## MATHEMATICS: PAPER II

## EXAMINATION NUMBER

$\square$
Time: 3 hours
150 marks

## PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 32 pages and an Information Sheet of 2 pages (i-ii). Please check that your question paper is complete.
2. Read the questions carefully.
3. Answer ALL the questions on the question paper and hand it in at the end of the examination. Remember to write your examination number in the space provided.
4. Four blank pages (pages 29 to 32) have been included at the end of the exam paper. If you run out of space for a question, use these pages. If you use this extra space, make sure that you indicate this clearly at the question to ensure that your answer is marked in full.
5. Diagrams are not necessarily drawn to scale.
6. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
7. Ensure that your calculator is in DEGREE mode.
8. Clearly show ALL calculations, diagrams, graphs etc. that you have used in determining your answers. Answers only will NOT necessarily be awarded full marks.
9. It is in your own interest to write legibly and to present your work neatly.
10. Round off to one decimal place unless otherwise stated.

FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

| Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | Q9 | Q10 | Q11 | Q12 | TOTAL |
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## SECTION A

## QUESTION 1

In the diagram below:

- $P$ is the centre of the circle.
- $A B$ and $C D$ pass through $P$.
- $A, B, C$ and $D$ lie on the circle.
- The equation of line $A B$ is $y=-2 x+3$.
- The equation of line $C D$ is $y=3 x-7$.

(a) Determine the coordinates of A.
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(b) Calculate the coordinates of point P , the centre of the circle.
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(c) Calculate the length of AP.
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(d) Write down the equation of the circle with centre $P$ in the form $(x-p)^{2}+(y-q)^{2}=r^{2}$.
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(e) Calculate the coordinates of B.
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(f) If $Q$ is the x-intercept of $A B$, determine the equation of the line that is parallel to $C D$ and passing through Q .
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## QUESTION 2

(a) If $\cos 25^{\circ}=m$, then, without the use of a calculator, determine the value(s) of the following in terms of $m$ :
(1) $\sin 25^{\circ}$
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(2) $\cos 50^{\circ}$
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(3) $\cos 55^{\circ}$
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(b) Simplify the following expression into a single term:
$\frac{\sin 21^{\circ} \cos w+\cos 21^{\circ} \sin w}{\sin \left(w+21^{\circ}\right)}-\tan ^{2} \beta \cdot \cos ^{2} \beta$
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(c) In $\triangle A B C$ it is given that $A B=A C=5$ units and $\hat{B}=72^{\circ}$.

Sketch $\triangle A B C$ and then calculate the area of $\triangle A B C$.
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## QUESTION 3

In the diagram below:
The graphs of $f(x)=3 \sin p x$ and $g(x)=2 \sin x$ have been drawn for $x \in\left[0^{\circ} ; 360^{\circ}\right]$.

(a) Write down the value of $p$.
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(b) What is the period of $f(x)$ ?
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(c) Write down the coordinates of B, correct to one decimal digit.
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(d) If $k$ is a positive real number, then for what values of $k$ will $f(x)=k$ have no real solutions?
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## QUESTION 4

(a) Use the diagram below to prove the statement that says, "The acute angle formed by a chord and a tangent at the point of contact is equal to the angle in the alternate segment."


Required to prove that $E \hat{A} C=A \hat{B} C$.
Construction: $\qquad$

Proof:
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(b) In the diagram below, $B, G, F$ and $E$ are points on the circle.

- EAG is a diameter.
- $C D$ is a tangent to the circle at $B$.
- $C \hat{B} E=55^{\circ}$.


Determine the size of $\hat{E}_{1}$.
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(c) In the diagram below, E, F, G and D lie on the circle with centre C .

- Lines ED and GD are extended to H and J respectively.
- HB and JB are drawn with $\mathrm{HBJ}=100^{\circ}$.
- $\mathrm{F} \hat{C} E=50^{\circ}$ and $\mathrm{F} \hat{G} \mathrm{C}=35^{\circ}$.


Prove that DHBJ is a cyclic quad.
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## QUESTION 5

(a) If $g(x)=\tan \frac{x}{2}$ then:
(1) Determine all values of $x$ (general solution) for which $g(x)$ is undefined.
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(2) On the set of axes provided below, sketch $g(x)$ if $x \in\left[-180^{\circ} ; 180^{\circ}\right]$.


## (b) Prove the following identity:


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## QUESTION 6

(a) The table below is a cumulative frequency table that summarises the ages of people that attended a music concert.

| Class interval (age in years) | Cumulative frequency |
| :---: | :---: |
| $0<x \leq 15$ | 12 |
| $15<x \leq 30$ | 28 |
| $30<x \leq 45$ | 50 |

How many people who attended the music concert were older than 15 but younger than or equal to thirty years in age?
(b) Refer to the cumulative frequency curve representing test results below.

(1) Show on the graph where you would find the median test result.
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(2) Determine the interquartile range.
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(c) For the data in the table below the line of best fit is $y=A+B x$.

| $x$ | 22 | 31 | 40 | 24 | 42 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $y$ | 43 | 58 | 81 | 45 | 80 |

(1) Calculate the values of $A$ and $B$ :

A = $\qquad$ $B=$
(2) If the correlation coefficient is 0,99 ; explain why it is not a wise idea to make a prediction for y if x is equal to 230 .
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(d) (1) A box-and-whisker diagram is shown below.


Draw in the median on the diagram above so that the plot represents data that is skewed to the right or positively skewed.
(2) The managing director of a sales department with 120 people gives an annual bonus to the top 40 performing individuals. Explain why the annual average income of his staff is positively skewed.
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(e) You have a machine that cuts wooden poles into ten-metre lengths. You take a random sample of poles and measure their lengths in metres and calculate the standard deviation of the lengths to be 0,8 .

Do you think the machine needs a service? Explain.
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## SECTION B

## QUESTION 7

The diagram below shows the relationship between the time spent on an electronic device and the amount of time spent doing physical activity during the day.

(a) Circle the correlation coefficient that best describes the data represented in the diagram above:
$r=1 \quad r=-1 \quad r=0,8 \quad r=-0,8$
(b) If $A$ and $B$ were removed from the data set above, what would happen to the:
(1) correlation coefficient?
(2) gradient of the line of best fit?
(3) Circle the line below which best describes the person represented by A .

- A person who has just bought an i-pad and plays computer games.
- A person who watches sport on television and likes to read books.
- A person who plays professional sport and studies via the internet.
(c) Please refer to the information in the table below and answer the questions that follow.

|  | COFFEE SHOP A | COFFEE SHOP B |
| :---: | :---: | :---: |
| Days of the week | Cups of coffee sold per day | Cups of coffee sold per day |
| Monday | low | fairly high |
| Tuesday | low | fairly high |
| Wednesday | low | fairly high |
| Thursday | low | fairly high |
| Friday | high | fairly high |
| Saturday | high | fairly high |
| Sunday | 350 cups/day | low |
| MEAN | $m$ cups/day | 350 cups/day |
| Standard Deviation | $p$ cups/day |  |

(1) Explain why the standard deviation at Coffee Shop B is smaller than the standard deviation at Coffee Shop A.
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(2) If Coffee Shop A decides to sell coffee at a higher price on the weekends, then how would this affect the mean and standard deviation?
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(3) What possible strategy could coffee shop B introduce so that the mean and standard deviation both increase? (Explain your answer.)
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## QUESTION 8

(a) In the diagram alongside:

- CD\|NH.
- NH intersects DE at G.
- K is a point on DE with $\mathrm{DG}=\mathrm{GK}$.
- HK||FE.
- CN:CE = 2:5.

(1) Determine $\frac{E K}{K G}$. (Show all your working.)
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(2) Calculate $\frac{\text { Area of } \triangle D G H}{\text { Area of } \triangle D E F}$.
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(b) In the diagram below, $\mathrm{H} ; \mathrm{E} ; \mathrm{N}$ and G lie on the circle with centre O .
- $\quad \mathrm{NC}$ is a tangent at N and passes through P .
- B lies on HC with GB\|NC.
- HP and GB intersect at $E$.
- $H G=H E$.
- GÔN $=2 x$.

(1) Prove that $\mathrm{GN}=\mathrm{NE}$.
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## (2) Prove that $\triangle G O N\|\| \Delta G H E$.

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(3) Show that $\frac{O N \times G E}{G N}=\frac{H B \times E P}{B C}$.
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## QUESTION 9

(a) A metal frame is built to help provide some shade to a triangular piece of land $A B C$.

- $A, B$ and $C$ are on the same horizontal plane.
- $A C=7$ metres; $C B=8$ metres and $A B=10$ metres.
- AF, BG and CH are vertical metal poles.
- $A F=B G=3$ metres and $C H=2$ metres.
- HF, FG and GH are metal poles that complete the metal frame.


Calculate the area of $\Delta F G H$. (The area of canvas required.)
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(b) In the diagram below, C and A are points that lie on the circle.

- $\quad C$ and $B$ lie on the $x$-axis.
- $A B$ is a tangent at point $A(5 ; 3)$.
- The equation of the circle is $x^{2}+y^{2}-6 x-4 y+8=0$.

(1) Find the coordinates of C .
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## (2) Calculate the length of CB.

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## QUESTION 10

In the diagram below, $\mathrm{A} ; \mathrm{B}$ and F lie on the circle.

- The equation of line EA is $3 y-2 x=8$.
- The gradient of line AF is -1 .

(a) Calculate the size of $E \hat{A} F$.
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(b) If $E A=\sqrt{52}$ and $F B=\sqrt{40}$ then calculate the length of $C B$ if the centre of the circle lies on $C B$ and $C B \perp A F$.
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## QUESTION 11

In the diagram below, C, D and M are points on the circle.

- $M C \hat{D}=x$.
- KD is a tangent to the circle at D .
- $E$ is a point on DK.
- EM is another tangent to the circle at M .
- $\mathrm{KME}=x+45^{\circ}$ and $\mathrm{E} \hat{K} M=2 x-40^{\circ}$.


Determine the value of $x$.
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## QUESTION 12

The diagram below is an aerial view of four wind turbines placed at $A, D, E$ and $B$.

- Line $A B$ has equation $5 x+12 y=60$.
- A lies on the $y$-axis.
- B lies on the $x$-axis.
- $E$ is the midpoint of DB.
- C lies on $A B$ and represents the control station.
- The area of $\triangle A D C: \triangle E C D$ is 8:9.

(a) Calculate the distance of $A B$.
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(b) Find the coordinates of C .
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## ADDITIONAL SPACE (ALL questions)

REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.
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