MATHEMATICS: PAPER II

Time: 3 hours   150 marks


PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1.   This question paper consists of 14 pages, a Diagram Sheet (pages i to iv), an Answer Sheet for Questions 4 and 5 and an Information Sheet (pages i to ii). Please check that your paper is complete.

2.   Write your examination number in the space provided in your Answer Book, the Diagram Sheet and the Answer Sheet.

3.   Answer ALL the questions. Answer Questions 4 and 5 on the Answer Sheet provided, and hand this in with your Answer Book.

4.   A Diagram Sheet is provided. Any changes made to a diagram must be shown on this sheet, and not on the question paper. Please hand in the Diagram Sheet with your Answer Book.

5.   Please note that diagrams are not necessarily drawn to scale.

6.   All necessary working details must be shown.

7.   Approved non-programmable and non-graphical calculators may be used, unless otherwise stated.

8.   Ensure that your calculator is in DEGREE mode.

9.   All answers must be given correct to one decimal digit, unless otherwise stated.

10.   It is in your own interest to write legibly and to present your work neatly.


SECTION A

QUESTION 1

(a)   In the diagram below, trapezium ABCD is drawn with vertices $A(-4;5)$, $B(7;2)$, $C(0;-2)$ and $D(t;1)$.
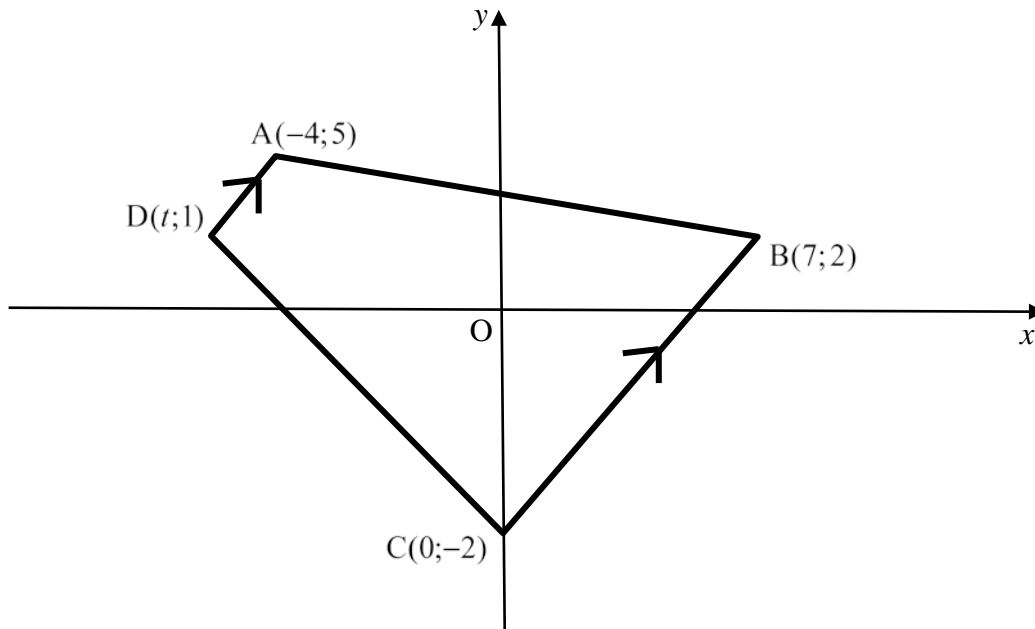
(1)   Determine the equation of the line passing through B and C.   (3)

(2)   Determine, correct to one decimal digit, the acute angle that the line BC makes with the x-axis.   (2)

(3)   Determine the value of t.   (3)

(b)   In the diagram, circle centre T is drawn.

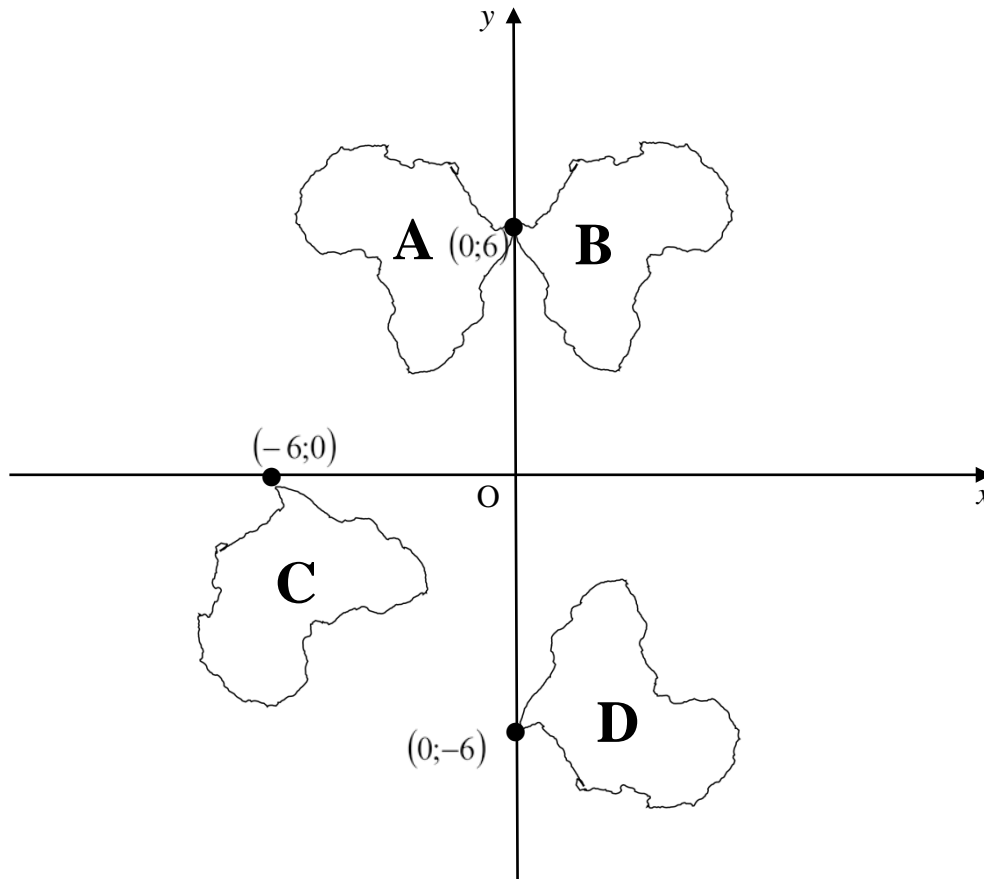   PQ is a diameter of the circle with $P(3;5)$ and $Q(-1;-7)$ on the circle.
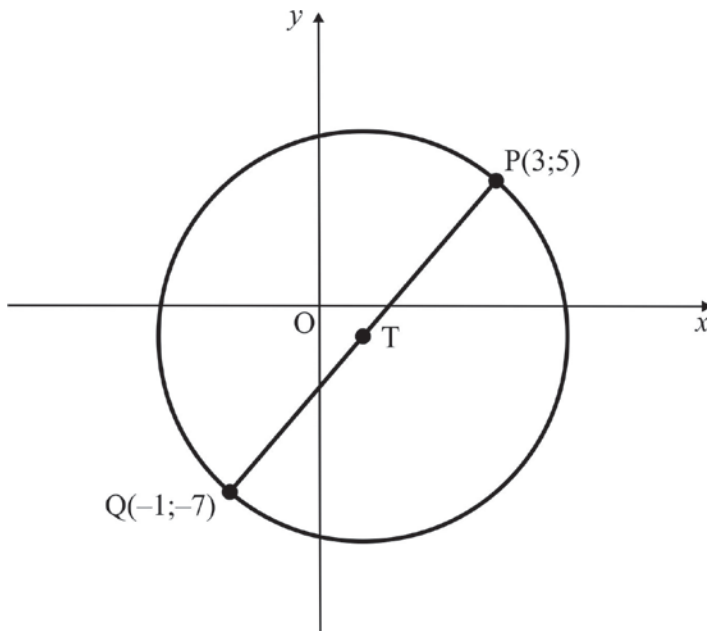
       Determine the equation of the circle.   (5)
                                                                                       [13]

QUESTION 2

(a)   In the diagram below, the outline of Africa is drawn and labelled A, with (0;6) a point on the outline.

   The outline of Africa undergoes different transformations to obtain images B, C and D.

       In each case, complete the rule in the form $(x; y) \to \dots$ if

       (1)   A is transformed to B.   (2)

       (2)   A is transformed to C.   (2)

       (3)   A is transformed to D.   (2)

(b)   In the diagram below, the outline of South Africa is drawn.

R$(-4;4)$ is a point on the outline. The points on the outline are enlarged through the origin by a factor k, to obtain a larger image with $R'(-12;12)$, the corresponding image point of R.
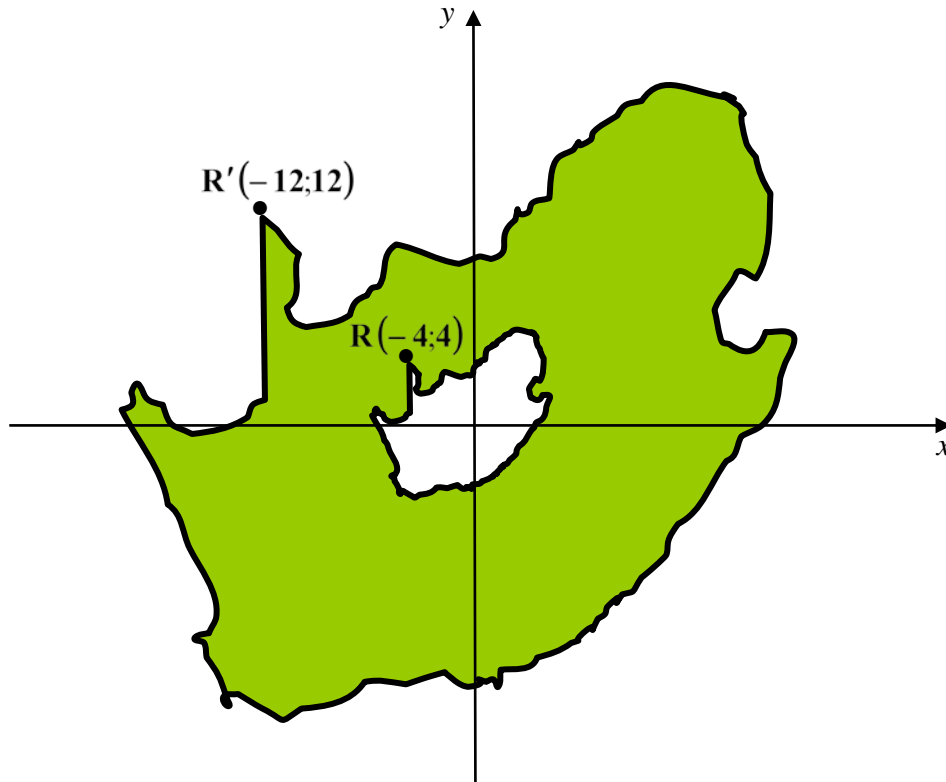
(1)   Write down the value of k.   (1)

(2)   If the perimeter of the smaller outline is x units, write down the perimeter of the larger outline.   (1)

(3)   Write down the value of $\dfrac{\text{Area of smaller outline}}{\text{Area of larger outline}}$.   (1)

(4)   If the larger outline was further enlarged by a factor of 2, write down the co-ordinates of the point on the smaller outline corresponding to the point T'' (–30;6) on the new enlarged outline.   (2)

[11]

QUESTION 3

PLEASE ENSURE THAT YOUR CALCULATOR IS IN DEGREE MODE.

(a)   Determine the value of p correct to two decimal digits if $\theta = 82°$ and

$$2p \tan\left(\frac{\theta}{2}\right) = \sin(2\theta). \tag{3}$$

(b)   (1)   Determine the general solution of:

$$\sin(3\alpha) = -0{,}5. \tag{3}$$

(2)   Hence solve for $\alpha \in [90°; 120°]$ and $\sin(3\alpha) = -0{,}5$. (1)

(c)   Simplify:

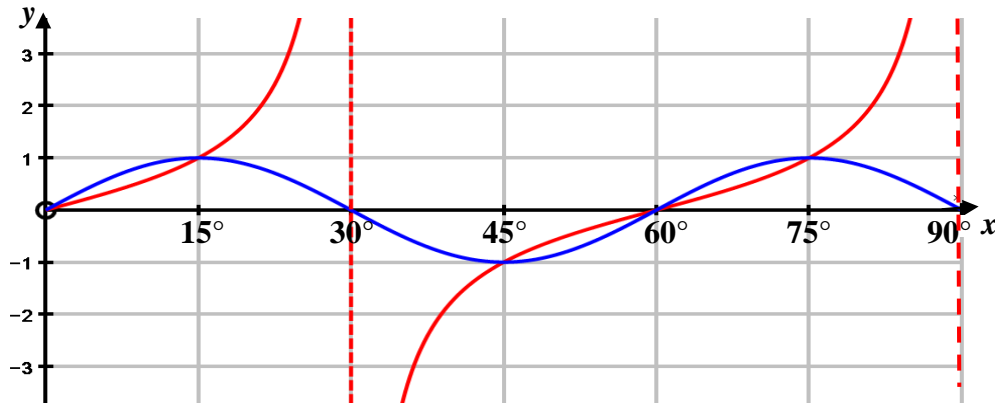$$\frac{\sin(-\beta) + \sin(360° - \beta)}{\sin(180° - \beta) + \sin 180°} \tag{5}$$

[12]

QUESTION 4

ANSWER THIS QUESTION ON THE ANSWER SHEET PROVIDED.

(a)   On the set of axes below, the graphs of

$f(x) = \tan 3x$ and $g(x) = \sin 6x$ for $x \in [0°; 90°]$ are drawn.

(1)   Write down the period of f.   (1)

(2)   Write down the period of g.   (1)

(3)   Use your graph to solve $\tan 3x = \sin 6x$ for $x \in [0°; 90°]$.   (2)

(4)   If the graph of g is shifted 2 units vertically up,

(i)   write down the equation of the resulting graph.   (1)

(ii)   write down the range of the resulting graph.   (1)

(b)   (1)   Simplify as far as possible: $\cos(\theta - 45°) - \dfrac{\sqrt{2}}{2}\cos\theta$.   (3)

(2)   Hence, or otherwise, state a value of θ for which

$\cos(\theta - 45°) - \dfrac{\sqrt{2}}{2}\cos\theta$ is as large as possible.   (1)

(3)   Calculate: $\cos(180° - 45°) - \dfrac{\sqrt{2}}{2}\cos 180°$.   (1)

(4)   On the grid provided on the Answer Sheet, sketch the graphs of

$j(\theta) = \dfrac{\sqrt{2}}{2}\cos\theta$ and $h(\theta) = \cos(\theta - 45°)$

for $\theta \in [-180°; 180°]$.

Label all intercepts and turning points clearly.   (7)
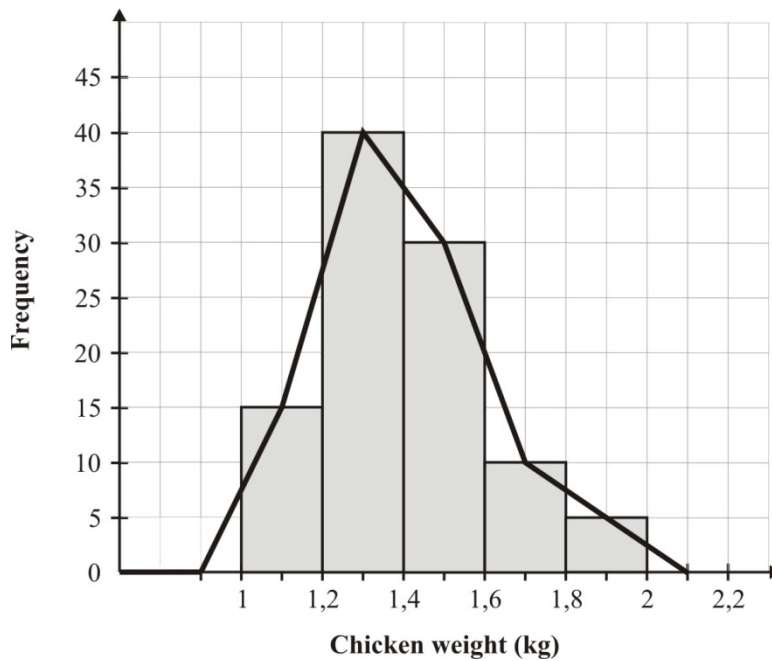
(5)   Use the letter A to show on your graph where you would read off the

maximum value of $\cos(\theta - 45°) - \dfrac{\sqrt{2}}{2}\cos\theta$.   (1)

[19]

QUESTION 5

ANSWER THIS QUESTION ON THE ANSWER SHEET PROVIDED.

In the diagram below, the frequency polygon and histogram summarise the weights of 100 chickens.

(a)   On the grid provided on the Answer Sheet, draw the cumulative frequency curve for the chicken weights of the 100 chickens.   (7)
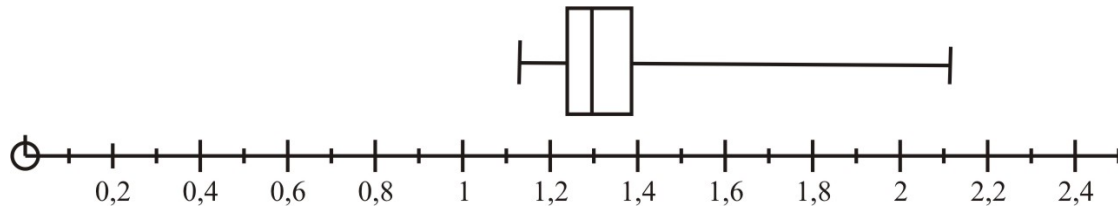
(b)   Given the intervals of chicken weights:

[1;1,2)   [1,2;1,4)   [1,4;1,6)   [1,6;1,8)   [1,8;2,0)

Write down the interval in which the following would lie:

(1)   lower quartile   (1)

(2)   median   (1)

(3)   upper quartile   (1)

(4)   the 90th percentile   (1)

(c)   The box and whisker plot below was drawn from the original data of chicken weights.

The box and whisker plot drawn below is *not* correct.

Give two different reasons why the given plot is *not* correct.   (2)

[13]

Total for Section A: 68 marks

SECTION B

QUESTION 6

Two different classes of students, class A and class B, wrote the same statistics test.

(a)   The marks obtained by class A are given in the table below:

69	66	64	63	62	61	60	60	60	60
59	59	58	58	57	53	51	48	47	40

Use a calculator to determine the mean and the variance from the mean for the marks obtained in class A. Give your answers correct to two decimal digits. (3)

(b)   The following information was obtained from the marks obtained by class B.

$$\sum_{n=1}^{22} x_n = 1320 \quad and \quad \sum_{n=1}^{22}(x_n - 60)^2 = 1012$$

(1)   How many students were in class B? (1)

(2)   Determine the mean mark for class B. (1)

(3)   Determine the standard deviation from the mean for class B.
        Give your answer correct to three decimal digits. (2)

(c)   Explain which class, class A or class B, in your opinion did better. Justify your answer. (2)

(d)   Suppose that all the students who obtained a mark of 60 in class A were moved to class B. What would be the new variance for class B? Leave your answer correct to two decimal digits. (2)

[11]

QUESTION 7

(a)   In the diagram below, the circle centre M is drawn.

The tangents to the circle drawn at R$(-8;3)$ and T$(a;0)$ have equations $4x - 7y + 53 = 0$ and $7x + 4y - 21 = 0$ respectively.
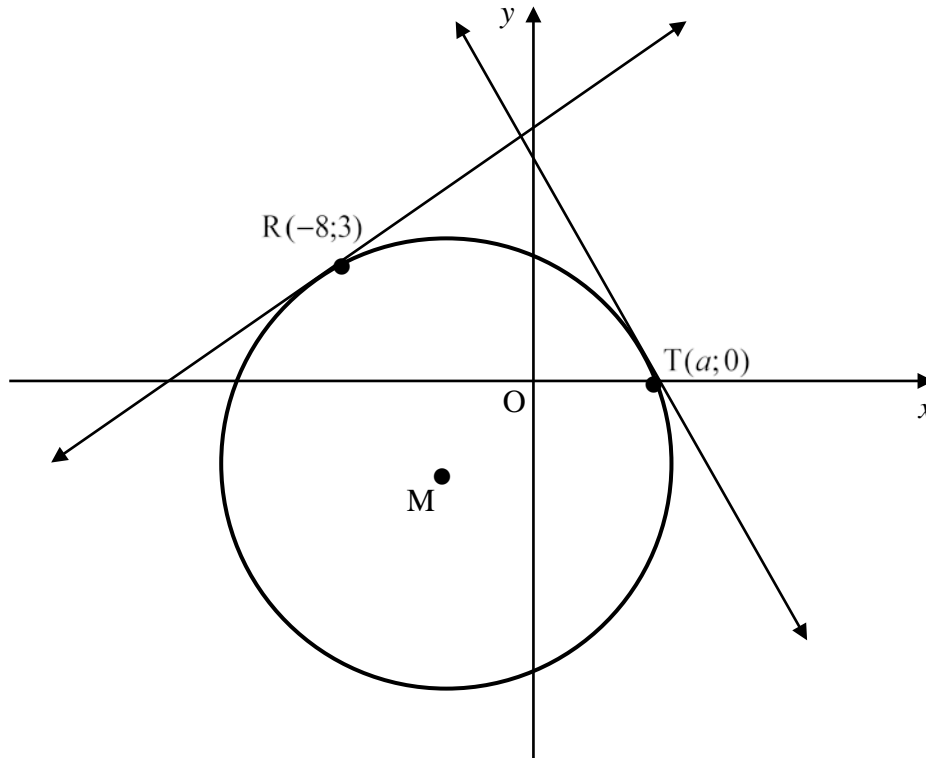
(1)   Show that $a = 3$.   (2)

(2)   Determine the co-ordinates of M.   (10)

(b)   Given: $x^2 + y^2 - 2kx + 4ky + 4 = 0$ is the equation of a circle.

(1)   Determine the co-ordinates of the centre and radius of the circle in terms of k.   (4)

(2)   If $(\sin\theta + k; \cos\theta - 2k)$ is a point on the circle for all values of θ, determine the value(s) of k.   (3)

(3)   For which values of k does the equation NOT represent a circle?   (3)

[22]

QUESTION 8

In the diagram below, point A $(a; a)$ is rotated clockwise about the origin to a point B $(1; 7)$ through an obtuse angle of θ degrees.
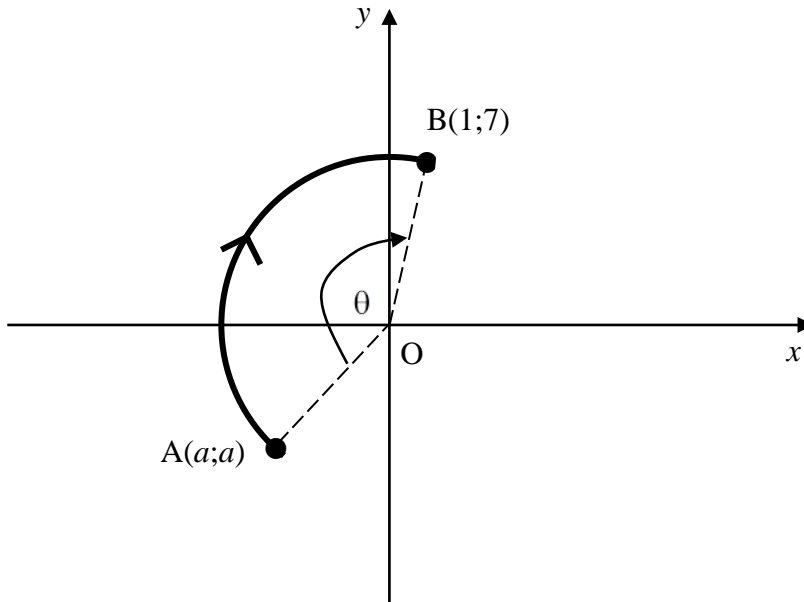
(a)   Show that $a = -5$. (3)

(b)   Hence determine the value of θ. (6)

[9]

QUESTION 9

(a)   If $\cos\theta > 0,$ determine the general solution to $\tan^2\theta = 2,$ correct to one decimal digit.   (4)

(b)   (1)   Prove that $\sin(3A) = 3\sin A - 4\sin^3 A.$ Hint: $3A = 2A + A.$ (6)

        (2)   Hence write down the minimum value of $\dfrac{\sin 3A}{\sin A}.$ (1)

(c)   If $\cos\beta = m$, express the following in terms of m, without using a calculator.

        (1) $\cos(\beta + 20°).\cos(\beta - 20°) - \sin(\beta + 20°).\sin(\beta - 20°)$ (4)

        (2) $\sin\left(\dfrac{\beta}{2} + 45°\right)\cos\left(\dfrac{\beta}{2} + 45°\right)$ (3)

[18]

QUESTION 10

(a)   In the diagram below, ΔABC is drawn. $A\hat{B}C$ is an obtuse angle. AB = 10, AC = 15 and $A\hat{C}B = 30°$.
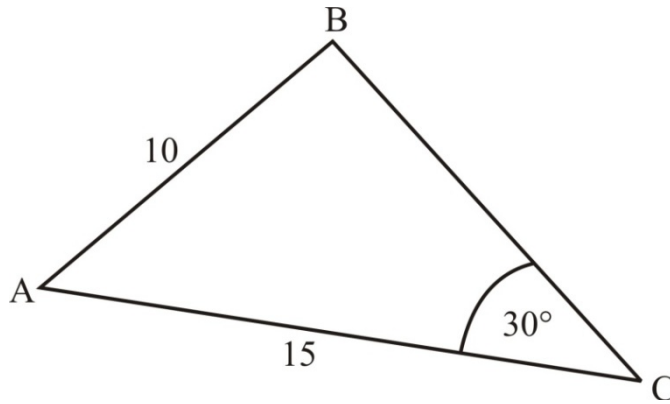
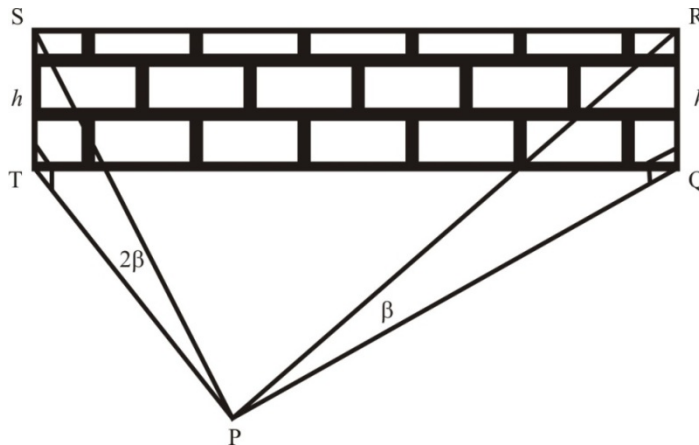Determine the area of ΔABC.   (6)

(b)   In the diagram below, QRST is a vertical wall of height h on level ground. P is a point on the ground in front of the wall.

The angle of elevation of R from P is β and the angle of elevation of S from P is 2β.

TP = 2,25 metres and QP = 6 metres.

(1)   Express $\tan \beta$ and $\tan 2\beta$ in terms of h.   (2)

(2)   Given: $\dfrac{\tan 2\beta}{\tan \beta} = \dfrac{2}{1 - \tan^2 \beta}$. Show that $\tan^2 \beta = \dfrac{1}{4}$.   (4)

(3)   Hence determine the value of β correct to two decimal digits.   (2)

(4)   Determine SR, the length of the wall, if $S\hat{P}R = 120°$ and $\beta = 26,6°$.   (8)

                                                                                      [22]

Total for Section B: 82 marks

Total: 150 marks