Answers to:

Mathematics
IEB 2017 Paper 1

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SECTION A

QUESTION 1

a.  
1. \((x - 1)^2 = 2(1 - x)\)
   \[x^2 - 2x + 1 = 2 - 2x\]
   \[x^2 - 1 = 0\]
   \[(x - 1)(x + 1) = 0\]
   \[x = 1 \text{ or } x = -1\]

2. \(5^{-x} \cdot 5^{x-2} = \frac{25^{2x}}{5}\)
   \[5^{-x+x-2} = 5^{4x-1}\]
   \[5^{-2} = 5^{4x-1}\]
   \[-2 = 4x - 1\]
   \[x = -\frac{1}{4}\]

b. \((x + 1)^2 < 9\)
   \[x^2 + 2x + 1 < 9\]
   \[x^2 + 2x - 8 < 0\]
   \[(x + 4)(x - 2) < 0\]
   Critical values: \(x = -4\) or \(x = 2\)
   \[-4 < x < 2\]

c. If \(x = 2\) and \(x = -4\) are roots of \(x^2 + bx + c = 0\), then we have:
   \((x - 2)(x + 4) = 0\)
   \[x^2 + 2x - 8 = 0\]
   \[b = 2\] and \(c = -8\)

d.  
1. Let \(y = x - 2\). Then, we have:
   \[x - 2 = -\frac{4}{x} - 4\]
   \[\therefore y = -\frac{4}{y} - 4\]
   \[\therefore y^2 = -4 - 4y\]
   \[\therefore y^2 + 4y + 4 = 0\]
2. \( x - 2 = -\frac{4}{x^2} - 4 \)
   \[
   \therefore (x - 2)^2 = -4 - 4(x - 2)
   \]
   \[
   \therefore x^2 - 4x + 4 = -4 - 4x + 8
   \]
   \[
   \therefore x^2 = 0
   \]
   \[
   \therefore x = 0, \text{ hence we have real and equal roots.}
   \]

QUESTION 2

a.

1.

2. Let \( A = \{HTT, THT, TTH\} \) (i.e. \( A \) is the event of getting two tails and one head in any order). Let \( S \) be the entire space of tossing three unbiased coins, i.e.

\[
S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}
\]

\[
P(A) = \frac{n(A)}{n(S)}
\]

\[
\therefore P(A) = \frac{3}{8}
\]
b.  

1. \( P(A \cap B) = 0 \)

2.  

(i) We cannot pick a R2 coin and a R5 coin at the same time.

(ii) Let \( A = \{ \text{picking a R2 coin} \} \) and \( B = \{ \text{picking a R5 coin} \} \). Then,

\[
P(A \text{ or } B) = P(A) + P(B) \text{ (since } A \text{ and } B \text{ are mutually exclusive)}
\]

\[
= 0.36 + 0.47
\]

\[
= 0.83
\]

c.  

1. 

2. Let \( A = \{ \text{Machine A presses a R5 coin} \} \) and \( B = \{ \text{Machine B presses a R5 coin} \} \).

Then, we have: \( P(\text{exactly one machine press a R5 coin}) \)

\[
= P(A \text{ and not } B) + P(B \text{ and not } A)
\]

\[
= 0.3 + 0.2
\]

\[
= 0.5
\]
QUESTION 3

a. \( 480163 \div 0.502 = R956500 \)

b. \( R956500 \times 5\% = R47825 \)

c. The cost of the machinery including import charges remains constant at: 
\[ R956500 + R47825 = R1004325. \]
Now, we have: 
\[ A = P(1 + i)^n \]
\[ \therefore 1004352 = 225450(1 + 0.095)^n \]
\[ \therefore \frac{1004352}{225450} = (1.095)^n \]
\[ \therefore \log_{1.095} \left( \frac{1004352}{225450} \right) = n \]
\[ \therefore n \approx 16.46 \text{ years} \]
\[ \therefore \text{approximately 17 years.} \]

d. 
1. The amount required for the loan is given by:
\[ R1004352 - R225450 = R778875 \]
Hence the monthly instalments are calculated as:
\[ P = X \left[ \frac{1 - (1 + i)^{-n}}{i} \right] \]
\[ \therefore 778875 = X \left[ \frac{1 - (1 + 0.01)^{-4\times12}}{0.01} \right] \]
\[ \therefore X = R20510,76607 \]
\[ \therefore X = R20510,77 \]

2. The outstanding balance at the end if two years is given by:
\[ \text{Outstanding balance} = X \left[ \frac{1 - (1 + i)^{-n}}{i} \right] \]
\[ = 20510,76607 \left[ \frac{1 - (1 + 0.01)^{-2\times12}}{0.01} \right] \]
\[ = R435718,1466 \]
\[ \approx R435718,15 \]
QUESTION 4

a. 1. We have: \( T_n = a n^2 + bn + c \), where
\[ 2a = 1 \]
\[ \therefore a = \frac{1}{2} \text{, and} \]
\[ 3a + b = T_2 - T_1 \]
\[ = 2 \]
\[ \therefore b = 2 - 3 \left( \frac{1}{2} \right) = \frac{1}{2} \text{, and} \]
\[ a + b + c = T_1 \]
\[ = 1 \]
\[ \therefore c = 1 - \frac{1}{2} - \frac{1}{2} = 0 \]
Hence \( T_n = \frac{1}{2} n^2 + \frac{1}{2} n \)

b. Let each term represent a step in the staircase. Then, we are given that:

\[ T_3 = 52 \text{ cm} \]
\[ T_7 = 78 \text{ cm} \]

Since this is given as an arithmetic sequence, we have:
\[ T_n = a + (n - 1)d \]
\[ \therefore T_3 = a + 2d = 52 \ldots \text{Eq}(1) \]
\[ \therefore T_7 = a + 6d = 78 \ldots \text{Eq}(2) \]
\[ \therefore \text{Eq}(2)-\text{Eq}(1): 4d = 26 \therefore d = \frac{26}{4} \]
Sub into Eq(1): \( a = 52 - 2 \left( \frac{26}{4} \right) = 39 \).

\[ \therefore T_{43} = 39 + 42 \left( \frac{26}{4} \right) \]
\[ \therefore T_{43} = 312 \text{ cm} \]
QUESTION 5

a. 1. We have \( f(x) = x^2 - 6x + 9 \)
   \[ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \]
   \[ = \lim_{h \to 0} \frac{(x+h)^2 - 6(x+h) + 9 - (x^2 - 6x + 9)}{h} \]
   \[ = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 6x - 6h + 9 - x^2 + 6x - 9}{h} \]
   \[ = \lim_{h \to 0} \frac{2xh + h^2 - 6h}{h} \]
   \[ = \lim_{h \to 0} (2x + h - 6) \]
   \[ = 2x - 6 \]

2. \( f'(-3) = 2(-3) - 6 = -12 \)

b. \( y = \pi x^{-1} + 3x^{\frac{1}{3}} \)
   \[ \frac{dy}{dx} = -\pi x^{-2} + x^{\frac{2}{3}} \]
SECTION B

QUESTION 6

a.
1. Domain of \( g \) is \( \{ x \in \mathbb{R} : x \neq 3 \} \)

2. Range of \( h \) is \( \{ y \in \mathbb{R} : y \neq -3 \} \)

3. (i) \( g \) must be shifted +5 units

(ii) \( g \) must be shifted −5 units

b.
1. We have: \( y = a(b)^x \). We substitute points \( A \left( 0; \frac{1}{4} \right) \) and \( B \left( 2; \frac{9}{4} \right) \)

\[ \therefore a(b)^0 = \frac{1}{4} \text{ (Sub point } A \left( 0; \frac{1}{4} \right) ) \]

\[ \therefore a = \frac{1}{4} \]

\[ \therefore \frac{1}{4} (b)^2 = \frac{9}{4} \]

\[ \therefore b^2 = 9 \]

\[ \therefore b = \pm 3, \text{ but we are told that } b > 0. \text{ Hence } b = 3. \]

2.
3. Range of $f$ is $\left[\frac{1}{4}; \infty\right)$

4. We have: $f(x) = \frac{1}{4}(3)^x$ for $x \geq 0$. To get $f^{-1}$, we make the following change:
   Interchanging the roles of $x$ and $y$ we get: $x = \frac{1}{4}(3)^y$, $y \geq 0$. Now solve for $y$.
   \[ \therefore 4x = 3^y \]
   \[ \therefore y = \log_3(4x) \]
   Remember that the domain of $f^{-1}$ is the same as the range of $f$. Hence:
   \[ y = \log_3(4x), \text{ for } x \geq \frac{1}{4}. \]

5. See the sketch of $f^{-1}$ in Question 6b. part 2.

QUESTION 7

a. We complete the square: We have $f(x) = x^2 + 6x + 5$
   \[ \therefore f(x) = x^2 + 6x + (3)^2 + 5 - (3)^2 \]
   \[ \therefore f(x) = (x + 3)^2 - 4 \]
   \[ \therefore T.P. (-3; -4) \]

b.
1. \[ x^2 + 6x + 5 = -x - 5 \]
   \[ \therefore x^2 + 7x + 10 = 0 \]
   \[ \therefore (x + 5)(x + 2) = 0 \]
   \[ \therefore x = -5 \text{ or } x = -2 \]
   Hence $A(-5; 0)$ and $B(-2; 0)$

2. Note that this is a horizontal shift of $t$ units to both graphs $f$ and $g$. If we want one positive and one negative root, we have to shift both graphs at the same time at least 2 units to the right and at the most 5 units to the right, hence we have that $-5 < t < -2$. 

c. 1. Note that point M lies on \( y = -x - 5 \) and N lies on \( y = x^2 + 6x + 5 \)
\[ \therefore \text{Length } MN = y_M - y_N \]
\[ = (-x - 5) - (x^2 + 6x + 5) \]
\[ = -x - 5 - x^2 - 6x - 5 \]
\[ = -x^2 - 7x - 10 \]
Now, to get the maximum length, we set the derivative of the length = 0
\[ \therefore D_x[-x^2 - 7x - 10] = -2x - 7 \]
\[ = 0 \]
\[ \therefore x = -\frac{7}{2} \]
Hence the max. length of MN = \( -\left(-\frac{7}{2}\right)^2 - 7 \left(-\frac{7}{2}\right) - 10 = \frac{9}{4} \) units.

2. Note that \( f(x) + k \) is a vertical shift of the graph \( f \). We calculated that the largest possible vertical distance between \( f \) and \( g \) is \( \frac{9}{4} \) units. We can see that shifting \( f \) vertically down will always intersect \( g \), so we don’t want to shift \( f \) vertically down. However, if we shift \( f \) more than \( \frac{9}{4} \) units vertically upwards then \( f \) will never intersect with \( g \). Hence \( k > \frac{9}{4} \).

QUESTION 8

a. 1. We are given the geometric series: \( (x + 3) + (x - 3) + (12 - x) + \cdots \) and we are told that the series converges, therefore we must have that \( -1 < r < 1 \). Assume that \( x = -\frac{3}{2} \), then we get:
\[ \left(-\frac{3}{2} + 3\right) + \left(-\frac{3}{2} - 3\right) + \left(12 - \left(-\frac{3}{2}\right)\right) + \cdots \]
\[ = \frac{3}{2} + \left(-\frac{9}{2}\right) + \left(\frac{27}{2}\right) + \cdots \]
\[ \therefore r = -3 \text{, which is a contradiction since } -1 < r < 1. \]
Hence \( x \neq -\frac{3}{2} \).
2. Since we have a geometric sequence, we know that:

\[
\frac{T_3}{T_2} = \frac{T_2}{T_1} \Rightarrow \frac{12-x}{x-3} = \frac{x-3}{x+3}
\]

\[
(12 - x)(x + 3) = (x - 3)^2
\]

\[
12x + 36 - x^2 - 3x = x^2 - 6x + 9
\]

\[
2x^2 - 15x - 27 = 0
\]

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
= \frac{-(-15) \pm \sqrt{(-15)^2 - 4(2)(-27)}}{2(2)}
\]

\[
∴ x = 9 \text{ or } x = \frac{-3}{2}, \text{ but from part 1. we know that } x \neq \frac{-3}{2}.
\]

Hence \( x = 9 \).

b. We are given that: A geometric series is such that

\[
S_4 = 7\frac{1}{2}
\]

\[
S_5 = 15\frac{1}{2}
\]

\[
S_6 = 31\frac{1}{2}
\]

Remember that to get the \( n^{th} \)-term from a sum, we use: \( T_n = S_n - S_{n-1} \).

Hence we have:

\[
T_5 = S_5 - S_4 = 15\frac{1}{2} - 7\frac{1}{2} = 8
\]

\[
T_6 = S_6 - S_5 = 31\frac{1}{2} - 15\frac{1}{2} = 16
\]

Now, remember the general formula for a geometric sequence is given by:

\[ T_n = ar^{n-1}, \text{ thus:} \]

\[
T_5 = ar^4 = 8 \text{ ...Eq(1)}
\]

\[
T_6 = ar^5 = 16 \text{ ...Eq(2)}
\]

\[
∴ \frac{T_6}{T_5} = r = 2.
\]

\[
∴ a = \frac{8}{2^4} = \frac{1}{2} \text{ (Using Eq(1))}
\]

Lastly, the sum formula for a geometric series to \( n \) terms is given by:

\[
S_n = \frac{a(r^n - 1)}{r - 1}
\]

\[
= \frac{1}{2}(2^n - 1)
\]

\[
= 2^{n-1} - \frac{1}{2}
\]
QUESTION 9

a. We are given two pieces of important information, namely:
   (i) \( f(x) = -x^3 + bx^2 + cx - 3 \), and
   (ii) \( f(1) = 4 \) and \( f''\left(\frac{1}{2}\right) = 1 \).

Then, we have:
\[
f(1) = -(1)^3 + b(1)^2 + c(1) - 3
= -1 + b + c - 3
= b + c - 4
\]
But from (ii), \( f(1) = 4 \)
\[
\therefore b + c - 4 = 4
\]
\[
\therefore b + c = 8 \quad \text{Eq}(1)
\]

Also, \( f'(x) = -3x^2 + 2bx + c \) and \( f''(x) = -6x + 2b \). From part (ii), we have:
\[
f''\left(\frac{1}{2}\right) = -6\left(\frac{1}{2}\right) + 2b
= -3 + 2b
= 1
\]
\[
\therefore 2b - 3 = 1
\]
\[
\therefore b = 2
\]
Sub. into Eq(1):
\[
\therefore 2 + c = 8
\]
\[
\therefore c = 6
\]

b. The graph \( f \) is concave up when: \( f''(x) > 0 \)
\[
\therefore -6x + 4 > 0
\]
\[
\therefore x < \frac{2}{3}
\]
QUESTION 10

We have a cup with a capacity of 340 ml. A dispenser fills the cup at a rate of $x$ ml/s. The dispenser now increases its rate to $(x + 2)$ ml/s and the new time to fill the cup is 3 seconds faster than the initial rate of $x$ ml/s. Mathematically, this can be expressed as:

(i) The time to fill the cup at a rate of $x$ ml/s is given by: $\frac{340}{x}$ seconds.
(ii) The time to fill the cup at a rate of $(x + 2)$ ml/s is: $\frac{340}{x+2}$ seconds.
(iii) The difference in time to fill the cup at a rate of $(x + 2)$ ml/s vs. $x$ ml/s is 3 seconds.

Hence using all this information, we arrive at:

\[
\frac{340}{x} - \frac{340}{x+2} = 3 \\
\Rightarrow \frac{340(x+2) - 340x}{x(x+2)} = 3 \\
\Rightarrow 340(x + 2) - 340x = 3x(x + 2) \\
\Rightarrow 3x^2 + 6x - 680 = 0 \\
\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
\Rightarrow x = \frac{-6 \pm \sqrt{(6)^2 - 4(3)(-680)}}{2(3)} \\
\Rightarrow x = 14.09 \text{ or } x = -16.09
\]

However, we cannot have a negative time, so $x = 14.09$.
Therefore the original time taken to fill the cup is given by:

\[
\frac{340}{x} = \frac{340}{14.09} \approx 24.13 \text{ seconds.}
\]
QUESTION 11

a.

1. Remember that the tangent to $f$ is horizontal when $f'(x) = 0$. From the given graph of $y = f'(x)$, we see that $f'(x) = 0$ when $x = -2$ or $x = 0$.

2.
b. We firstly have to calculate the equation of the tangent to \( y \) at the point \( F(0; 3) \). So:

\[
y = \frac{1}{15}x^3 + \frac{3}{4}x + 3
\]

\[
\Rightarrow \frac{dy}{dx} = \frac{1}{5}x^2 + \frac{3}{4}
\]

Hence the gradient of the tangent at the point \( F \) is given by:

\[
\frac{dy}{dx} \bigg|_{x=0} = \frac{1}{5}(0)^2 + \frac{3}{4} = \frac{3}{4}.
\]

So, the equation of the tangent is: \( y = \frac{3}{4}x + c \) and upon sub. of \( F(0; 3) \) we get \( c = 3 \).

Hence \( y = \frac{3}{4}x + 3 \).

Now, to calculate the point of intersection between the tangent and the line BC, we substitute \( x = 2 \) into \( y = \frac{3}{4}x + 3 \). Therefore we get \( y = 4\frac{1}{2} \). Hence the point is \( \left(2; \frac{9}{2}\right) \).

Lastly, we can calculate the areas of each person’s region (both shapes are trapezium)

Hence:

Area of Busi’s region = \( \frac{1}{2} \left( \frac{25}{2} + \frac{3}{2} \right) \times 2 = 8\frac{1}{2} \) units\(^2 \)

Area of Khanya’s region = \( \frac{1}{2} \left( 3 + 4\frac{1}{2} \right) \times 2 = 7\frac{1}{2} \) units\(^2 \)

Hence Busi’s region is larger.