Answers to:

Mathematics
IEB 2017 Paper 1

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SECTION A

QUESTION 1

a.

1. $(x-1)^2 = 2(1-x)$

$\therefore x^2 - 2x + 1 = 2 - 2x$

$\therefore x^2 - 1 = 0$

$\therefore (x-1)(x+1) = 0$

$\therefore x = 1$ or $x = -1$

2. $5^{-x}.5^{x-2} = \frac{25^{2x}}{5}$

$\therefore 5^{-x+x-2} = 5^{4x-1}$

$\therefore 5^{-2} = 5^{4x-1}$

$\therefore -2 = 4x - 1$

$\therefore x = -\frac{1}{4}$

b. $(x+1)^2 < 9$

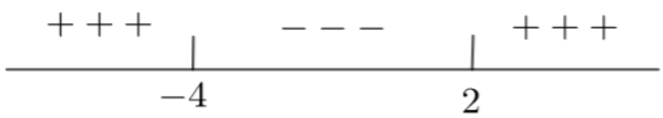
$\therefore x^2 + 2x + 1 < 9$

$\therefore x^2 + 2x - 8 < 0$

$\therefore (x+4)(x-2) < 0$

Critical values: $x = -4$ or $x = 2$

$\therefore -4 < x < 2$



c. If $x = 2$ and $x = -4$ are roots of $x^2 + bx + c = 0$, then we have:

$(x-2)(x+4) = 0$

$\therefore x^2 + 2x - 8 = 0$

$\therefore b = 2$ and $c = -8$

d.

1. Let $y = x - 2$. Then, we have:

$x - 2 = -\frac{4}{x-2} - 4$

$\therefore y = -\frac{4}{y} - 4$

$\therefore y^2 = -4 - 4y$

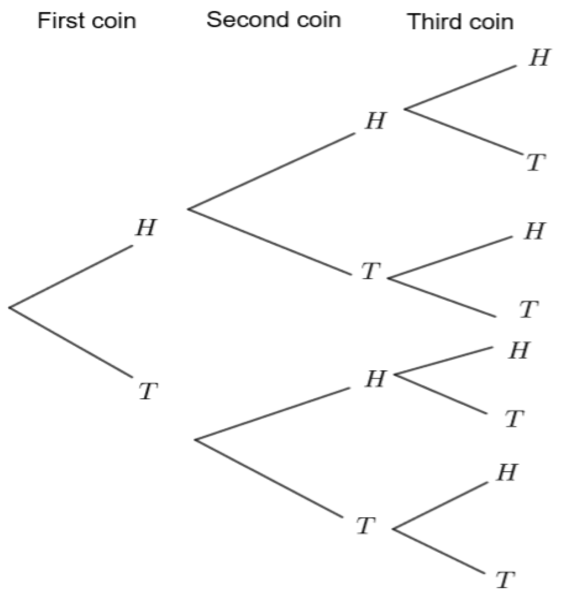
$\therefore y^2 + 4y + 4 = 0$

2. $x - 2 = -\frac{4}{x-2} - 4$
   $\therefore (x-2)^2 = -4 - 4(x-2)$
   $\therefore x^2 - 4x + 4 = -4 - 4x + 8$
   $\therefore x^2 = 0$
   $\therefore x = 0$, hence we have real and equal roots.

QUESTION 2

a.

1.



2. Let $A = \{HTT, THT, TTH\}$ (i.e. A is the event of getting two tails and one head in any order). Let S be the entire space of tossing three unbiased coins, i.e.
   $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$
   $P(A) = \frac{n(A)}{n(S)}$
   $\therefore P(A) = \frac{3}{8}$

b.

1. $P(A \cap B) = 0$

2.

(i) We cannot pick a R2 coin and a R5 coin at the same time.

(ii) Let $A = \{\text{picking a R2 coin}\}$ and $B = \{\text{picking a R5 coin}\}$. Then,

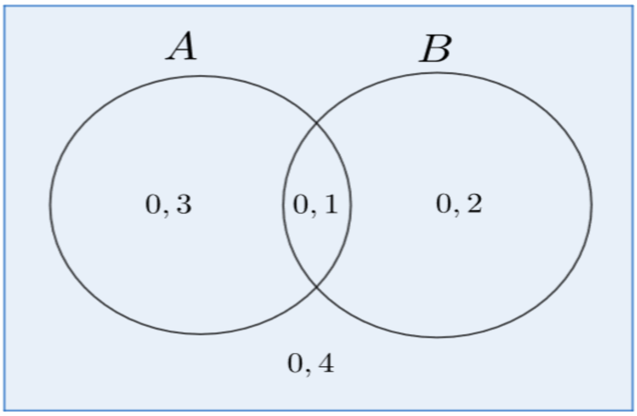
$P(A \text{ or } B) = P(A) + P(B)$ (since A and B are mutually exclusive)

$= 0{,}36 + 0{,}47$

$= 0{,}83$

c.

1.



2. Let $A = \{\text{Machine A presses a R5 coin}\}$ and

$B = \{\text{Machine B presses a R5 coin}\}$

Then, we have: $P(\text{exactly one machine press a R5 coin})$

$= P(A \text{ and not } B) + P(B \text{ and not } A)$

$= 0{,}3 + 0{,}2$

$= 0{,}5$

QUESTION 3

a. $480163 \div 0{,}502 = \text{R}956500$

b. $\text{R}956500 \times 5\% = \text{R}47825$

c. The cost of the machinery including import charges remains constant at:
$\text{R}956500 + \text{R}47825 = \text{R}1004325$. Now, we have:
$A = P(1+i)^n$
$\therefore 1004352 = 225450(1+0{,}095)^n$
$\therefore \frac{1004352}{225450} = (1{,}095)^n$
$\therefore \log_{1{,}095}\left(\frac{1004352}{225450}\right) = n$
$\therefore n \approx 16{,}46$ years
$\therefore$ approximately 17 years.

d.

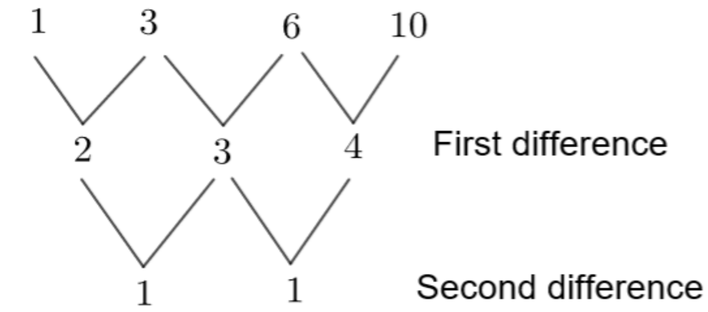
1. The amount required for the loan is given by:
$\text{R}1004352 - \text{R}225450 = \text{R}778875$
Hence the monthly instalments are calculated as:
$$P = x\left[\frac{1-(1+i)^{-n}}{i}\right]$$
$$\therefore 778875 = x\left[\frac{1-(1+0{,}01)^{(-4\times 12)}}{0{,}01}\right]$$
$$\therefore x = \text{R}20510{,}76607$$
$$\therefore x = \text{R}20510{,}77$$

2. The outstanding balance at the end if two years is given by:
$$\text{Outstanding balance} = x\left[\frac{1-(1+i)^{-n}}{i}\right]$$
$$= 20510{,}76607\left[\frac{1-(1+0{,}01)^{(-2\times 12)}}{0{,}01}\right]$$
$$= \text{R}435718{,}1466$$
$$\approx \text{R}435718{,}15$$

QUESTION 4

a.

1.



Constant second difference

2. We have: $T_n = an^2 + bn + c$, where

$$2a = 1$$
$$\therefore a = \frac{1}{2}, \text{ and}$$
$$3a + b = T_2 - T_1$$
$$= 2$$
$$\therefore b = 2 - 3\left(\frac{1}{2}\right) = \frac{1}{2}, \text{ and}$$
$$a + b + c = T_1$$
$$= 1$$
$$\therefore c = 1 - \frac{1}{2} - \frac{1}{2} = 0$$

Hence $T_n = \frac{1}{2}n^2 + \frac{1}{2}n$

b. Let each term represent a step in the staircase. Then, we are given that:

$T_3 = 52$ cm

$T_7 = 78$ cm

Since this is given as an arithmetic sequence, we have:

$T_n = a + (n-1)d$

$\therefore T_3 = a + 2d = 52$… Eq(1)

$\therefore T_7 = a + 6d = 78$… Eq(2)

$\therefore$ Eq(2)−Eq(1): $4d = 26 \therefore d = \frac{26}{4}$. Sub into Eq(1): $a = 52 - 2\left(\frac{26}{4}\right) = 39.$

$\therefore T_{43} = 39 + 42\left(\frac{26}{4}\right)$

$\therefore T_{43} = 312$ cm

QUESTION 5

a.

1. We have $f(x) = x^2 - 6x + 9$

$$\therefore f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0} \frac{(x+h)^2-6(x+h)+9-(x^2-6x+9)}{h}$$
$$= \lim_{h\to 0} \frac{x^2+2xh+h^2-6x-6h+9-x^2+6x-9}{h}$$
$$= \lim_{h\to 0} \frac{2xh+h^2-6h}{h}$$
$$= \lim_{h\to 0} \frac{h(2x+h-6)}{h}$$
$$= \lim_{h\to 0}(2x+h-6)$$
$$= 2x - 6$$

2. $f'(-3) = 2(-3) - 6 = -12$

b. $y = \pi x^{-1} + 3x^{\frac{1}{3}}$

$$\therefore \frac{dy}{dx} = -\pi x^{-2} + x^{-\frac{2}{3}}$$

SECTION B

QUESTION 6

a.

1. Domain of g is $\{x \in \mathbb{R} : x \neq 3\}$

2. Range of h is $\{y \in \mathbb{R} : y \neq -3\}$

3.

   (i) g must be shifted $+5$ units

   (ii) g must be shifted -5 units

b.

1. We have: $y = a(b)^x$. We substitute points $A\left(0; \frac{1}{4}\right)$ and $B\left(2; \frac{9}{4}\right)$

   $\therefore a(b)^0 = \frac{1}{4}$ (Sub point $A\left(0; \frac{1}{4}\right)$)

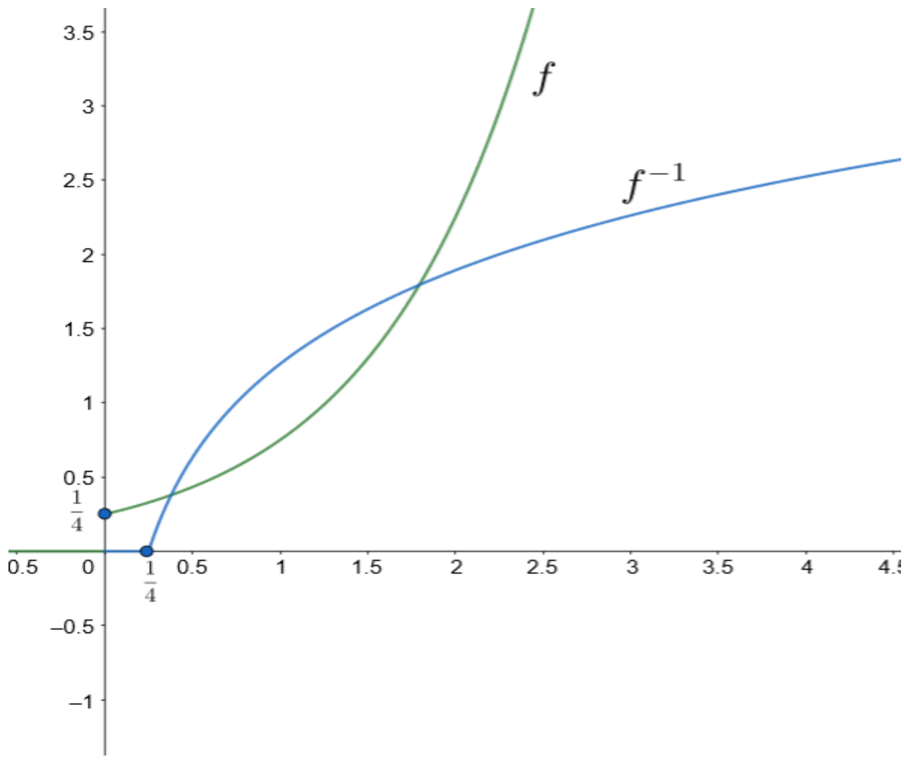
   $\therefore a = \frac{1}{4}$

   $\therefore \frac{1}{4}(b)^2 = \frac{9}{4}$

   $\therefore b^2 = 9$

   $\therefore b = \pm 3$, but we are told that $b > 0$. Hence $b = 3$.

2.



3. Range of f is $\left[\frac{1}{4};\infty\right)$

4. We have: $f(x)=\frac{1}{4}(3)^x$ for $x\geq 0$. To get f^{-1}, we make the following change:
   Interchanging the roles of x and y we get: $x=\frac{1}{4}(3)^y$, $y\geq 0$. Now solve for y.
   $\therefore 4x=3^y$
   $\therefore y=\log_3(4x)$.
   Remember that the domain of f^{-1} is the same as the range of f. Hence:
   $y=\log_3(4x)$, for $x\geq\frac{1}{4}$.

5. See the sketch of f^{-1} in Question 6b. part 2.

QUESTION 7

a. We complete the square: We have $f(x)=x^2+6x+5$
$\therefore f(x)=x^2+6x+(3)^2+5-(3)^2$
$\therefore f(x)=(x+3)^2-4$
$\therefore T.P.(-3;-4)$

b.

1. $x^2+6x+5=-x-5$
   $\therefore x^2+7x+10=0$
   $\therefore (x+5)(x+2)=0$
   $\therefore x=-5$ or $x=-2$
   Hence $A(-5;0)$ and $B(-2;0)$

2. Note that this is a horizontal shift of t units to both graphs f and g. If we want one positive and one negative root, we have to shift both graphs at the same time **at least 2 units to the right** and **at the most 5 units to the right**, hence we have that $-5<t<-2$.

c.

1. Note that point M lies on $y = -x - 5$ and N lies on $y = x^2 + 6x + 5$

$\therefore$ Length MN $= y_M - y_N$

$$= (-x - 5) - (x^2 + 6x + 5)$$
$$= -x - 5 - x^2 - 6x - 5$$
$$= -x^2 - 7x - 10$$

Now, to get the maximum length, we set the derivative of the length $= 0$

$$\therefore D_x[-x^2 - 7x - 10] = -2x - 7$$
$$= 0$$
$$\therefore x = -\frac{7}{2}$$

Hence the max. length of MN$= -\left(-\frac{7}{2}\right)^2 - 7\left(-\frac{7}{2}\right) - 10 = \frac{9}{4}$ units.

2. Note that $f(x) + k$ is a vertical shift of the graph f. We calculated that the largest possible vertical distance between f and g is $\frac{9}{4}$ units. We can see that shifting f vertically down will always intersect g, so we don't want to shift f vertically down. However, if we shift f more than $\frac{9}{4}$ units vertically upwards then f will never intersect with g. Hence $k > \frac{9}{4}$.

QUESTION 8

a.

1. We are given the geometric series: $(x + 3) + (x - 3) + (12 - x) + \cdots$ and we are told that the series converges, therefore we must have that $-1 < r < 1$. Assume that $x = -\frac{3}{2}$, then we get:

$$\left(-\frac{3}{2} + 3\right) + \left(-\frac{3}{2} - 3\right) + \left(12 - \left(-\frac{3}{2}\right)\right) + \cdots$$
$$= \frac{3}{2} + \left(-\frac{9}{2}\right) + \left(\frac{27}{2}\right) + \cdots$$

$\therefore r = -3$, which is a contradiction since $-1 < r < 1$.

Hence $x \neq -\frac{3}{2}$.

2. Since we have a geometric sequence, we know that:

$\frac{T_3}{T_2} = \frac{T_2}{T_1} \Rightarrow \frac{12-x}{x-3} = \frac{x-3}{x+3}$

$\therefore (12 - x)(x + 3) = (x - 3)^2$

$\therefore 12x + 36 - x^2 - 3x = x^2 - 6x + 9$

$\therefore 2x^2 - 15x - 27 = 0$

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-15) \pm \sqrt{(-15)^2 - 4(2)(-27)}}{2(2)}$

$\therefore x = 9$ or $x = -\frac{3}{2}$, but from part 1. we know that $x \neq -\frac{3}{2}$.

Hence $x = 9$.

b. We are given that: A geometric series is such that

$S_4 = 7\frac{1}{2}$

$S_5 = 15\frac{1}{2}$

$S_6 = 31\frac{1}{2}$

Remember that to get the n^{th}-term from a sum, we use: $T_n = S_n - S_{n-1}$.

Hence we have:

$T_5 = S_5 - S_4$
$= 15\frac{1}{2} - 7\frac{1}{2}$
$= 8$

and

$T_6 = S_6 - S_5$
$= 31\frac{1}{2} - 15\frac{1}{2}$
$= 16$

Now, remember the general formula for a geometric sequence is given by:
$T_n = ar^{n-1}$, thus:

$T_5 = ar^4 = 8$ ...Eq(1)

$T_6 = ar^5 = 16$ ...Eq(2)

$\therefore \frac{T_6}{T_5} = r = 2.$

$\therefore a = \frac{8}{2^4} = \frac{1}{2}$ (Using Eq(1))

Lastly, the sum formula for a geometric series to n terms is given by:

$S_n = \frac{a(r^n - 1)}{r - 1}$

$= \frac{\frac{1}{2}(2^n - 1)}{2 - 1}$

$= \frac{1}{2}(2^n - 1)$

$= 2^{n-1} - \frac{1}{2}$

QUESTION 9

a. We are given two pieces of important information, namely:

(i) $f(x) = -x^3 + bx^2 + cx - 3$, and

(ii) $f(1) = 4$ and $f''\left(\frac{1}{2}\right) = 1$.

Then, we have:

$f(1) = -(1)^3 + b(1)^2 + c(1) - 3$

$= -1 + b + c - 3$

$= b + c - 4$

But from (ii), $f(1) = 4$

$\therefore b + c - 4 = 4$

$\therefore b + c = 8$ ...Eq(1)

Also, $f'(x) = -3x^2 + 2bx + c$ and $f''(x) = -6x + 2b$. From part (ii), we have:

$f''\left(\frac{1}{2}\right) = -6\left(\frac{1}{2}\right) + 2b$

$= -3 + 2b$

$= 1$

$\therefore 2b - 3 = 1$

$\therefore b = 2$

Sub. into Eq(1):

$\therefore 2 + c = 8$

$\therefore c = 6$

b. The graph f is concave up when: $f''(x) > 0$

$\therefore -6x + 4 > 0$

$\therefore x < \frac{2}{3}$

QUESTION 10

We have a cup with a capacity of 340 ml. A dispenser fills the cup at a rate of x ml/s. The dispenser now increases its rate to $(x + 2)$ ml/s and the new time to fill the cup is 3 seconds faster than the initial rate of x ml/s. Mathematically, this can be expressed as:

(i) The **time** to fill the cup at a **rate of $\boldsymbol{x}$ ml/s** is given by: $\frac{340}{x}$ seconds.
(ii) The **time** to fill the cup at a **rate of $(\boldsymbol{x+2})$ ml/s** is: $\frac{340}{(x+2)}$ seconds.
(iii) The **difference in time** to fill the cup at a rate of $(x + 2)$ ml/s vs. x ml/s is 3 seconds.

Hence using all this information, we arrive at:

$$\frac{340}{x} - \frac{340}{x+2} = 3$$
$$\therefore \frac{340(x+2) - 340x}{x(x+2)} = 3$$
$$\therefore 340(x + 2) - 340x = 3x(x + 2)$$
$$\therefore 3x^2 + 6x - 680 = 0$$
$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(6) \pm \sqrt{(6)^2 - 4(3)(-680)}}{2(3)}$$
$$\therefore x = 14{,}09 \text{ or } x = -16{,}09$$

However, we cannot have a negative time, so $x = 14{,}09$.
Therefore the original time taken to fill the cup is given by:

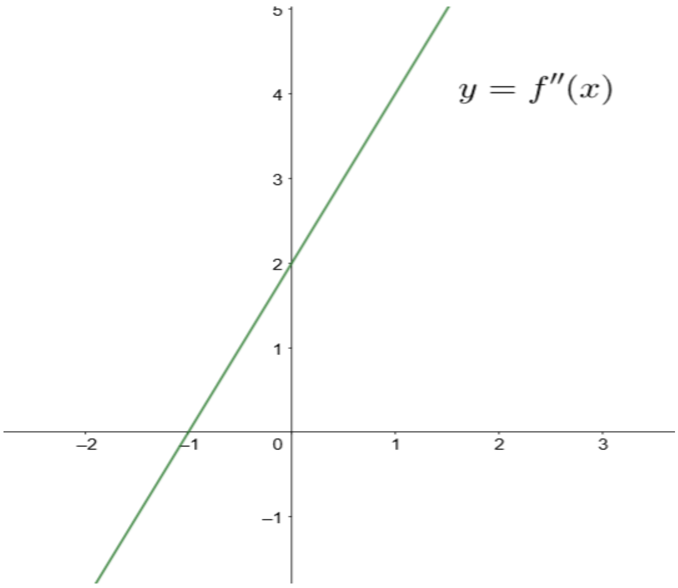
$$\frac{340}{x} = \frac{340}{14{,}09} \approx 24{,}13 \text{ seconds.}$$

QUESTION 11

a.

1. Remember that the tangent to f is horizontal when $f'(x) = 0$. From the given graph of $y = f'(x)$, we see that $f'(x) = 0$ when $x = -2$ or $x = 0$.

2.



b. We firstly have to calculate the equation of the tangent to y at the point $F(0;3)$. So:

$$y = \frac{1}{15}x^3 + \frac{3}{4}x + 3$$

$$\therefore \frac{dy}{dx} = \frac{1}{5}x^2 + \frac{3}{4}$$

Hence the gradient of the tangent at the point F is given by:

$$\frac{dy}{dx}|_{x=0} = \frac{1}{5}(0)^2 + \frac{3}{4} = \frac{3}{4}.$$

So, the equation of the tangent is: $y = \frac{3}{4}x + c$ and upon sub. of $F(0;3)$ we get $c = 3$. Hence $y = \frac{3}{4}x + 3$.

Now, to calculate the point of intersection between the tangent and the line BC, we substitute $x = 2$ into $y = \frac{3}{4}x + 3$. Therefore we get $y = 4\frac{1}{2}$. Hence the point is $\left(2; 4\frac{1}{2}\right)$.

Lastly, we can calculate the areas of each person's region (both shapes are trapezium) Hence:

Area of Busi's region $= \frac{1}{2}\left(5 + 3\frac{1}{2}\right) \times 2 = 8\frac{1}{2}$ units^2

Area of Khanya's region $= \frac{1}{2}\left(3 + 4\frac{1}{2}\right) \times 2 = 7\frac{1}{2}$ units^2

Hence Busi's region is larger.