GRADE 12 EXAMINATION
NOVEMBER 2017

ADVANCED PROGRAMME MATHEMATICS: PAPER II

Time: 1 hour | 100 marks

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1. This question paper consists of 14 pages, an Answer Sheet (Module 4) of 2 pages (i–ii) and an Information Booklet of 4 pages (i–iv). Please check that your question paper is complete.

2. This question paper consists of THREE modules:

   Choose **ONE** of the **THREE** modules:

   MODULE 2: STATISTICS (100 marks) OR
   MODULE 3: FINANCE AND MODELLING (100 marks) OR
   MODULE 4: MATRICES AND GRAPH THEORY (100 marks)

3. Non-programmable and non-graphical calculators may be used.

4. All necessary calculations must be clearly shown and writing should be legible.

5. Diagrams have not been drawn to scale.

6. **Rounding of final answers.**

   MODULE 2: **Four** decimal places, unless otherwise stated.
   MODULE 3: **Two** decimal places, unless otherwise stated.
   MODULE 4: **Two** decimal places, unless otherwise stated.

MODULE 2 STATISTICS

QUESTION 1

1.1 James has 12 friends, seven girls and five boys. He is only allowed to invite five friends to his birthday party. What is the probability that three of the friends he randomly invites to his party are boys? (6)

1.2 Arieb notes that in his suburb, 70% of households have an electric fence on the perimeter of their property. Find the probability that in a random sample of 10 households in his suburb, 7 of them have an electric fence. (5)

1.3 Three identical cups of coffee, two identical cups of latte and two identical cups of hot chocolate are arranged in a row. Calculate the number of arrangements of the seven cups of hot drinks if:

(a) the first and last cups in the row are the same type of hot drink. (6)

(b) the three cups of coffee are all next to one another and no other hot drink is next to the same type of hot drink. (6)

[23]

QUESTION 2

2.1 A random variable X has a probability mass function

$$P(X = x) = \begin{cases} 0{,}3 \times (0{,}7)^x & \text{for } x \in \{1, 2, 3, 4\} \\ C & \text{for } x = 5 \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine the value of C. (6)

(b) Find $P(X > 3)$. (3)

2.2 (a) Which formula should be used to determine a confidence interval for the population proportion? (A or B)

(A) $p \pm z\sqrt{\dfrac{p(1-p)}{n}}$ (B) $\bar{x} \pm z\dfrac{\sigma}{\sqrt{n}}$ (1)

(b) Carla conducted a survey on the preference of dark chocolate over milk chocolate. Her findings are as follows:

Using a sample of 500 people, the α% confidence limits for the proportion of people preferring dark chocolate are 0,2278 and 0,2922.

(i) Show that the number of people in the sample of 500 who preferred dark chocolate is 130. (4)

(ii) Hence, using the correct confidence interval formula, find α. (6)

[20]

QUESTION 3

The lengths of new crayons are normally distributed with a mean of 9 cm, and a standard deviation 0,1 cm.

3.1 Find the probability that a crayon chosen at random has a length greater than 8,9 cm. (6)

3.2 Find the probability that, in a random sample of six crayons, at least two have lengths greater than 8,9 cm. (8)

3.3 4% of the crayons are considered too short and cannot be sold. What is the minimum length of a crayon that can be sold? (6)

[20]

QUESTION 4

4.1 Five pairs of values for the variables x and y are given in the table below.

x	1	m	$m + 1$	4	5
y	5	$t - 1$	4	3	t

Given that $\Sigma x^2 = 55$ and $\bar{y} = 3$

(a) Show that $m = t = 2$. (8)

(b) Hence, determine the correlation coefficient for the set of data. (1)

(c) (i) Find the equation of the least squares regression line. (3)

(ii) Estimate the y value if $x = 6$ and comment on the reliability of this estimation. (3)

4.2 The number of hours Basi spends on her new business in a week is normally distributed with a standard deviation of 4,8 hours. In the past, the average working hours per week were 49,5 hours. Due to a change in her schedule, Basi wishes to test whether the average working hours per week has decreased. She chooses a random sample of 40 weeks and notes the total number of hours she spent on her business during these weeks is 1 920 hours.

Test Basi's hypothesis at the 7% level of significance. Ensure you state a conclusion based on your test. (10)

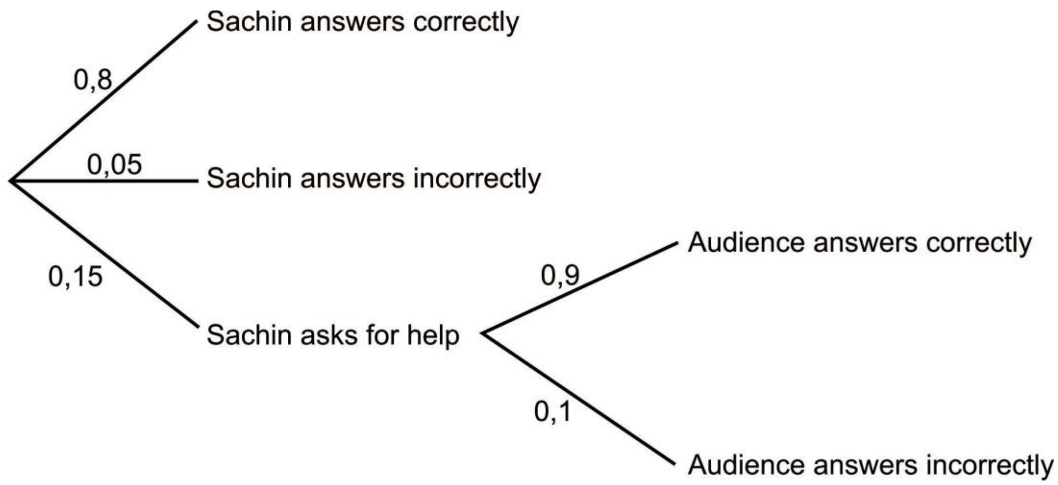
[25]

QUESTION 5

Sachin is attempting the *Who Wants To Be A Millionaire* TV quiz. He has to answer questions one after another. The quiz ends when a question is answered incorrectly.

- The probability that Sachin, himself, gives the correct answer to any question is 0,8.
- The probability that Sachin, himself, gives a wrong answer to any question is 0,05.
- The probability that Sachin decides to ask for help for any question is 0,15.

On the first occasion that Sachin decides to ask for help, he asks the audience. The probability that the audience gives the correct answer to any question is 0,9. The information is shown in the tree diagram below.



5.1 Find the probability that the first question asked is correctly answered. (4)

5.2 On the second occasion that Sachin decides to ask for help, he phones a friend. The probability that his friend gives the correct answer to any question is 0,7.

Looking at all the possible options, calculate the probability of getting the first two questions correct. (8)

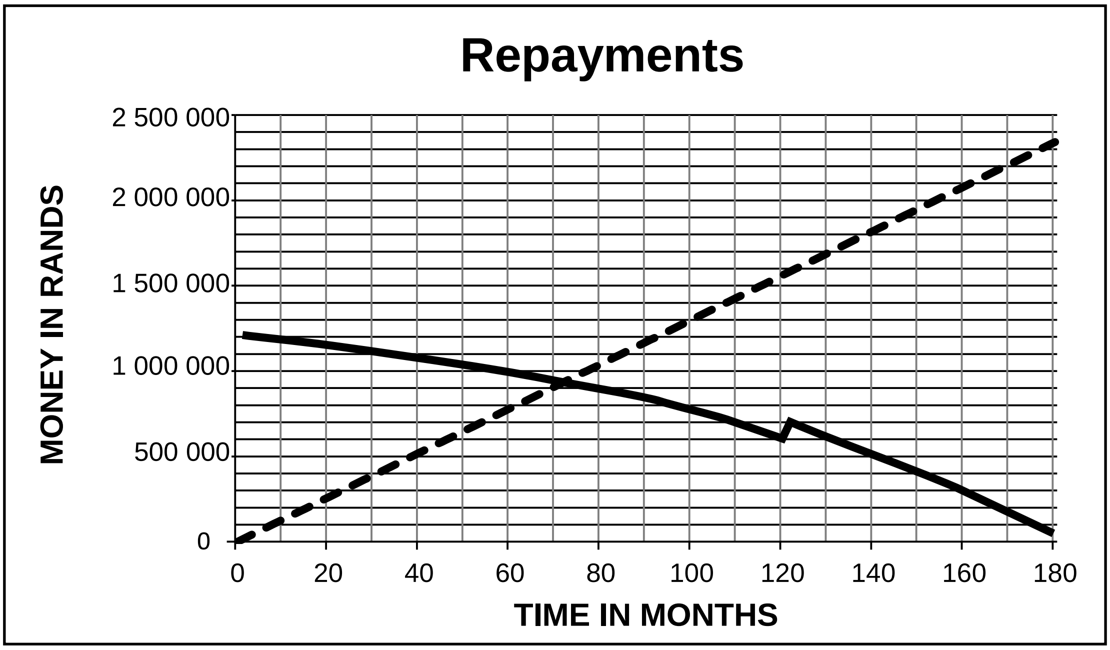
[12]

Total for Module 2: 100 marks

MODULE 3 FINANCE AND MODELLING

QUESTION 1

Ntsiko has taken out a home loan. In the graph below, the solid curve represents her outstanding balance on the loan over the agreed time period. The accumulation of her corresponding equal monthly payments over the same time period is represented by the broken line.



1.1 Write down the value of Ntsiko's initial home loan. (1)

1.2 After how many monthly payments is R1 000 000 of Ntsiko's loan still outstanding? (1)

1.3 Approximately how much interest will Ntsiko have paid by the time she has amortised the loan? (2)

1.4 What is the balance outstanding, when she has paid as much as she still owes? (2)

1.5 At 120 months, clearly the initial conditions of the loan were changed.

(a) How do we know Ntsiko's monthly payment was **not** changed? (2)

(b) What was changed? And how was it changed? (2)

[10]

QUESTION 2

Jude obtained a loan at an annual interest rate of 5,68%, compounded monthly. He will amortise the loan with equal monthly instalments of R5 154,26, starting in three months' time and completing the repayment within three years of the date of the loan being granted.

2.1 Calculate the value of the loan, correct to the nearest rand. (6)

2.2 Calculate the balance outstanding on the loan, two years after it was approved. (5)

2.3 Calculate how much interest Jude paid during the second year, if R116 674,09 was the balance outstanding, one year after the loan was approved. (5)

[16]

QUESTION 3

Eight years ago a company bought office equipment for R3 400 000. The equipment is now valued at R2 000 000. This figure is calculated using reducing-balance depreciation, and takes into account the increased value of the equipment due to inflation.

3.1 Calculate the depreciated value of the equipment, **without** inflation being taken into account. Assume that the rate of inflation has been constant at 6,8% per annum, compounded annually. (4)

3.2 Calculate the annual rate of reducing-balance depreciation on the value of the equipment (that is, **without** inflation being taken into account), as a percentage, correct to two decimals. (4)

3.3 The company decides to create a sinking fund (to replace the equipment) that accrues to R5 500 000 over the next six years. **Bi-annual** payments (that is, every six months) will be made into the fund, starting immediately and ending two years before the fund matures. A once-off withdrawal of R300 000, for maintenance of the current equipment is to be made three years after the fund is started. Calculate the value of the bi-annual payments, if the fund earns interest at a rate of 7,64% per annum, compounded **monthly**. (18)

[26]

QUESTION 4

In the Augrabies Game Reserve (Northern Cape), quiver trees are self-propagating with about 5% of new trees surviving each year. However, game rangers have noted that the older trees are dying at a rate of 50 per year. There are currently about 6 500 quiver trees in the reserve.

Rangers found that aardvarks were scratching themselves against the older trees, thus damaging the bark, making the trees more prone to infection. There are currently about 1 300 aardvarks in the reserve, with no real predators.

The rangers have calculated that, as long as the aardvark population remains less than 25% of the quiver tree population, no permanent ecological damage will be done. The table below incompletely records the population of quiver trees and aardvarks:

	Quiver trees	**Aardvarks**
P_0	6 500	1 300
P_1	6 775	1 396
P_2	7 064	1 500
P_3	7 367	1 610
P_4	7 685	1 730
P_5	**A**	1 858
P_6	**B**	1 995
P_7	**C**	2 143
P_8	**D**	2 301
P_9	**E**	2 472
P_{10}	**F**	2 655

4.1 Using information from the table, calculate the annual intrinsic growth rate for the aardvarks, as a percentage, correct to one decimal place. (4)

4.2 Give a recursive formula that models the quiver tree population. (4)

4.3 Complete the table by calculating the values A to F for the quiver tree population. Round off to the nearest whole value. (3)

4.4 According to these figures, during which year will the aardvarks start causing permanent ecological damage to the quiver tree population? (2)

4.5 From the given aardvark population figures, explain how it is evident that a Malthusian model, rather than a Logistic model, has been used. (3)

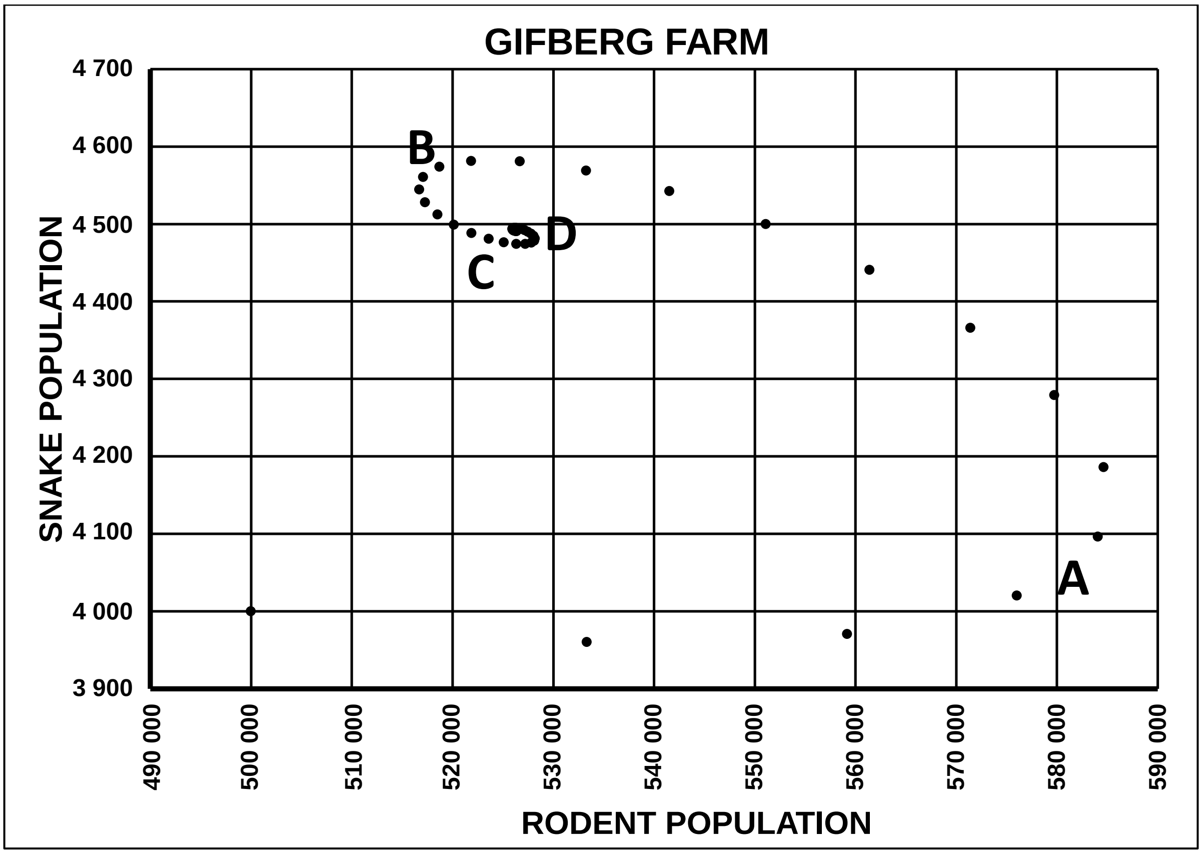
[16]

QUESTION 5

Gifberg is a popular hikers' destination in the isolated mountains near Van Rhynsdorp, in the Western Cape. The phase plot below records the populations of snakes and rodents at one-year intervals over a century, according to the Lotka-Volterra model.

Two out of every three rodents are female, and each bears a litter of about eight young every four months. The survival rate of the young is rather low at 5% per annum. The population of rodents is estimated to be about 500 000 and 533 300 in Year 0 and Year 1 respectively.

The life expectancy of snakes is five years, and their initial population is about 4 000.



5.1 Use the phase plot to answer the following questions:

(a) Read off the equilibrium points for the predator and prey populations. (2)

(b) Give the snake population when the rodent population is at its maximum. (2)

(c) Give the region (A, B, C or D) on the phase plot where both populations are decreasing. (2)

5.2 If 760 snakes were born in the first year, calculate the snake population at the end of the first year. (5)

5.3 Calculate the annual intrinsic growth rate of the rodents. (5)

5.4 About 40% of the rodent population was killed within the first year. Calculate, to the nearest thousand, the carrying capacity of the farm for rodents. (6)

[22]

QUESTION 6

Theos was a Greek mathematician of the second century, BC. He dabbled with irrational numbers and came up with some fascinating conjectures, among them:

Conjecture 1: If $T_n = \dfrac{a}{b}$ approximates $\sqrt{2}$, then $\dfrac{a+2b}{a+b}$ is a better approximation.

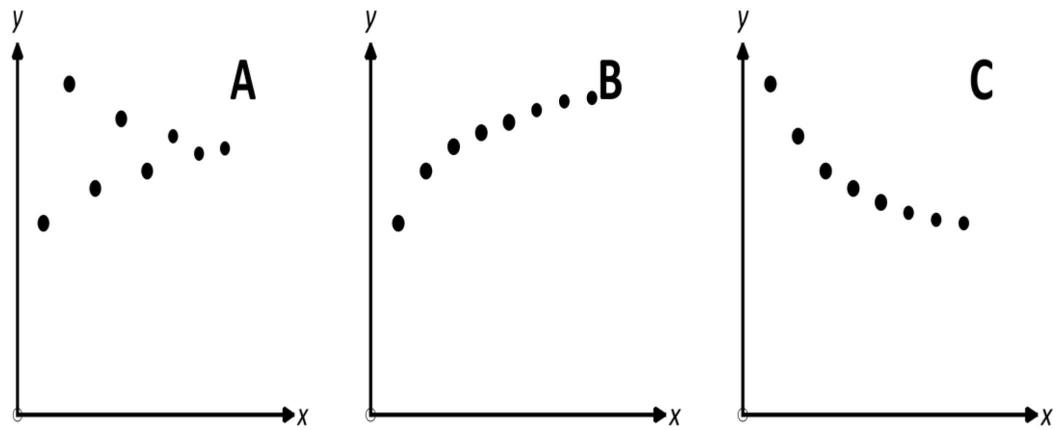
Conjecture 2: If $T_n = \dfrac{a}{b}$ approximates $\sqrt{3}$, then $\dfrac{2a+3b}{a+2b}$ is a better approximation.

6.1 Theos wrote Conjecture 1 as a first order recursive formula: $T_{n+1} = 1 + \dfrac{1}{1+T_n}$

Conjecture 1: $\sqrt{2} \approx 1{,}414\,214$ (correct to six decimal places)

(a) Using T_0 = 1,5: Which iteration is the first to accurately approximate the value of $\sqrt{2}$ correct to six decimal places? (2)

(b) Which graph below best describes the iteration process as it converges to $\sqrt{2}$?



(2)

6.2 Design a first order recursive formula for Theos to generate the value of $\sqrt{3}$. (6)

[10]

Total for Module 3: 100 marks

MODULE 4 MATRICES AND GRAPH THEORY

QUESTION 1

1.1 Consider the matrix equation $\begin{pmatrix} k & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 \\ 3 & 0 & 2 \\ 0 & 2 & 1 \end{pmatrix}\begin{pmatrix} k \\ 3 \\ 1 \end{pmatrix} = M.$

(a) Give the dimension of matrix *M*. (2)

(b) Calculate the value of *k,* if *M* is the zero matrix. Show relevant working. (8)

1.2 Three 3 × 3 matrices are given, such that: det(A) = *p*, det(B) = *q*, det(C) = *r.*
Express the following as a numerical value, or in terms of *p, q* and/or *r*:

(a) $\det(B.B^{-1})$ (2)

(b) det(A.C) (2)

(c) det(3A) (2)

(d) $\det(AB^{-1}C^{T})$ (2)

[18]

QUESTION 2

Draco (D), Jinny (J), Luna (L) and Neville (N) have each been given a different set of three equations to solve simultaneously. By using Gaussian reduction, they obtain the following augmented matrices:

$$D = \begin{pmatrix} 1 & -2 & 0 & 4 \\ 0 & 2 & 5 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad J = \begin{pmatrix} 1 & 2 & 3 & -2 \\ 0 & -1 & 3 & 2 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$L = \begin{pmatrix} 1 & -2 & 0 & 4 \\ 0 & 1 & 4 & -2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad N = \begin{pmatrix} 1 & -3 & 6 & 4 \\ 0 & 3 & -1 & 1 \\ 0 & 2 & 2 & -18 \end{pmatrix}$$

2.1 Neville realises that his set of equations will produce a unique solution. Complete his row reduction, and state the solution to his set of simultaneous equations. (6)

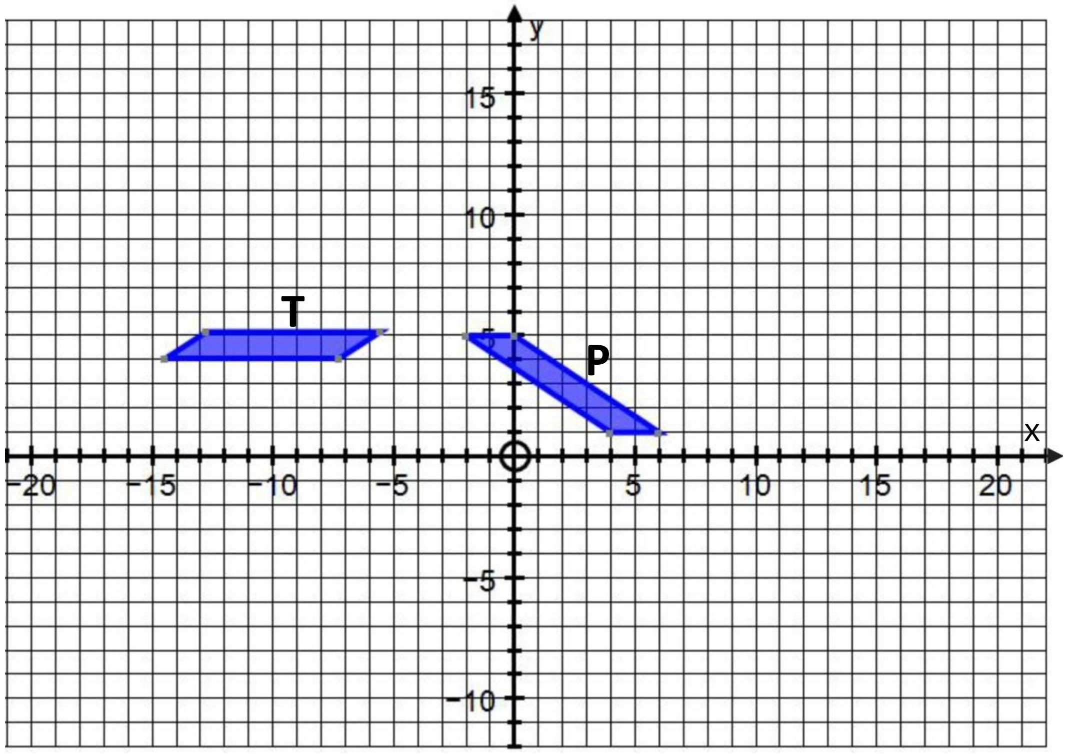
2.2 Whose set of equations produces no solutions? Justify your choice. (3)

2.3 Whose set of equations produces infinite solutions? Justify your choice. (3)

[12]

QUESTION 3

In the sketch below, P and T are two parallelograms in a Cartesian plane.
The coordinates of the vertices of P are (–2; 5), (0; 5), (6; 1) and (4; 1).



3.1 Sketch and label the following transformations of P on the **ANSWER SHEET**:

(a) A, which is P transformed by the matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$. (3)

(b) R, which is P transformed by the matrix $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$. (3)

(c) M, which is P transformed by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix}$. (4)

(d) S, which is P transformed by the matrix $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$. (4)

3.2 T is an image of P, after P has been rotated about the origin through an acute angle of ϴ, and then translated 10 units left and 1 unit up. The point on P with coordinates (6; 1) now lies on T at (–5,55; 5,15). Calculate the angle ϴ through which P was rotated, correct to one decimal place. (12)

[26]

QUESTION 4

The four adjacency matrices *P, Q, R* and *S* given below represent four graphs.
A '1' in the matrix indicates the two vertices are directly connected to each other by an edge.
A '0' indicates the two vertices are not directly connected to each other by an edge.

P =

	A	**B**	**C**	**D**	**E**
A	0	1	1	1	0
B	1	0	0	1	1
C	1	0	0	0	1
D	1	1	0	0	0
E	0	1	1	0	0

Q =

	A	**B**	**C**	**D**	**E**
A	0	1	0	1	0
B	1	0	0	1	1
C	0	0	0	0	1
D	1	1	0	0	0
E	0	0	0	0	0

R =

	A	**B**	**C**	**D**	**E**
A	1	1	0	1	0
B	1	1	0	1	1
C	0	0	1	0	1
D	1	1	0	1	0
E	0	0	0	0	1

S =

	A	**B**	**C**	**D**	**E**
A	0	0	0	1	1
B	0	0	1	0	0
C	0	1	0	0	0
D	1	0	0	0	1
E	1	0	0	1	0

4.1 How many edges does Graph P have? (2)

4.2 How do we know from the adjacency matrix that Graph Q is a directed graph? (2)

4.3 How do we know from the adjacency matrix that Graph R is a multi-graph? (2)

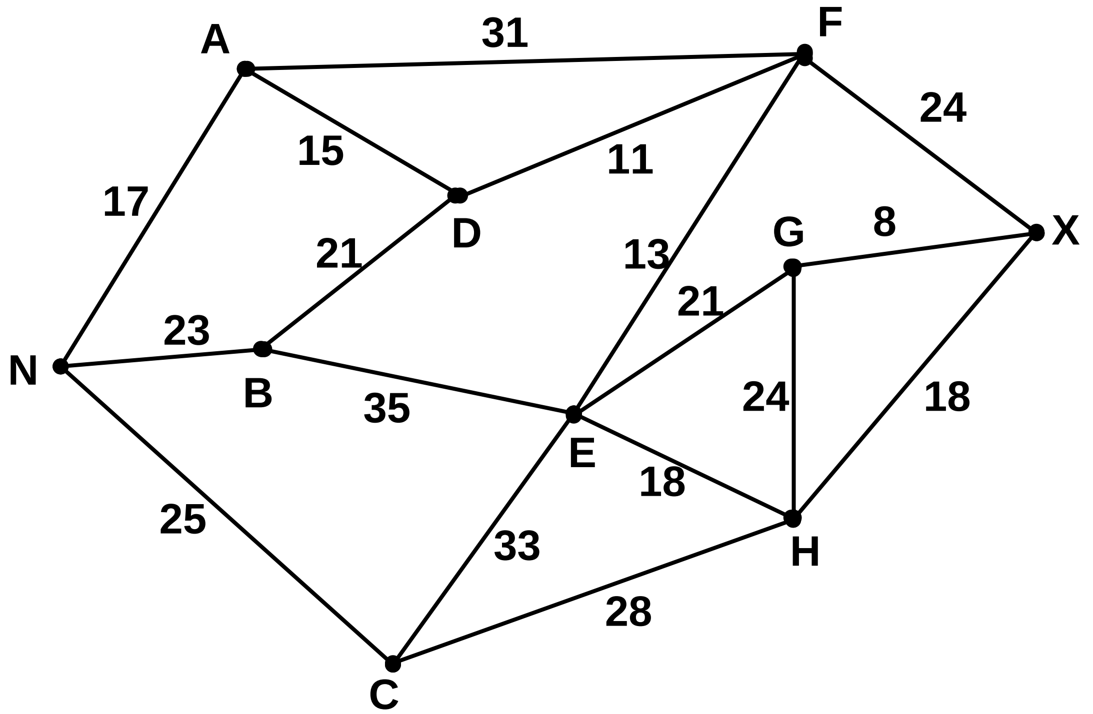
4.4 Draw Graph S to show that it is **not** a connected graph. (4)

[10]

QUESTION 5

Cave diving has become a popular sport, especially in provinces that are not close to the sea. Divac decides to visit the Komati Springs in Mpumalanga, with its eight submerged caves.

In the graph below, vertices A to H represent the eight caves, with the entrance and exit to the network indicated as N and X respectively. The weight of the edges represents the time taken in minutes for an average diver to swim from one cave to the next, allowing for some exploration time at each cave visited.



5.1 Divac wants to estimate the minimum time it will take to visit all eight caves, starting from when he enters the network until he exits it. Use **Prim's algorithm** to determine a lower bound, starting at N, by initially leaving out E. (10)

5.2 Answer this question on the **ANSWER SHEET**, showing evidence of your thought processes.

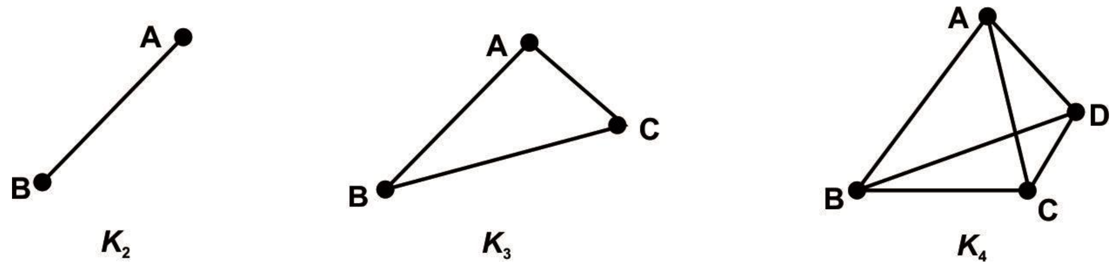
Divac can only take enough oxygen for 90 minutes, but wants to visit at least half the caves. Use **Dijkstra's algorithm** to find an optimal time route for him, starting at N and visiting at least half the caves before ending at X. Clearly state the optimal route, as well as its time. (12)

[22]

QUESTION 6

In a **complete** graph every vertex is directly connected to every other vertex. A complete graph is notated by K_n, where n is the number of vertices in the graph.

Some examples of complete graphs are given below:



6.1 Answer in terms of n:

(a) State the degree of K_n. (2)

(b) State the sum of the degrees of the vertices of K_n. (2)

6.2 Starting at A, it can be seen that:

K_2 has one Hamiltonian circuit, namely, A B A.
K_3 has two Hamiltonian circuits, namely, A B C A and A C B A.
K_4 has six Hamiltonian circuits, two of which are A B C D A and A D C B A.

(a) Name the remaining four Hamiltonian circuits in K_4. (5)

(b) If it is known that K_5 and K_6 have 24 and 120 Hamiltonian circuits respectively, express the number of Hamiltonian circuits of the complete graph K_n in terms of n. (3)

[12]

Total for Module 4: 100 marks