## MATHEMATICS: PAPER III



Time: 2 hours
100 marks

## PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 16 pages and a Diagram and Information Sheet of 4 (i - iv) pages. Please check that your paper is complete.
2. Read the questions carefully.
3. Answer ALL the questions on the question paper and hand this in at the end of the examination. You may work on the Diagram Sheet, but need not hand it in at the end of the examination.
4. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
5. Round off your answers to four decimal digits where necessary.
6. All the necessary working details must be clearly shown.
7. It is in your own interest to write legibly and to present your work neatly.

| Question | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | Total |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Marks |  |  |  |  |  |  |  |  |  |  |  |  |

## SECTION A

## QUESTION 1

(a) The sequence of numbers: $1 ; 3 ; 7 ; 15 ; 31 ; \ldots$ can be generated by the explicit formula $\mathrm{T}_{\mathrm{k}}=2^{\mathrm{k}}-1$.
(1) Find the eighth term in the sequence.
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$\qquad$
(2) Write down a recursive formula to generate this sequence in the form $\mathrm{T}_{\mathrm{k}+1}=\ldots$
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$\qquad$
$\qquad$
$\qquad$
$\qquad$
(b) A sequence is defined by $T_{1}=3 x$ and $T_{k}=\frac{T_{k-1}+2}{k-1}$, for $k \geq 2 ; \mathrm{k} \in \mathrm{N}$.

Find the next two terms of the sequence.
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$\qquad$
$\qquad$
$\qquad$
$\qquad$

## QUESTION 2

All answers containing factorials must be calculated e.g.: 4! $=\mathbf{2 4}$.
(a) How many 4 digit numbers larger than 6000 can be made using the digits $0,1,2,5,6$, 8,9 if you may not repeat digits?
$\qquad$
$\qquad$
$\qquad$
(b) A box contains 5 red discs, 3 white discs and 2 green discs.

A disc is drawn at random from the box.

- If the disc is red or green, it is kept out of the box and then a second disc is drawn.
- If the disc is white, it is put back in the box and then a second disc is drawn.

Find the probability that the second disc drawn is white.
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$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$ (6)
(c) Three brothers Owen, Robert and Richard are to run in a race which has eight runners in total. The eight competitors line up one to a lane, in the lanes numbered $1-8$.
(1) Write down the total number of arrangements at the starting line.
$\qquad$
$\qquad$
(2) Find the total number of arrangements in which the three brothers are all next to each other.
$\qquad$
$\qquad$
$\qquad$
(3) Find the probability that Owen is in lane 1, Robert in lane 2 and Richard is in lane 3.
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$\qquad$
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$\qquad$

## QUESTION 3

A school organised a dance for their 150 Grade 12 learners. The learners were asked to indicate their preference for the theme. They had to choose from Casino (C), France (F) and Winter Wonderland (W). The information collected is shown in the Venn Diagram below.

$$
S=150
$$


(a) Calculate the probability that a learner, chosen at random:
(1) does not prefer the Casino or the France theme.
$\qquad$
$\qquad$
(2) prefers only TWO of the given theme choices.
$\qquad$
$\qquad$
$\qquad$
(b) Show with all working, whether the events preferring Casino (C) and preferring France (F) are independent or not.
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$\qquad$
$\qquad$
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$\qquad$

## QUESTION 4

In the table below, the number of households, the number of households headed by single parents and the mean family size have been tabulated for 10 areas in Cape Town.

| Number of households $(\boldsymbol{x})$ | Number of households <br> headed by a single parent $\boldsymbol{y})$ | Mean family size |
| :---: | :---: | :---: |
| 5302 | 423 | 3,31 |
| 6359 | 516 | 3,14 |
| 7256 | 657 | 2,98 |
| 8077 | 798 | 2,7 |
| 8729 | 1008 | 2,63 |
| 9028 | 1023 | 2,59 |
| 9205 | 1275 | 2,45 |
| 9379 | 1398 | 2,34 |
| 9412 | 1456 | 2,21 |
| 9517 | 1667 | 2,15 |

(a) State whether there is a positive or negative correlation between the number of households headed by single parents and the mean family size.
$\qquad$
$\qquad$
(b) When predicting the number of households headed by single parents $(y)$ on the number of households ( $x$ ), the equation of the regression line (drawn below) is given by:
$y=A+B x$ with a correlation coefficient $r$

(1) Use your calculator to determine the values of A, B and $r$. Give answers correct to 4 decimal digits.

A = $\qquad$
B = $\qquad$
$r=$
(2) Use this regression line equation to predict the number of households (to the nearest unit) headed by single parents if there are 15000 households.
$\qquad$
$\qquad$
$\qquad$
(3) Would you have confidence in this prediction for 15000 ? Justify your answer.
$\qquad$
$\qquad$
$\qquad$

## QUESTION 5

In 1995 studies based on 1000 women taking an oral contraceptive pill, found that one woman had suffered a thrombosis. The next year 1000 women who were taking a new contraceptive pill were tested, and two were found to have suffered a thrombosis. This resulted in the following sensational newspaper headline.

## NEW CONTRACEPTIVE PILL INCREASES THE RISK OF THROMBOSIS BY 100\%.

(a) Do you think that this editorial comment is misleading? Write a sentence to justify your decision.
$\qquad$
$\qquad$
(b) Give one effect that this headline might have had, once it was published.
$\qquad$
$\qquad$

## QUESTION 6

The marks for 130 learners were recorded, grouped and then represented by the Frequency Distribution below.

(a) Complete the table for the first few class intervals.

| Class Interval | Midpoint of Interval | Frequency |
| :---: | :---: | :---: |
| $0-10$ |  | 5 |
| $10-20$ |  | 15 |
| $20-30$ |  | 25 |

(b) Estimate the mean for the above Frequency Distribution of a population of 130 data points.
$\qquad$
(c) Write down the estimated standard deviation.
$\qquad$
(d) State whether the following are TRUE or FALSE:
(1) The median interval is $40-50$.
(2) The data is positively skewed.
(3) If each of the data values was increased by 5, the mean and the standard deviation would increase by 5 .
$\qquad$
$\qquad$
$\qquad$
(4) The ogive drawn below matches the given frequency distribution.


## QUESTION 7

The quantity of a certain cooldrink in a plastic container is found to be normally distributed with a mean of 375 ml and a standard deviation of 1.5 ml .
Determine the expected number of containers in a sample of 200 that lie between 372 ml and 376.5 ml .

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$\qquad$

## SECTION B

## QUESTION 8

In the figure, $\mathrm{AB} / / \mathrm{EC}$ and $\mathrm{AD} / / \mathrm{BC} . \mathrm{BFE}, \mathrm{AFD}$ and EDC are all straight lines.
$\mathrm{AD}=15$ units and $\mathrm{FD}=6$ units.

(a) Complete the following statement: $\triangle \mathrm{ABF} / / /$
(b) Find, with reasons, the numerical value of $\frac{\mathrm{DC}}{\mathrm{DE}}$.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$ (4)
(c) Find the length of FE if Area $\triangle \mathrm{BEC}=172,6$ units $^{2}$ and $\hat{\mathrm{B}}_{2}=67^{\circ}$. Give your answer correct to the nearest unit.
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$\qquad$

## QUESTION 9

In the diagram, VN and VY are tangents to the circle at N and Y .
$\mathrm{NA} / / \mathrm{VS}$ and $\hat{\mathrm{A}}=65^{\circ}$.

(a) Write down, with reasons, three other angles also equal to $65^{\circ}$.
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$\qquad$
$\qquad$
$\qquad$
(b) Prove that VYSN is a cyclic quadrilateral.
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$\qquad$
$\qquad$ (2)
(c) Prove that $\triangle A S N$ is isosceles.
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$\qquad$

## QUESTION 10

In the figure, VQ is a diameter of the circle with centre O .
The chord EU is drawn parallel to the tangent to the circle at Q and cuts VQ at N .
The point R lies on the tangent to the circle such that $R E \perp Q E$.


Complete the following table:

| Statement | Reason |  |
| :--- | :--- | :--- |
| (a) $\quad \hat{\mathrm{Q}}_{1}+\hat{\mathrm{Q}}_{2}=90^{\circ}$ |  |  |
| (b) $\quad \mathrm{ENQ}=90^{\circ}$ |  |  |
| $\therefore N U=E N$ |  |  |
|  |  |  |

(c) Prove $\triangle V N U / / / \Delta Q E R$.
$\qquad$
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$\qquad$
(d) Prove that $\frac{E N}{V N}=\frac{E R}{E Q}$.
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$\qquad$
(e) Hence, if $N U=6 ; V N=12$ and $E R=4$, calculate the length of $R Q$. Leave your answer in simplest surd form.
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$\qquad$
$\qquad$
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$\qquad$

## QUESTION 11

In the figure equidistant parallel lines divide a right angled triangle into 4 areas.

Triangle \begin{tabular}{|c|c|}
\hline

 and trapeziums 

2 <br>
\hline
\end{tabular}


(a) Determine Area 1 : Area 2 : Area 3 : Area 4. Show all working.
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$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(b) Hence, if the area of the entire figure is $1200 \mathrm{~mm}^{2}$, find the area of the quadrilateral labelled 3 in the diagram.
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$\qquad$

