Answers to:

Mathematics
IEB 2017 Paper 2

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QUESTION 1

How to use a Casio calculator for Regression modelling

Press:

MODE → 3:STAT → 2: A + Bx

Enter data into the x and y columns

Press: AC

To find A:

SHIFT → 1 → 5:Reg → 1:A → =

To find B:

SHIFT → 1 → 5:Reg → 2:B → =

To find r (correlation coefficient)

SHIFT → 1 → 5:Reg → 3:r → =

To find $\hat{y}$ given $\hat{x}$:

Enter $\hat{x}$ - value → SHIFT → 1 → 5:Reg → 5: $\hat{y}$ → =

To find the mean point $(\bar{x}; \bar{y})$

SHIFT → 1 → 4:Var → 2: $\bar{x}$ → =

SHIFT → 1 → 4:Var → 5: $\bar{y}$ → =

How to use a Casio calculator to find Mean and Standard Deviation

Press:

MODE → 3:STAT → 1: 1 - VAR

Enter data into the x and FREQ columns

If **no** FREQ column then PRESS:

SHIFT → SET UP → page down → 4: STAT → 1: ON

Press: AC: →

To find the mean:

SHIFT → 1 → 4: Var → 2: $\bar{x}$

To find the standard deviation:

SHIFT → 1 → 4: Var → 3: σx

Remember: $variance = (\sigma x)^2$

NB: It is important that you are familiar with using your calculator for statistics questions such as regression modelling, finding means/variances, etc. We have attached above a step-by-step instruction guide on how to use your Casio calculator to compute these statistical operations.

a. $r = 0{,}7337$

b. C

c. $A = 0{,}6268$ and $B = 0{,}0264$

d. No, we should not use the regression line for a 30 day rest day, as this would be extrapolating data (since our given data values are relatively low and 30 is too high)

QUESTION 2

a. $m_{OA} = \frac{y_A - y_O}{x_A - x_O}$

$= \frac{4-0}{2-0}$

$= \frac{4}{2}$

$= 2$

and $\tan A\hat{O}B = m_{OA}$

$\therefore \tan A\hat{O}B = 2$

$\therefore A\hat{O}B = 63{,}43°$

b. To find the equation of the perpendicular bisector of line OA, we first need to see where the midpoint of line OA is (so that the perpendicular line cutting OA will bisect). Hence,

$$midpt(OA) = \left(\frac{x_O + x_A}{2}; \frac{y_O + y_A}{2}\right)$$
$$= (1; 2).$$

Now, for the line to be perpendicular to line OA, we must have:

$m_{\perp} = -\frac{1}{m_{OA}}$

$= -\frac{1}{2}$

$\therefore y = -\frac{1}{2}x + c$. Now sub. $(1; 2)$

$\therefore 2 = -\frac{1}{2}(1) + c$

$\therefore c = \frac{5}{2}$

$\therefore y = -\frac{1}{2}x + \frac{5}{2}$

c. We proceed in a similar manner to find the perpendicular bisector of OB:

$$midpt(OB) = \left(\frac{x_O+x_B}{2}; \frac{y_O+y_B}{2}\right)$$
$$= (3; 0)$$

Note that line OB is just a straight line on the x −axis, so the perpendicular bisector will be a vertical straight line passing through $x = 3$. Hence the equation of the perpendicular bisector is $x = 3$.

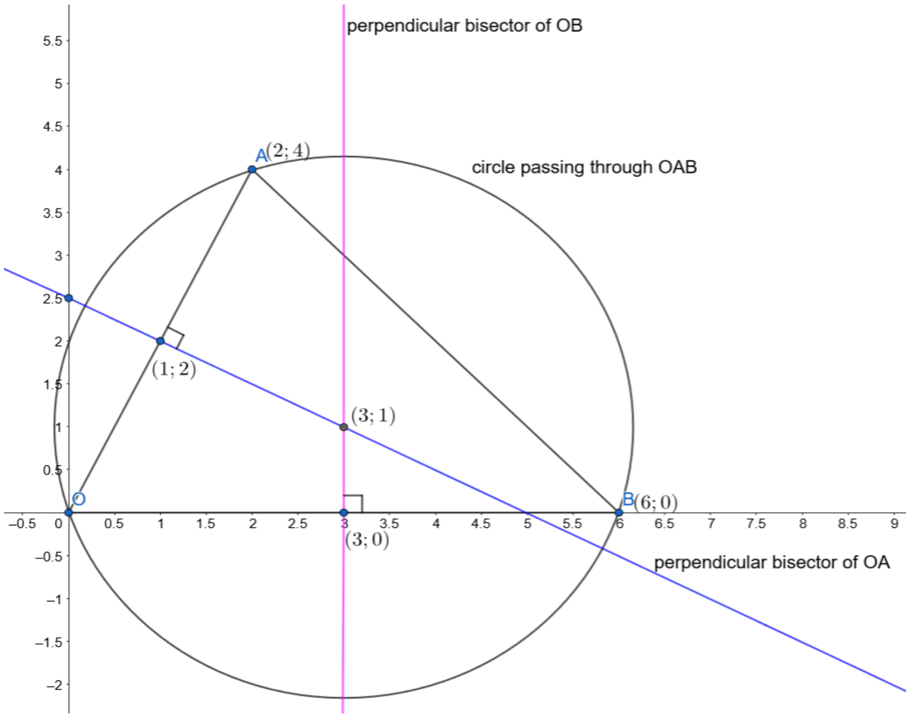
d. To get the equation of the circle passing through points O, A and B, we use the two perpendicular bisectors to get the centre of the circle (we do this because of the important theorem, i.e., **The line segment joining the centre of a circle to the midpoint of a chord is perpendicular to the chord**), i.e., equate $x = 3$ and $y = -\frac{1}{2}x + \frac{5}{2}$. So, we get:

$$y = -\frac{1}{2}(3) + \frac{5}{2}$$
$$\therefore y = 1$$

Hence the centre of the circle is the point $(3; 1)$. We can now sub. any point on the circle to complete the equation $(x-3)^2 + (y-1)^2 = r^2$. For convenience, we will sub. point $O(0; 0)$.

Thus: $(0-3)^2 + (0-1)^2 = r^2$

$$\therefore r^2 = 10$$
$$\therefore (x-3)^2 + (y-1)^2 = 10$$



NB: The diagram above is just to help you visualise the solutions better, it is not required for you to draw for marks but it is always super helpful to visualise.

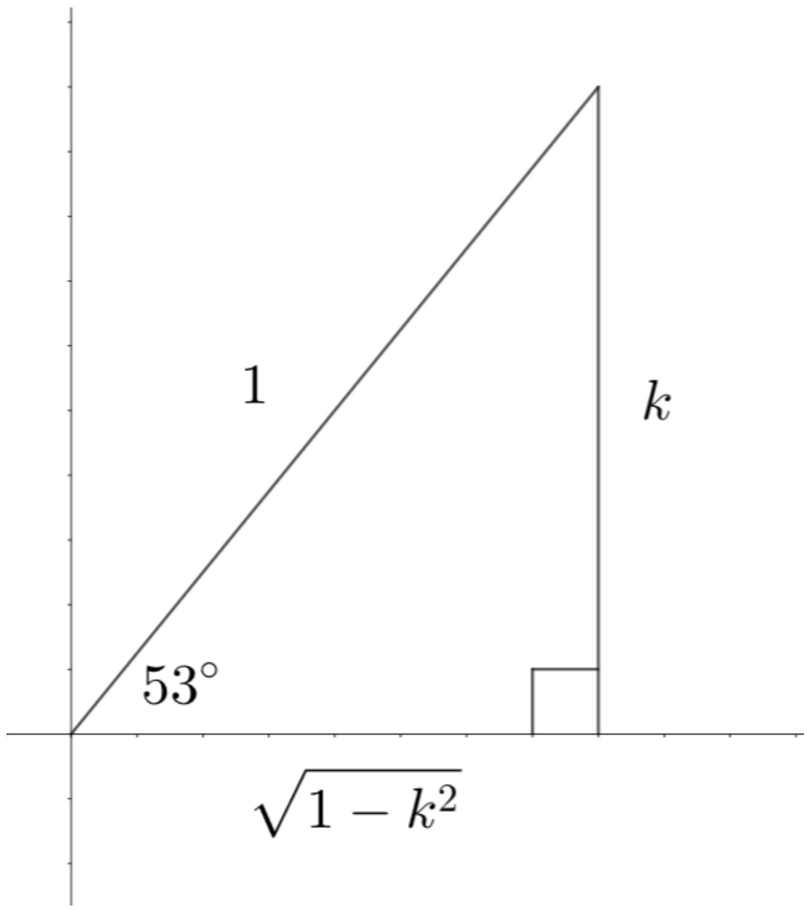
QUESTION 3

a.

1. $$\begin{aligned}\sin(53^\circ) &= \sin(31^\circ + 22^\circ)\\ &= \sin(31^\circ)\cos(22^\circ) + \cos(31^\circ)\sin(22^\circ)\\ &= k\end{aligned}$$

2. $$\begin{aligned}\cos(143^\circ) &= \cos(90^\circ + 53^\circ)\\ &= -\sin(53^\circ)\\ &= -k\end{aligned}$$

3. $$\begin{aligned}\sin(75^\circ)\sin(22^\circ) + \cos(75^\circ)\cos(22^\circ) &= \cos(75^\circ - 22^\circ)\\ &= \cos(53^\circ)\\ &= \sqrt{1-k^2}\end{aligned}$$



b. R.T.P: $\frac{\cos\theta}{\sin 2\theta} - \frac{\cos 2\theta}{2\sin\theta} = \sin\theta$

Proof:

$$
\begin{aligned}
\text{LHS} &= \frac{\cos\theta}{\sin 2\theta} - \frac{\cos 2\theta}{2\sin\theta} \\
&= \frac{\cos\theta}{2\sin\theta\cos\theta} - \frac{\cos^2\theta - \sin^2\theta}{2\sin\theta} \\
&= \frac{1}{2\sin\theta} - \frac{\cos^2\theta - \sin^2\theta}{2\sin\theta} \\
&= \frac{1-(\cos^2\theta - \sin^2\theta)}{2\sin\theta} \\
&= \frac{1-\cos^2\theta + \sin^2\theta}{2\sin\theta} \\
&= \frac{\sin^2\theta + \sin^2\theta}{2\sin\theta} \qquad (\text{Since } 1-\cos^2\theta = \sin^2\theta) \\
&= \frac{2\sin^2\theta}{2\sin\theta} \\
&= \sin\theta
\end{aligned}
$$

$\therefore$ LHS = RHS ■

c.

$$
\begin{aligned}
3\sin^2\theta - 2\sin\theta &= 0 \\
\therefore \sin\theta(3\sin\theta - 2) &= 0 \\
\therefore \sin\theta = 0 \text{ or } 3\sin\theta - 2 &= 0 \\
\therefore \sin\theta = 0 \text{ or } \sin\theta &= \frac{2}{3}
\end{aligned}
$$

For $\sin\theta = 0 : \theta = 0° + k.360°, k \in \mathbb{Z}$ or $\theta = 180° + k.360°, k \in \mathbb{Z}$

For $\sin\theta = \frac{2}{3} : \theta = 41{,}8° + k.360°, k \in \mathbb{Z}$ or $\theta = 138{,}2° + k.360°, k \in \mathbb{Z}$

QUESTION 4

a. $M(3; -1)$

b. Since point C lies on the y-axis, we sub $x = 0$ into the circle equation:

$$
\begin{aligned}
\therefore (0-3)^2 + (y+1)^2 &= 25 \\
\therefore y^2 + 2y + 1 + 9 - 25 &= 0 \\
\therefore y^2 + 2y - 15 &= 0 \\
\therefore (y+5)(y-3) &= 0 \\
\therefore y = -5 \text{ or } y &= 3
\end{aligned}
$$

Now, point C lies above the x-axis, hence $C(0; 3)$.

c. Firstly, we have $m_{CM} = \frac{y_C - y_M}{x_C - x_M}$

$$= \frac{3-(-1)}{0-3}$$

$$= -\frac{4}{3}$$

Therefore, $m_{AC} = -\frac{1}{m_{CM}}$

$$= \frac{3}{4}$$

Hence $y = \frac{3}{4}x + c$. Sub the point $C(0; 3)$.

$\therefore y = \frac{3}{4}x + 3$

d. To determine the length of AB, we first need the co-ordinates of A and B.

For point A, we have: Sub $y = 0$ into $y = \frac{3}{4}x + 3$. Then:

$\frac{3}{4}x + 3 = 0$

$\therefore x = -4$

Hence we have $A(-4; 0)$.

Now, for point B, we have: Sub $y = 0$ into $(x-3)^2 + (y+1)^2 = 25$. Then:

$(x-3)^2 + (0+1)^2 = 25$

$\therefore (x-3)^2 = 24$

$\therefore x - 3 = \pm\sqrt{24}$

$\therefore x = 3 \pm \sqrt{24}$ (i.e. $x = 7{,}9$ or $x = -1{,}9$)

Now, from our diagram it follows that: $B(-1{,}9; 0)$.

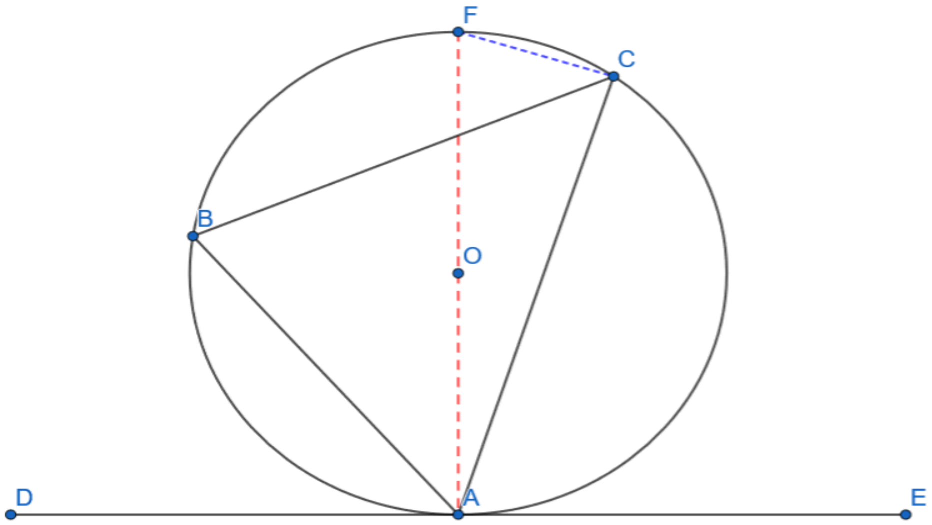
Thus $dist(AB) = 4 - 1{,}9$

$= 2{,}1$ units.

QUESTION 5

a. R.T.P: $C\hat{A}E = A\hat{B}C$

Construction: We draw diameter AOF and chord FC.



Proof:

We have: $O\hat{A}C + C\hat{A}E = 90°$ (Tangent is ⊥ to line through centre)

$F\hat{C}A = 90°$ (Angles in a semi-circle)

$O\hat{F}C + O\hat{A}C = 90°$ (Angles in ΔAFC)

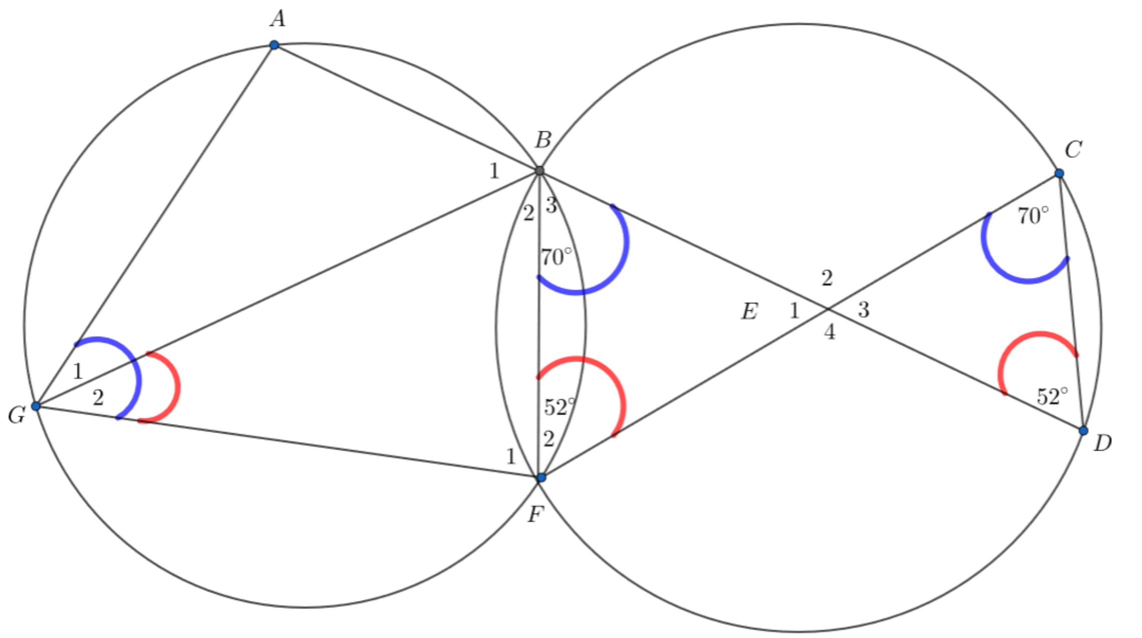
Therefore

$O\hat{F}C = C\hat{A}E$ (Since $O\hat{F}C + O\hat{A}C = C\hat{A}E + O\hat{A}C$)

However $O\hat{F}C = A\hat{B}C$ (Angles in the same segment AC)

Hence $C\hat{A}E = A\hat{B}C$ ■

b.



NB: In Euclidean Geometry, it is always super helpful to re-draw any given diagrams and fill in as much information you can that is given to you in the question.

Now, we have:

$\hat{B}_3 = 70°$ (Angles in the same segment FD

$\hat{F}_2 = 52°$ (Angles in the same segment FD)

$\hat{B}_3 = \hat{G}_1 + \hat{G}_2$ (Ext. angle of a cyclic quad $GABF$ = Int. opp. angle)

$\therefore \hat{G}_1 + \hat{G}_2 = 70°$

$\therefore \hat{F}_2 = \hat{G}_2 = 52°$ (tan-chord theorem)

Hence $\hat{G}_1 = 70° - \hat{G}_2 = 18°$

QUESTION 6

a. $A = 50$

b. 400 (see your ogive curve)

c. $P = 50$ and $M = 100$ (read off your ogive to see this)

d. We have $300 \leq Q_3 \leq 325$ (approx.) and $Q_1 = 200$. Then IQR ≈ 110

e. $\bar{x} = 250$

f.

1. This would not affect the median as it’s only the upper 25% of data that would be affected.

2. The difference between the mean and the data would decrease, hence the standard deviation would also decrease (since they depend on each other)

3. This would skew the data to the left since we would have mean < median.

SECTION B

QUESTION 7

a. Firstly we have that $m_{OA} = \frac{y_O - y_A}{x_O - x_A}$

$$= \frac{0-6}{0-2}$$
$$= 3$$

Now, the equation of line OA is given by: $y = 3x + c$. Sub point $(0;0)$. Therefore $y = 3x$. The equation of line EF is given by: $2y + x = 10$, i.e., $y = -\frac{1}{2}x + 5$. Now, point B is a point of intersection, thus:

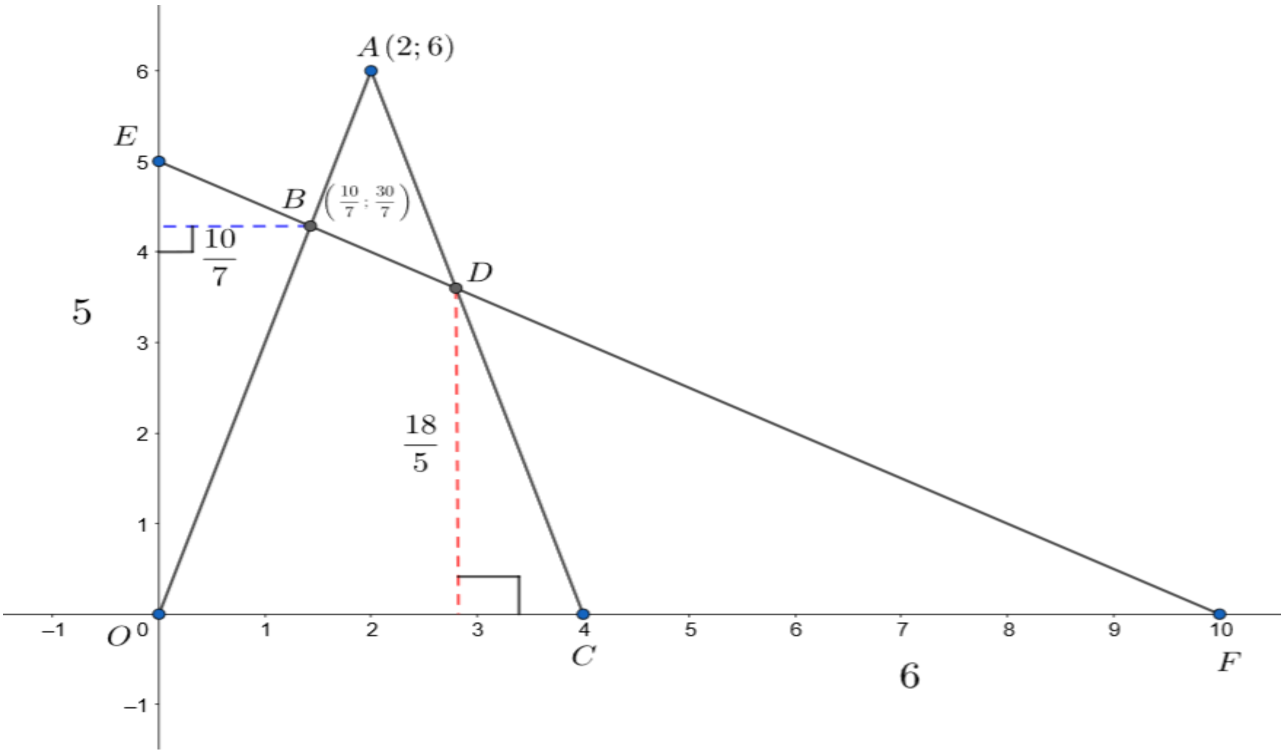
$$-\frac{1}{2}x + 5 = 3x$$
$$\therefore \frac{7}{2}x = 5$$
$$\therefore x = \frac{10}{7}$$

Hence point $B\left(\frac{10}{7};\frac{30}{7}\right)$.

Now, to calculate the area of ΔEBO, we need the base and height. We can see from our diagram that the height of ΔEBO is $\frac{10}{7}$ and the base is the length of line OE. Hence, to get point E, sub $x = 0$ into $2y + x = 10$. Hence $y = 5$. So $E(0;5)$.

Thus:

$$\text{Area of } \Delta EBO = \frac{1}{2} \times 5 \times \frac{10}{7}$$
$$= \frac{25}{7} \text{ units}^2$$



b. We are given that $D\left(x;\frac{18}{5}\right)$. Hence:

Area of $\Delta DCF = \frac{1}{2} \times CF \times \perp$height.

Note: The perpendicular height is $\frac{18}{5}$. Thus we have:

$$\text{Area of } \Delta DCF = \frac{1}{2} \times 6 \times \frac{18}{5}$$
$$= \frac{54}{5} \text{ units}^2$$

(See diagram on pg. 11)

QUESTION 8

a.

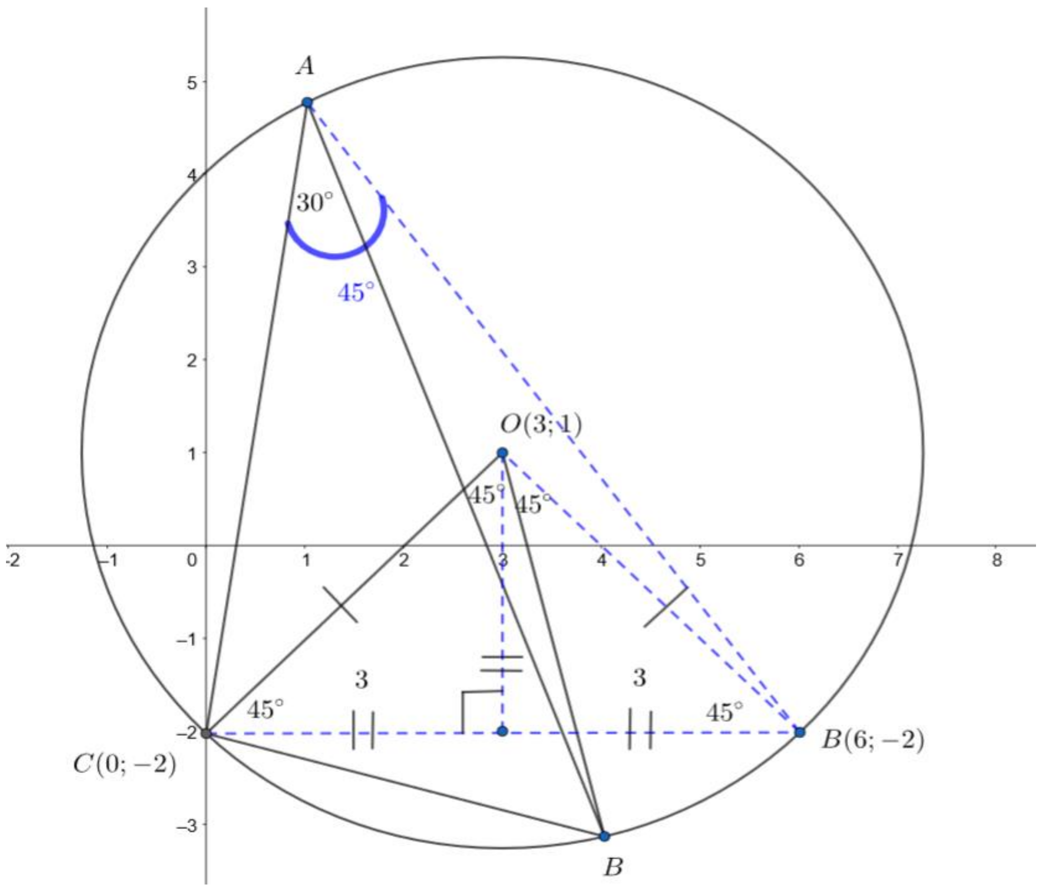
1. The length of OC is given by:

$$dist(OC) = \sqrt{(x_O - x_C)^2 + (y_O - y_C)^2}$$
$$= \sqrt{(0-3)^2 + (-2-1)^2}$$
$$= \sqrt{18} = 3\sqrt{2}$$

2. Note that if point B moves along the circle to be parallel to the x-axis, then we will have chord CB which is bisected by the line from the centre O, hence we get $B(6;-2)$

3. Firstly note that $m_{OC} = \frac{y_O - y_C}{x_O - x_C}$
$$= \frac{1-(-2)}{3-0}$$
$$= \frac{3}{3}$$
$$= 1$$

$\therefore O\hat{C}B = 45°$ (since line $CB \parallel x$-axis)
$\therefore C\hat{O}B = 90°$ (Angles in ΔOCB)
$\therefore C\hat{A}B = 45°$ (Angle at centre = 2x Angle at circum.)



b. We are given that point B moves **anti-clockwise** along the circle until the area of ΔOBC is $\frac{9}{2}$ units^2. We note the following:

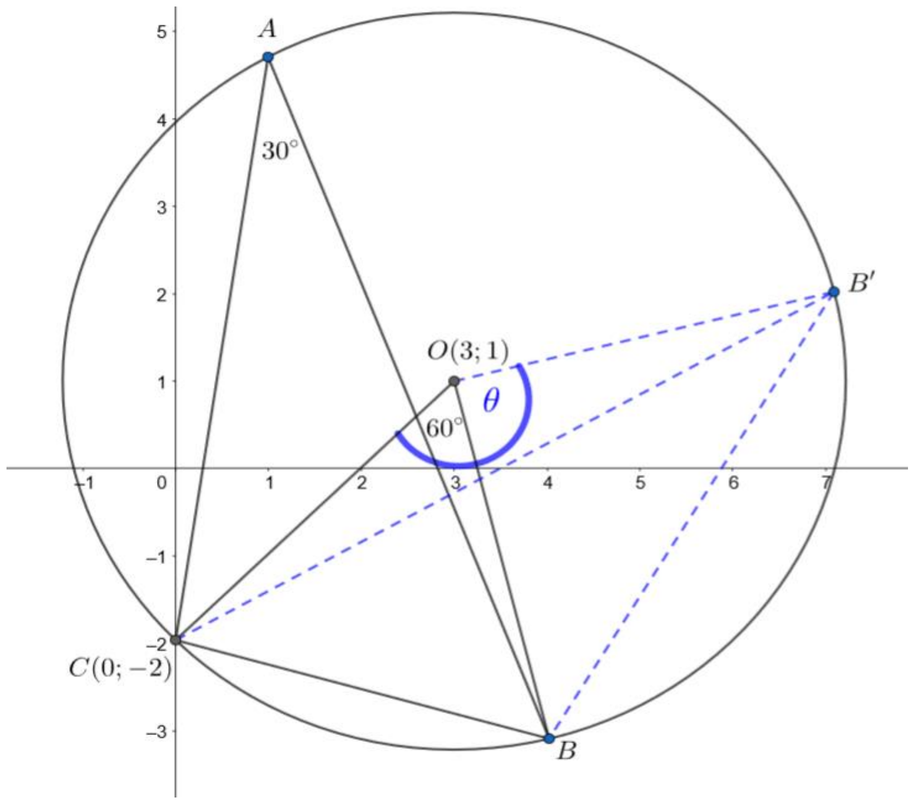
The circumference of the circle is given by: $2\pi r = 2\pi\sqrt{18}$ units.
(Note: $r = dist(OC)$).
We were also given that: $C\hat{A}B = 30°$
$\therefore C\hat{O}B = 60°$ (Angle at centre = 2x Angle at circum.)

Now, we calculate the angle θ after B moves to the new position B'.
From the sine area rule, we get:

Area of $\Delta O\widehat{B'}C = \frac{1}{2} \times OC \times OB' \times \sin\theta = \frac{9}{2}$
$\therefore \frac{1}{2} \times \sqrt{18} \times \sqrt{18} \times \sin\theta = \frac{9}{2}$
$\therefore \sin\theta = \frac{1}{2}$
$\therefore$ Key angle $= 30°$
However, we have an obtuse triangle, hence $\theta = 180° - 30° = 150°$.
Hence B needs to move $150° - 60° = 90°$ anti-clockwise.

Lastly, to calculate the **shortest distance** required for this area is given by:

$2\pi r \times \frac{(angle\ you\ are\ moving)}{360°} = 2\pi\sqrt{18} \times \frac{90°}{360°}$ units.



QUESTION 9

a. R.T.P: $\Delta ADC \mid\mid\mid \Delta DBC$

Proof:

Firstly, we have that $\hat{C}$ is a common angle

Next, $\widehat{D}_2 = \hat{A}$ (tan-chord theorem)

Lastly, $\hat{B}_2 = A\widehat{D}C$ (Angles in a Δ).

Hence we have that $\Delta ADC \mid\mid\mid \Delta DBC$ (A.A.A) ■

b. R.T.P: $AB.BC = DC^2 - BC^2$

Proof:

Firstly, we have that $\frac{DC}{BC} = \frac{AC}{DC}$ (since $\Delta ADC \mid\mid\mid \Delta DBC$)

Therefore we get: $DC^2 = AC.BC$ ...Eq(1)

However, we note that: $AC = AB + BC$

Substituting this into Eq(1) yields:

$$DC^2 = (AB + BC).BC$$
$$\therefore DC^2 = AB.BC + BC^2$$
$$\therefore AB.BC = DC^2 - BC^2$$ ■

QUESTION 10

a. R.T.P: $DL \parallel CB$

Proof:

Firstly, we have that: $A\widehat{D}L = 90°$ (Angles in a semi-circle)

$A\hat{C}B = 90°$ (Angles in a semi-circle)

Therefore $DL \parallel CB$ (via corresponding angles)

b. R.T.P: $2SD = LC$

Proof:

Firstly, we have that: $LC = LA$ (radii of the larger circle)

$SD = SL = SA$ (radii of the smaller circle)

However, we that: $LA = SA + SL$

Therefore, we get: $LC = LA$

$= SA + SL$

$= SD + SD$

$= 2SD$ ■

c. We know that $AS = SL$ and $AL = LB$ (radii) and $AB = 2AL$ and $AL = 2SL$.

Hence we get: $\frac{SL}{AB} = \frac{SL}{4SL} = \frac{1}{4}$

d. We are given that $AB = 30$ units and $\frac{BN}{NC} = \frac{7}{9}$. We use the proportionality theorem:

Note that $LB = 15$ units (since $LB = \frac{1}{2}AB$).

Let $BN = 7k$ and $NC = 9k$ for some positive integer k $\left(since\ \frac{BN}{NC} = \frac{7}{9}\right)$.

Then, we have:

$$\frac{LM}{LB} = \frac{NC}{BC}$$

$$\therefore \frac{LM}{15} = \frac{9k}{16k} \quad \text{(Note: } BC = BN + NC\text{)}$$

$$\therefore LM = \frac{9}{16} \times 15 = 8{,}44 \text{ units}$$

QUESTION 11

a.

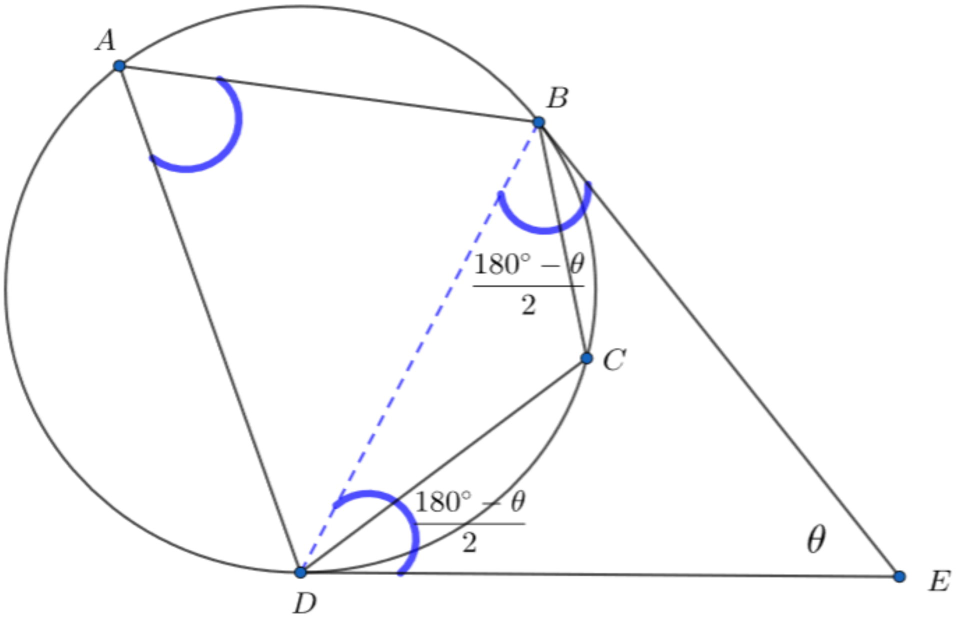
1. The length of BE is the length of the diameter – length of ED.
   Also, OA is a radius of the circle. Hence $BE = 2OA - ED$

2. R.T.P: $(2AO - ED)^2 = BC^2 - AE^2$
   Proof:
   We have that: $AE = EC$ (line from centre is $\perp$ to chord theorem)
   $BE^2 + EC^2 = BC^2$ (Pythagoras)
   $\therefore BE^2 = BC^2 - EC^2$
   $= BC^2 - AE^2$ (since $AE = EC$)
   $\therefore (2OA - ED)^2 = BC^2 - AE^2$ ■ (From part a. 1. $BE = 2OA - ED$)

b. R.T.P: $B\hat{C}D = 90° + \frac{\theta}{2}$

Proof:

We make the following construction: Draw chord DB



Now, we have: $D\hat{B}E = B\widehat{D}E$ (Tangents drawn from a common point)

Then, $D\hat{B}E + B\widehat{D}E + B\hat{E}D = 180°$ (Angles in a Δ)

$\therefore 2D\hat{B}E + \theta = 180°$ (since $D\hat{B}E = B\widehat{D}E$)

$\therefore D\hat{B}E = \frac{180°-\theta}{2} = B\widehat{D}E$

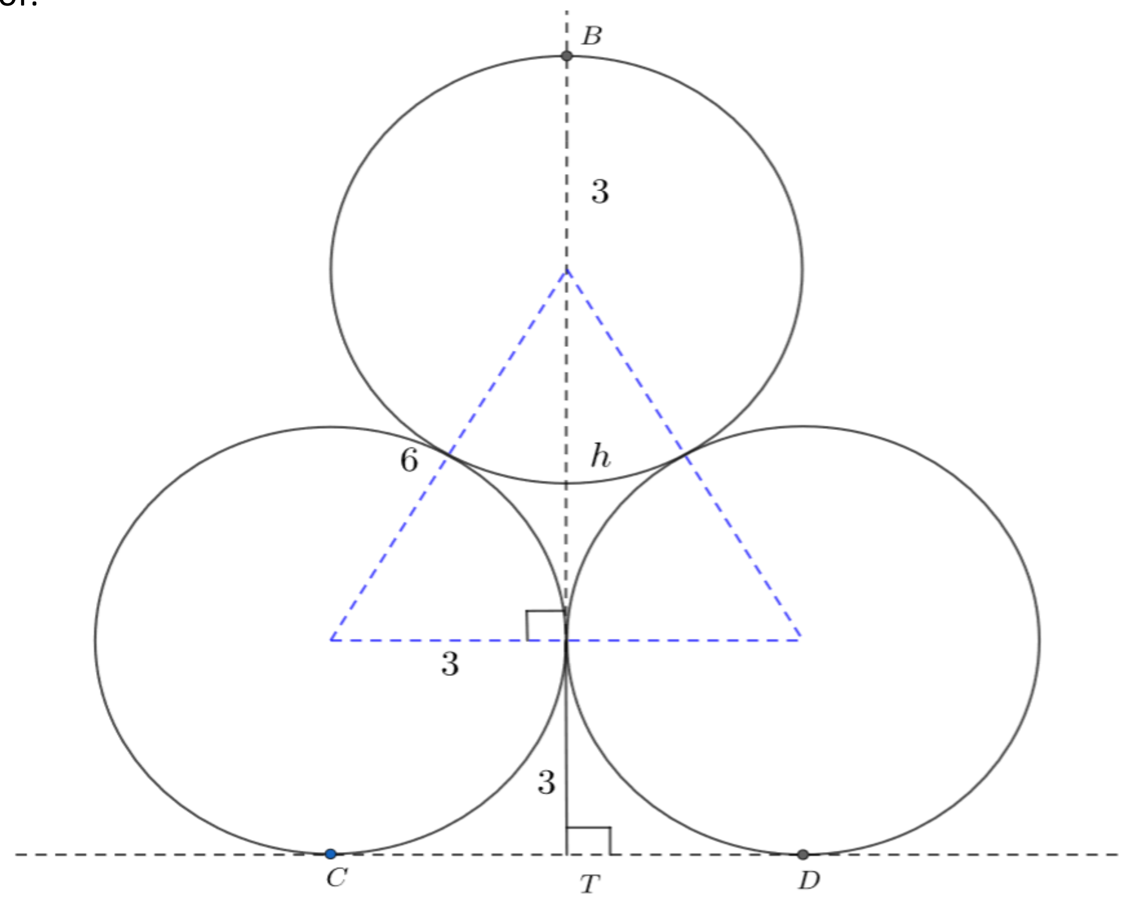
Also, we have: $\hat{A} = B\widehat{D}E = \frac{180°-\theta}{2}$ (tan-chord theorem)

$\hat{A} + \hat{C} = 180°$ (Opp. angles of a cyclic quad are supp.)

$$\begin{aligned}
\therefore \hat{C} &= 180° - \hat{A} \\
&= 180° - \frac{180°-\theta}{2} \\
&= \frac{360°-180°+\theta}{2} \\
&= \frac{180°+\theta}{2} \\
&= 90° + \frac{\theta}{2} \ \blacksquare
\end{aligned}$$

QUESTION 12

a. R.T.P: $BT = 3\sqrt{3} + 6$
Proof:



To calculate the length of BT, we see that: $BT = 3 + h + 3 = 6 + h.$
To solve for h, we use Pythagoras' theorem: $h^2 + 3^2 = 6^2$

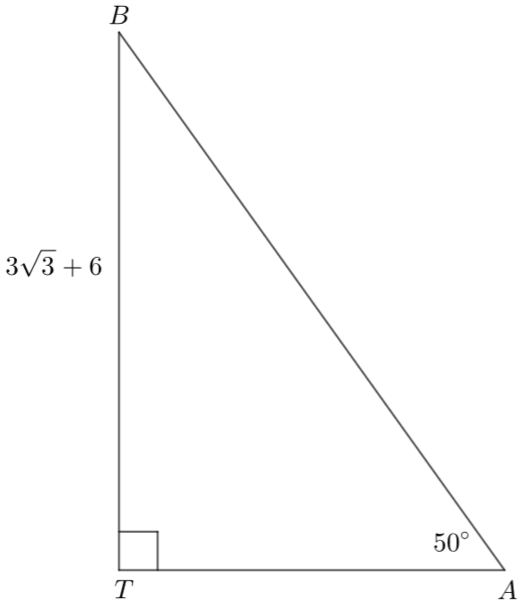
$$\therefore h = \sqrt{36 - 9}$$
$$= \sqrt{27}$$
$$= 3\sqrt{3}$$

Hence: $BT = 3\sqrt{3} + 6$ ■

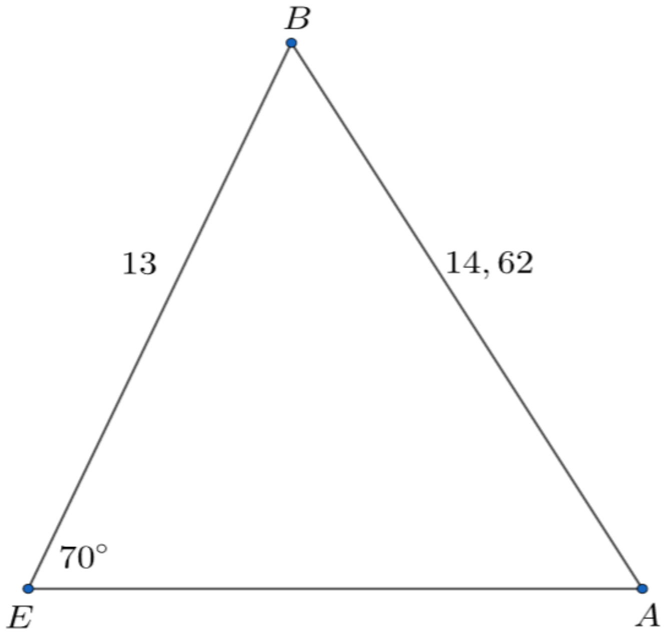
b.

1. If we looked at this diagram from the front view, we would get the diagram in Q12 a. This will help us in determining the length of AB. From the side view we can see that we get a right-angled triangle ΔBTA with height $BT = 3\sqrt{3} + 6$ and we are given that $B\hat{A}T = 50°$. Hence:

$$\sin 50° = \frac{BT}{AB}$$
$$\therefore AB = (3\sqrt{3} + 6)(\sin 50°) = 14{,}62 \text{ metres}$$



2. We are given that $EB = 13$ metres long. Then, if we view ΔBEA from the front:



Now, using the sine rule we get:

$$\frac{\sin A}{13} = \frac{\sin 70°}{14{,}62}$$
$\therefore \hat{A} = 56{,}68°$ and so $\hat{B} = 180° - (70° + 56{,}68°) = 53{,}32°$

Hence, to get EA, we use the cosine rule:

$$\begin{aligned} EA^2 &= EB^2 + AB^2 - 2 \times EB \times AB \times \cos \hat{B} \\ &= 13^2 + 14{,}62^2 - 2(13)(14{,}62) \cos 53{,}32° \end{aligned}$$
$$\begin{aligned} \therefore EA &= \sqrt{155{,}6815} \\ &= 12{,}48 \text{ metres.} \end{aligned}$$