NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2011

MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 3 hours 150 marks

These marking guidelines were used as the basis for the official IEB marking session. They were prepared for use by examiners and sub-examiners, all of whom were required to attend a rigorous standardisation meeting to ensure that the guidelines were consistently and fairly interpreted and applied in the marking of candidates' scripts.

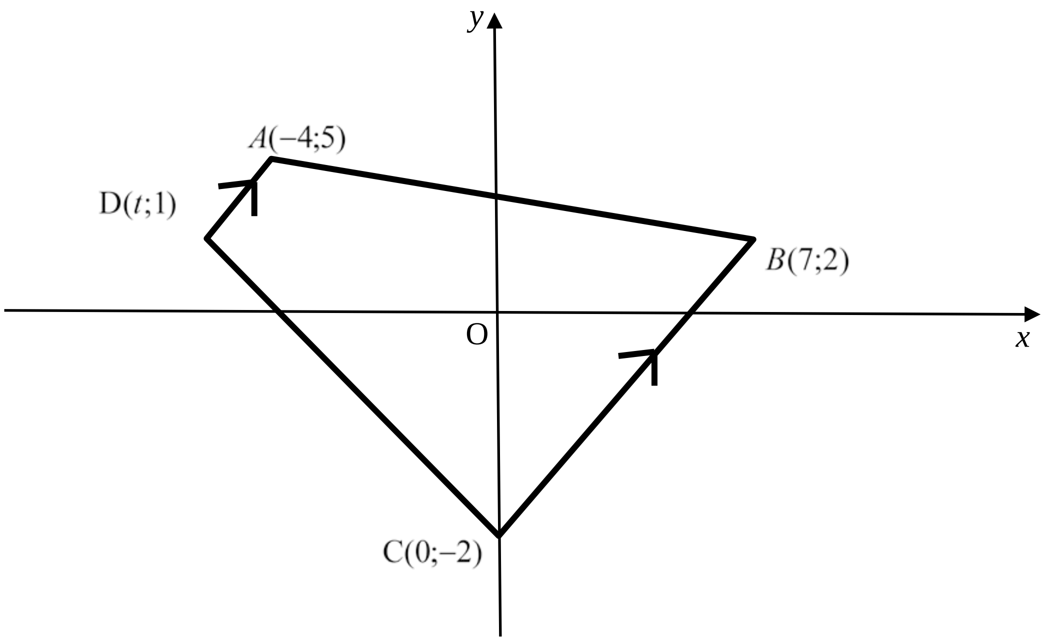
At standardisation meetings, decisions are taken regarding the allocation of marks in the interests of fairness to all candidates in the context of an entirely summative assessment.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines, and different interpretations of the application thereof. Hence, the specific mark allocations have been omitted.

SECTION A

QUESTION 1

(a)



(1) $m_{BC} = \overset{m}{\dfrac{2-(-2)}{7-0}} = \dfrac{4}{7}$ a

$\therefore y = \dfrac{4}{7}x - 2$ a (3)

(2) $\therefore \tan\theta = \dfrac{4}{7}$ m

$\therefore \theta = 29{,}7°$ a a (2)

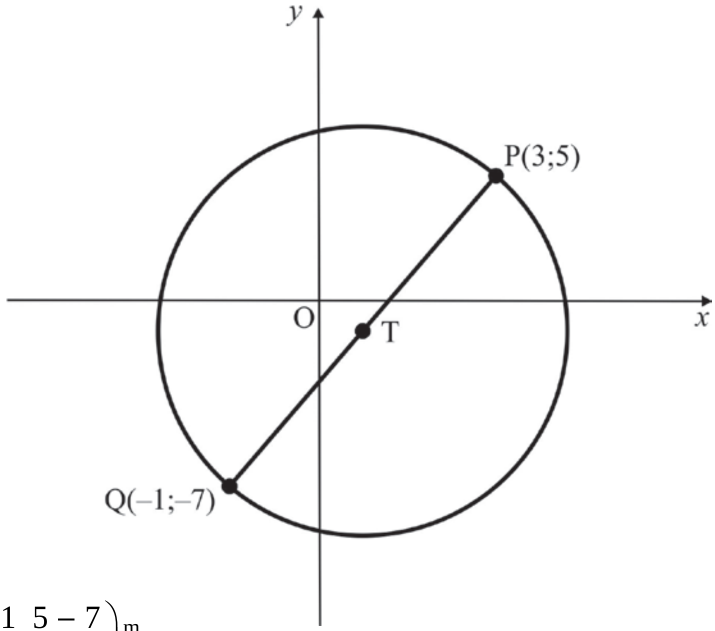
(3) $m_{AD} = \dfrac{5-1}{-4-t} = \dfrac{4}{7}$ since $m_{AD} = m_{BC}$ a

$\therefore -16 - 4t = 28$

$\therefore -4t = 44$

$\therefore t = -11$ a (3)

(b)



$(x_T; y_T) = \left(\frac{3-1}{2}; \frac{5-7}{2}\right)$ m

$T(1;-1)$ a

$(x-1)^2 + (y+1)^2 = r^2$ m

$\therefore (3-1)^2 + (5+1)^2 = r^2$ m

$\therefore r^2 = 40$

$\therefore (x-1)^2 + (y+1)^2 = 40$ a (5)

[13]

QUESTION 2

(a) (1) $(x;y) \to (\overset{a}{-}\overset{a}{x};y)$ (2)

(2) $(x;y) \to (\overset{a}{-}\overset{a}{y};x)$ (2)

(3) $(x;y) \to (\overset{a}{-}\overset{a}{x};-y)$ (2)

$\left.\right\}$ No partial credit

(b) (1) $k = 3$ a (1)

(2) $3x$ units a (1)

(3) $\frac{\text{Area of smaller outline}}{\text{Area of larger outline}} = \frac{1}{9}$ a (1)

(4) T $(x;y) \to$ T'' $(6x;6y)$ m

$\therefore$ T (–5;1) a (2)

[11]

QUESTION 3

(a) $2p\tan\left(\frac{\theta}{2}\right)=\sin(2\theta)$

$\therefore 2p\tan 41° = \sin 164°$ a $\qquad \therefore\ p=\frac{\sin 64°}{2\tan 41°}$ a

$\therefore\ p=0{,}16$ a (3)

(b) (1) $\sin(3\alpha)=-0{,}5$

ref angle $=30°$ a

$3\alpha = 210° + k.360°$ or $3\alpha = 330° + k.360°$ a; $k\in\mathbb{Z}$

$\therefore \alpha = 70° + k.120°$ or $\alpha = 110° + k.120°$ a divide by 3; $k\in\mathbb{Z}$ (3)

(2) $\alpha = 110°$ a (1)

(c) $\frac{\sin(-\beta)+\sin(360°-\beta)}{\sin(180°-\beta)+\sin 180°}$

a a

$=\frac{-\sin\beta-\sin\beta}{\sin\beta+0}$ a a

$=-2$ a (5)

[12]

QUESTION 4

(a) (1) $60°$ a (1)

(2) $60°$ a (1)

(3) $x\in\{0°;15°45°;60°;75°\}$ aa (2)

(4) (i) $y=\sin 6x+2$ a (1)

(ii) $y\in[1;3]$ a (1)

(b) (1) $\cos(\theta - 45°) - \frac{\sqrt{2}}{2}\cos\theta$

$= \cos\theta.\cos 45° \overset{a}{+} \sin\theta.\sin 45° - \frac{\sqrt{2}}{2}\cos\theta$

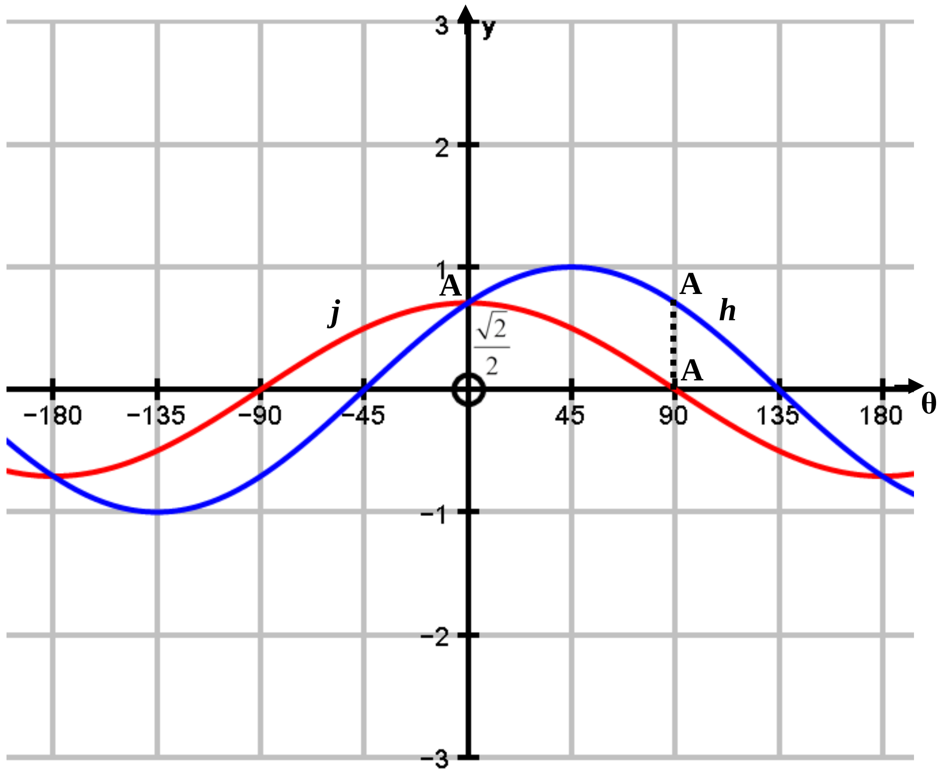
$= \cos\theta.\frac{\sqrt{2}}{2}^{a} + \sin\theta.\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\cos\theta$

$= \frac{\sqrt{2}}{2}\sin\theta^{a}$ (3)

(2) $\theta = 90°^{a}$ (1)

(3) O [a] (1)

(4)



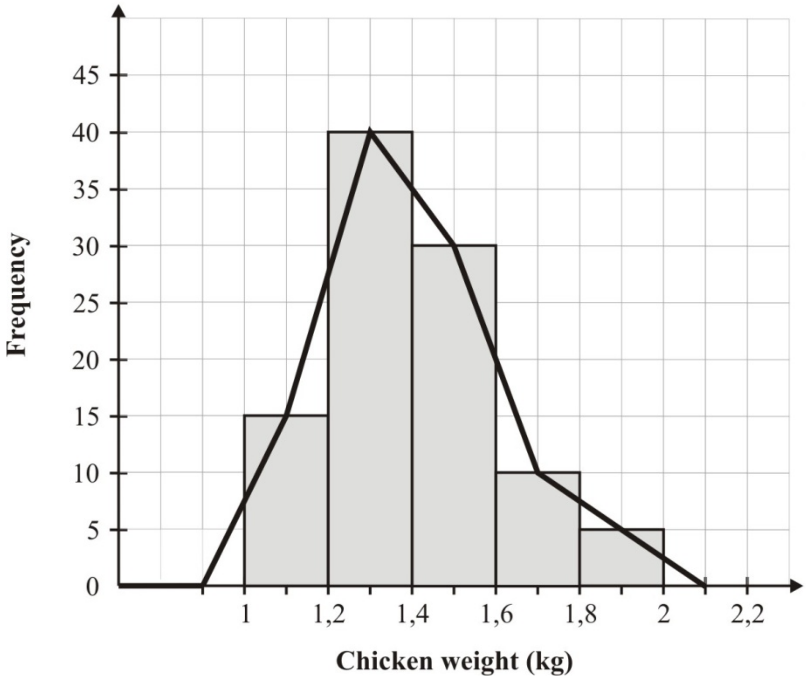
$y = \frac{\sqrt{2}}{2}\cos x$: correct shape
Correct intercepts
Correct turning points

$y = \cos(\theta - 45°)$: correct shape
Correct intercepts
Correct turning points (7)

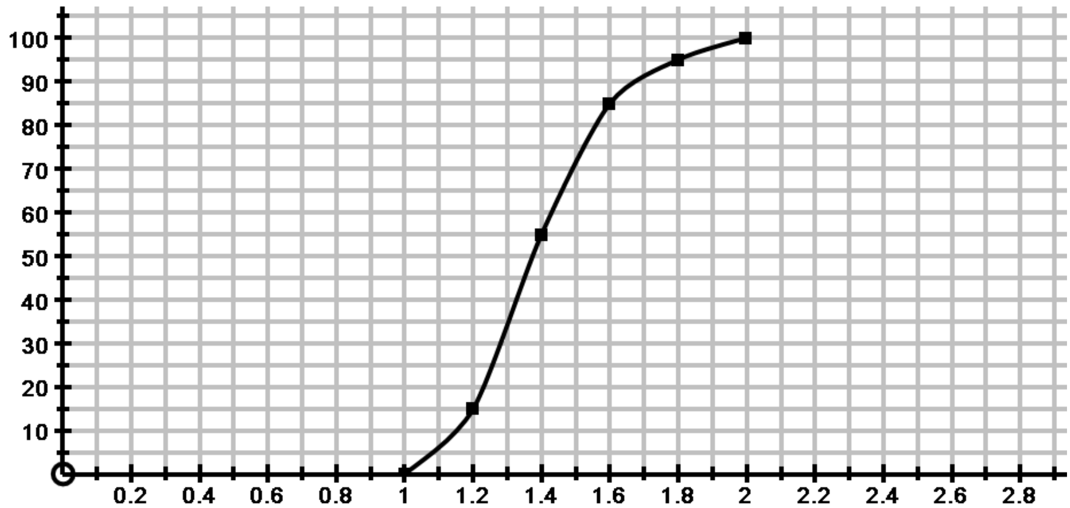
(5) Indicated by dotted line and the letter A (1)

[19]

QUESTION 5



(a)



(1;0) (1,2;15) (1,4;55) (1,6;85) (1,8;95) (2;100)
Joining the points (7)

(b) (1) [1,2;1,4) (1)
(2) [1,2;1,4) (1)
(3) [1,4;1,6) (1)
(4) [1,4;1,6) (1)

(c) Maximum cannot exceed 2
The upper quartile cannot be less than 1,4 (2)

[13]

Total for Section A: 68 marks

SECTION B

QUESTION 6

(a) mean = 57,75 [a]
standard deviation = 6,737024 … [a]
Therefore variance = 45,39 [ca] (3)

(b) (1) 22 [a] (1)

(2) $\frac{1320}{22} = 60$ [a] (1)

(3) Variance $= \frac{1012}{22} = 46$ [a]

Therefore, standard deviation = $\sqrt{46} = 6{,}782$ [a] (2)

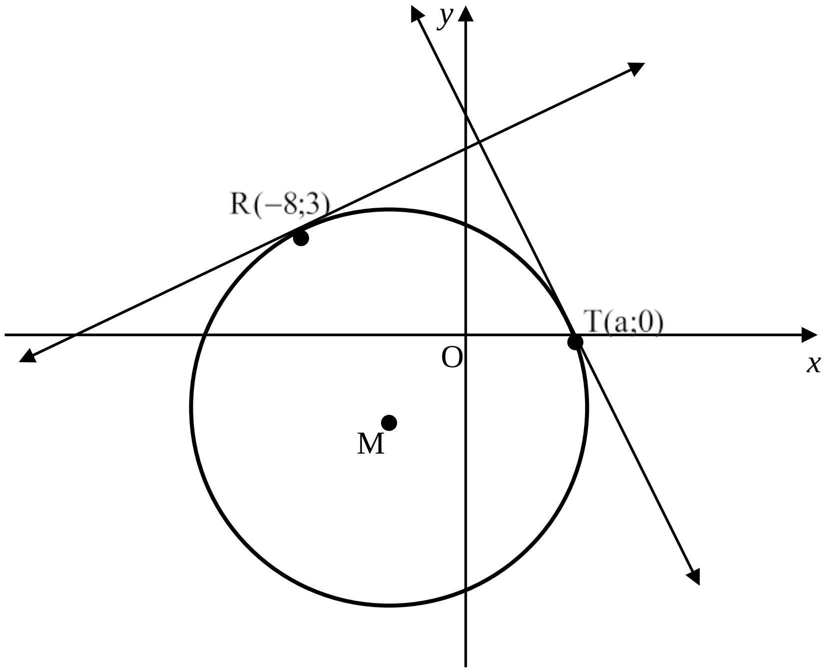
(c) Class B [a] did better as their average is higher. According to the standard deviations, Class A marks are not more spread out than the Class B marks. [a] (2)

(d) Variance $= \frac{1012}{26} = 38{,}92$ [a] [a] (2)

[11]

QUESTION 7

(a)



(1) $7x + 4y - 21 = 0$

Let $y = 0$

$7x + 4(0) - 21 = 0$

$7x = 21$

$x = 3$ (2)

(2) $4x - 7y + 53 = 0$

$7y = 4x + 53$

$y = \frac{4}{7}x + \frac{53}{7}$

$\mathrm{M}_{\tan} = \frac{4}{7}$

$\mathrm{M}_{\mathrm{RM}} = -\frac{7}{4}$

$y - 3 = -\frac{7}{4}(x + 8)$

$y - 3 = -\frac{7}{4}x - 14$

$y = -\frac{7}{4}x - 11$

$7x + 4y - 21 = 0$

$4y = -7x + 21$

$y = -\frac{7}{4}x + \frac{21}{4}$

$\mathrm{M}_{\tan} = -\frac{7}{4}$

$\mathrm{M}_{\mathrm{MT}} = \frac{4}{7}$

$y - 0 = \frac{4}{7}(x - 3)$

$y = \frac{4}{7}x - \frac{12}{7}$

$\therefore -\frac{7}{4}x - 11 = \frac{4}{7}x - \frac{12}{7}$

$\therefore -49x - 308 = 16x - 48$

$\therefore -65x = 260$

$\therefore x = -4$

$$\therefore\ y = -\frac{7}{4}(-4) - 11$$
$$= 7 - 11$$
$$= -4$$
$$\therefore\ \text{M}\ (-4; -4)$$ (10)

(b) (1) $x^2 + y^2 - 2kx + 4ky + 4 = 0 \qquad x^2 - 2kx + k^2 + y^2 + 4ky + 4k^2 = -4$ [a]

$\therefore\ (x - k)^2 + (y + 2k)^2 = 5k^2 - 4$ [a]

$\therefore$ centre $(k; -2k)$ [a]

Radius $\sqrt{5k^2 - 4}$ [a] (4)

(2) $(\sin\theta + k - k)^2 + (\cos\theta - 2k + 2k)^2 = 5k^2 - 4$ [a]

$\therefore \sin^2\theta + \cos^2\theta = 5k^2 - 4$

$\therefore \quad 1 = 5k^2 - 4$ [a]

$\therefore \quad 1 = k^2$

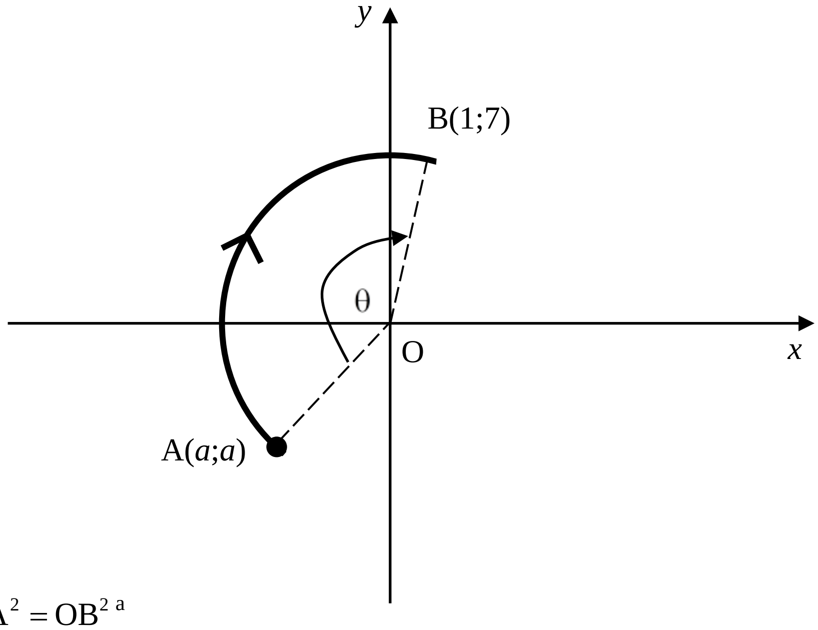
$\therefore \quad k = \pm 1$ [a] (3)

(3) $5k^2 - 4 > 0$ [m]

$\therefore\ k^2 > \frac{4}{5} \quad \therefore\ k > \frac{2}{\sqrt{5}}$ or $k < \frac{-2}{\sqrt{5}}$ [a] (3)

[22]

QUESTION 8



(a) $OA^2 = OB^2$ a

$\therefore (a-0)^2 + (a-0)^2 = (7-0)^2 + (1-0)^2$ a

$\therefore 2a^2 = 50$

$\therefore a^2 = 25$

$\therefore a = 5 \text{ or } a = -5$ (3)

since $a < 0$, $a = -5$ a

(b) $x' = x\cos\theta + y\sin\theta$ a $\qquad y' = -x\sin\theta + y\cos\theta$ a

$1 = -5\cos\theta - 5\sin\theta \ \ldots$ a

and $7 = 5\sin\theta - 5\cos\theta \ldots$ a

ALTERNATE:

(1)+(2): $\qquad$ aa $\theta = 45° + 90° + \alpha$ a

$-10\cos\theta = 8$ a $\qquad \tan\alpha = \frac{1}{7}$ a

$\therefore \cos\theta = -\frac{4}{5}$ $\qquad \therefore \alpha = 8{,}1°$ a

$\therefore \theta = 143{,}1°$ a $\qquad \therefore \alpha = 143{,}1°$ a (6)

[9]

QUESTION 9

(a) $\tan^2\theta = 2$

$\tan\theta = \pm\sqrt{2}$ a

$\theta = 54,7° + k.360°$ a or $\theta = 305,3 + \mathrm{k}.360$ a where k is an integer. a (4)

(b) (1) $= \sin(3\mathrm{A}) = \sin(2\mathrm{A} + \mathrm{A})$ a

$= \sin 2\mathrm{A}.\cos\mathrm{A} + \sin\mathrm{A}.\cos 2\mathrm{A}$ a

a

$= 2\sin\mathrm{A}.\cos\mathrm{A}.\cos\mathrm{A} + \sin\mathrm{A}.(1 - 2\sin^2\mathrm{A})$ a

a

$= 2\sin\mathrm{A}(1 - \sin^2\mathrm{A}) + \sin\mathrm{A} - 2\sin^3\mathrm{A}$

$= 2\sin\mathrm{A} - 2\sin^3\mathrm{A} + \sin\mathrm{A} - 2\sin^3\mathrm{A}$ a

$= 3\sin\mathrm{A} - 4\sin^3\mathrm{A}$ (6)

(2) $\dfrac{\sin 3\mathrm{A}}{\sin\mathrm{A}} = 3 - 4\sin^2\mathrm{A}$ Therefore minimum value is -1 a (1)

(c) (1) $\cos(\beta + 20°).\cos(\beta - 20°) - \sin(\beta + 20°).\sin(\beta - 20°)$

$= \cos\left[(\beta + 20°) + (\beta - 20°)\right]$

$= \cos 2\beta$ a

$= 2\cos^2\beta - 1$ a

$= 2\mathrm{m}^2 - 1$ a (4)

(2) $\sin\left(\dfrac{\beta}{2} + 45°\right)\cos\left(\dfrac{\beta}{2} + 45°\right) = \dfrac{1}{2}\sin(\beta + 90°) = \dfrac{1}{2}\cos\beta = \dfrac{1}{2}\mathrm{m}$ a a a (3)

[18]

QUESTION 10

(a) $\dfrac{\sin\mathrm{B}}{\mathrm{AC}} = \dfrac{\sin\mathrm{C}}{10}$ a

$\therefore \sin\mathrm{B} = \dfrac{\sin 30 \times 15}{10}$

$\therefore \sin\mathrm{B} = \dfrac{3}{4}$ a

$\therefore \mathrm{B} = 131,4°$ a

$\therefore \mathrm{A} = 18,6°$ a

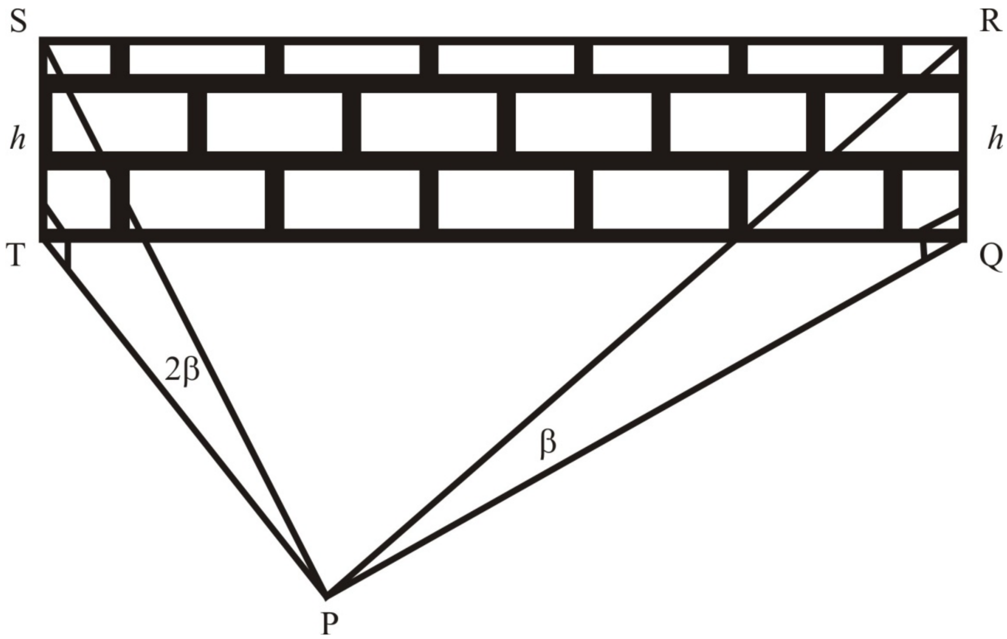
$\therefore$ Area of $\Delta\mathrm{ABC}$

$= \dfrac{1}{2} \times \mathrm{AB} \times \mathrm{AC} \times \sin\mathrm{A}$ a

$= \dfrac{1}{2} \times 10 \times 15 \times \sin\mathrm{A}$

= 23,9 square units a (6)

(b)



(1) $\tan 2\beta = \frac{h}{2,25}$ a

$\tan\beta = \frac{h}{6}$ a (2)

(2) $\frac{\tan 2\beta}{\tan\beta} = \frac{6}{2,25} = \frac{24}{9}$ am

$\frac{\tan 2\beta}{\tan\beta} = \frac{24}{9} \therefore \frac{2}{1 - \tan^2\beta} = \frac{24}{9}$ a

$\therefore 24 - 24\tan^2\beta = 18$

$\therefore -24\tan^2\beta = -6$

$\tan^2\beta = \frac{1}{4}$ a (4)

(3) $\tan\beta = \frac{1}{2}$ a

$\therefore \beta = 26,6°$ a (2)

(4) $\dfrac{PT}{PS} = \cos 2\beta$

$\therefore PS = \dfrac{2,25}{\cos 53,2^\circ}$

$\therefore PS = 3,75$

$\therefore PR = \dfrac{6}{\cos 26,6^\circ}$ a

$\therefore PR = 6,71$

$\therefore SR^2 = PS^2 + PR^2 - 2.PS.PR.\cos \hat{P}$ a

$\therefore SR^2 = (3,75)^2 + (6,71)^2 - 2(3,75)(6,71).\cos 120^\circ$

$\therefore SR^2 = 84,2491$

$\therefore SR = 9,2$ (8)

[22]

Total for Section B: 82 marks

Total: 150 marks