$
$k = 8(-2)+2(-2)$
$= -20$
$\therefore$ No solution for $k > 34$ or $k < -20$ (6)
[15]

QUESTION 7

(a) (1) $T_8 = 50 \times 1,2^8$
$= 214,990848$
≈ 215 mm (2)

(2) $50 \times 1,2^n > 400$
$1,2^n > 8$
$n > \log_{1,2} 8$
$n > 11,40535 \ldots$
$\therefore$ 12 times (4)

(b) (1) Total no. of squares $= 32$
Total no. of dots $= 45$ (2)

(2) (i) n^2 (1)

(ii) $4n$ (1)

(iii) $(n+1)^2 + 4(n+1)$
$= (n+1)(n+1+4)$
$= (n+1)(n+5)$ (2)

(3) $(n+1)(n+5) = 320$
$n^2 + 6n + 5 = 320$
$n^2 + 6n - 315 = 0$
$(n+21)(n-15) = 0$
$n = -21$ or $n = 15$
✘N.V.
No. of grey squares $= 15^2$
$= 225$
No. of white squares $= 60$ (6)
[18]

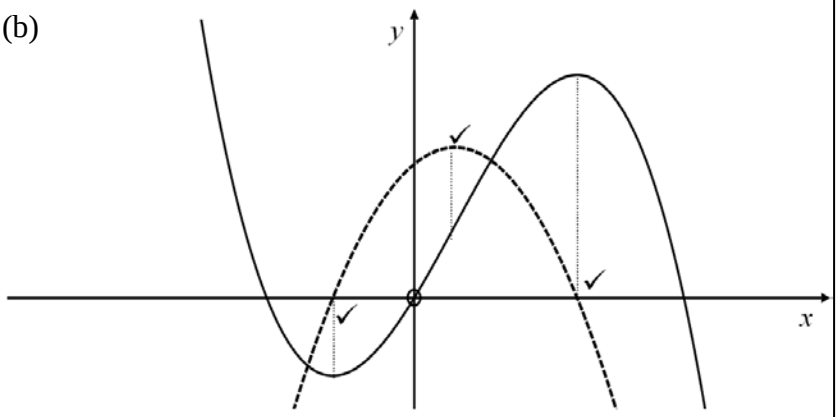
QUESTION 8

(a)
$$x + y = 60$$
$$x = 60 - y$$

$$K = xy^3$$
$$= (60 - y)y^3$$
$$= 60y^3 - y^4$$
$$\frac{dK}{dy} = 180y^2 - 4y^3 = 0$$
$$4y^2(45 - y) = 0$$
$$y = 0 \text{ or } y = 45 \quad (\text{✗ Gives Min})$$
$$\xrightarrow{} x = 15$$

(5)

(b)



(3)

(c)
$$y = a(x+4)(x+2)(x+0,5)$$
$$8 = a(0+4)(0+2)(0+0,05)$$
$$= a(4)$$
$$a = 2$$
$$y = 2(x+4)(x+2)(x+0,5)$$

(3)

(d)
$$PQ = (2x^3 + 13x^2 + 22x + 8) - (-4x^3 + 6x^2 + 26x)$$
$$= 6x^3 + 7x^2 - 4x + 8$$
$$\frac{dPQ}{dx} = 18x^2 + 14x - 4$$
$$9x^2 + 7x - 2 = 0$$
$$(9x - 2)(x + 1) = 0$$
$$\therefore x = \frac{2}{9} \text{ or } x = -1 \quad (\text{✗ (N.V. } x > 0))$$

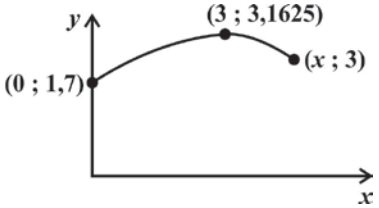
$$\text{Min. } PQ = 6\left(\frac{2}{9}\right)^3 + 7\left(\frac{2}{9}\right)^2 - 4\left(\frac{2}{9}\right) + 8$$
$$= 7,5226 \ldots$$
$$\approx 7,5$$

(7)

[18]

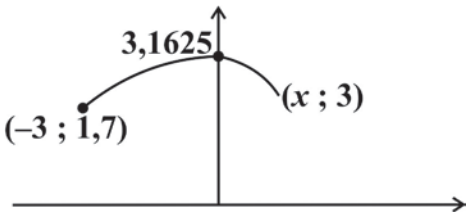
QUESTION 9

Place Tashmira on *y*-axis as shown in diagram.



$y = a(x-3)^2 + 3{,}1625$
$1{,}7 = a(9) + 3{,}1625$
$9a = -1{,}4625$
$a = -0{,}1625$
$-0{,}1625(x-3)^2 = -0{,}1625$
$-0{,}1625(x-3)^2 = -0{,}1625$
$(x-3)^2 = 1$
$x-3 = \pm 1$
$x = 3+1$ or $x = 3-1$
$= 4$ or $= 2$ ✓
N.V. (Before T.P)

ALTERNATIVELY
Place *y*-axis to go through T.P.



$y = ax^2 + 3{,}1625$
$1{,}7 = a(-3)^2 + 3{,}1625$
$= 9a + 3{,}1625$
$9a = -1{,}4625$
$a = -0{,}1625$
$y = -0{,}1625x^2 + 3{,}1625$
$3 = -0{,}1625x^2 + 3{,}1625$
$0{,}1625x^2 = 0{,}1625$
$x^2 = 1$
$x = 1$ (Net is right of *y*-axis)
$\therefore$ Distance from Tashmira to net is 4 m ✓

[6]

73 marks

Total: 150 marks