Answers to:

Mathematics
IEB 2016 Paper 2
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SECTION A

QUESTION 1

a. We are given $\angle OAB = 135^\circ$ and $\angle OCB = 90^\circ$, so $\angle OAB + \angle OCB = 225^\circ$.
   Hence opp. angles do not add up to $180^\circ$.

b. Using tan-gradient, we have $m_{AB} = \tan 45^\circ = 1$. Since $OA = 8$ units, then $A(0; 8)$.
   Substitute point $A$ into $y = mx + c$, then $y = x + 8$.

c. 1. Since $OC = 6$ units, then $C(6; 0)$. Hence $x = 6$ is the equation of $BC$.

2. $OCBA$ is a parallelogram with base lengths of $OA$ and $BC$. The perpendicular height is given by $OC$. Note $B(6; 14)$, hence:
   Area $= \frac{1}{2} (OA + BC)(OC) = \frac{1}{2} (8 + 14)(6) = 66 \text{ units}^2$

QUESTION 2

a. 1. $M = \frac{2 \sin^2 \theta + 2 \sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta}$
   $\therefore M = \frac{2 \sin \theta \cos \theta}{2 \sin \theta (\sin \theta + \cos \theta)}$
   $\therefore M = \frac{\sin \theta}{\cos \theta - \sin \theta}$
   $\therefore M = P$

2. $P$ is undefined when: $\cos \theta - \sin \theta = 0$
   $\therefore \cos \theta = \sin \theta$
   $\therefore 1 = \tan \theta$
   $\therefore$ Reference angle $= 45^\circ$
   $\therefore \theta = -135^\circ \text{ or } 45^\circ \text{ or } 225^\circ$

b. 1. Quadrant 2
2. \[
\tan \beta = \frac{y}{x} = \frac{\sqrt{11}}{-3}
\]
c. 1. \[
cos(\alpha - 30^\circ) - \cos(\alpha + 30^\circ)
= \cos \alpha \cos 30^\circ + \sin \alpha \sin 30^\circ - (\cos \alpha \cos 30^\circ - \sin \alpha \sin 30^\circ)
= \cos \alpha \cos 30^\circ + \sin \alpha \sin 30^\circ - \cos \alpha \cos 30^\circ + \sin \alpha \sin 30^\circ
= 2 \sin \alpha \sin 30^\circ = 2 \sin \alpha \times \left(\frac{1}{2}\right)
= \sin \alpha
\]
2. \[
\sin \alpha = 2 \sin^2 \alpha
\]
\[
0 = \sin \alpha (2 \sin \alpha - 1)
\]
\[
\sin \alpha = 0 \quad \text{or} \quad \sin \alpha = \frac{1}{2}
\]
\[
\alpha = 0^\circ + k \cdot 180^\circ, k \in \mathbb{Z} \quad \text{or} \quad \alpha = 30^\circ + k \cdot 360^\circ, k \in \mathbb{Z}
\]
\[
\alpha = 150^\circ + k \cdot 360^\circ, k \in \mathbb{Z}
\]

**QUESTION 3**

a. We have: Radius of circle Q is 9 - 5 = 4 units. Now, 
\[
x_Q \text{ of the centre of circle Q is } 9 + 5 = 14 \text{ units,}
\]
\[
y_Q \text{ of the centre of circle Q is } 5 \text{ units.}
\]
Hence the equation of circle Q is given by: \( (x - 14)^2 + (y - 5)^2 = 16 \)

b. We are given: \( (x - p)^2 + y^2 - 22y = -117 \). We complete the square on the LHS:
\[
\therefore (x - p)^2 + y^2 - 22y + 121 = -117 + 121
\]
\[
\therefore (x - p)^2 + (y - 11)^2 = 4
\]
Hence the length of RQ is 4 + 2 = 6 units.
c. To get the length of AB, we first calculate:

Length of: \( PR^2 = PQ^2 + QR^2 \)

\[ \therefore PR = \sqrt{(14 - 5)^2 + (11 - 5)^2} \]

\[ \therefore PR = \sqrt{117} \]

Length of: \( PA = 5 \)
Length of: \( BR = 2 \)
Therefore the length of line \( AB = PR - PA - BR \)
\[ = \sqrt{117} - 5 - 2 \]
\[ = 3.82 \text{ units} \]

**QUESTION 4**

a. Draw \( AO \) and \( OC \). Then,

\[ \text{R.T.P: } \hat{B} + \hat{D} = 180^\circ \]

Proof:

\[ \hat{O}_2 = 2 \times \hat{B} \]  (Angle at centre)
\[ \hat{O}_1 = 2 \times \hat{D} \]  (Angle at centre)
\[ \hat{O}_1 + \hat{O}_2 = 360^\circ \]  (Angle around a pt.)

\[ \therefore 2\hat{D} + 2\hat{B} = 360^\circ \]
\[ \therefore \hat{B} + \hat{D} = 180^\circ . \]

b. We have: \( \hat{ABC} = 62^\circ \)  (tan-chord thm.)
\[ \hat{AOC} = 124^\circ \]  (Angle at centre = 2\times Angle at circum.)
\[ \hat{C}_2 = \hat{A}_3 = 28^\circ \]  (\( OC = OA \), radii are equal)
\[ \hat{A}_2 = 25^\circ \]  (Given)

\[ \therefore \hat{C}_1 = 180^\circ - (\hat{A}_2 + \hat{A}_3 + \hat{B} + \hat{C}_2) \]  (Angles in a \( \Delta \))
\[ \therefore \hat{C}_1 = 180^\circ - (25^\circ + 28^\circ + 62^\circ + 28^\circ) \]
\[ = 37^\circ \]

c.

1. We can conclude that \( N = Q \).

2. Proof: \( \hat{D}_1 = \hat{B} \)  (ext. angle of a cyclic quad)
\[ \hat{D}_1 = \hat{A}_1 + \hat{C}_2 \]  (ext. angle of \( \Delta = \) sum of two int. opp. angles)
\[ \therefore \hat{B} = \hat{A}_1 + \hat{C}_2. \]
QUESTION 5

a. Sub. \( x = 360^\circ \) into \( y = 3 \sin x + 1 \):
\[
\therefore y = 3 \sin 360^\circ + 1 \\
\therefore y = 1 \\
\therefore B(360^\circ; 1)
\]

b. \( 3 \sin x + 1 = -1 \)
\[
\therefore 3 \sin x = -2 \\
\therefore \sin x = -\frac{2}{3} \\
\text{Key angle} = 41.81^\circ
\]

Hence \( x = 221.81^\circ \) or \( x = 318.19^\circ \).

c. From the graph, we can see that any straight line \( g(x) = k \) will cut through \( f(x) \) in the interval \([0^\circ; 180^\circ]\) in between the values of \( 1 \leq k \leq 4 \). Hence there will be no solutions when \( k > 4 \) or \( k < 1 \).

QUESTION 6

a. Using you calculator, we get: \( r = 0.9755 \), therefore a very strong relationship.

b. Remember that: \( y = a + bx \), where: \( A = 2788.26 \) and \( B = 1658.39 \). Hence
\[
y = 2788.26 + 1658.39x
\]

c. \( y = R \, 34297.67 \).

Hence the managers projected income based on the line of best fit is \( R \, 34297.67 \) and the actual sales was \( R \, 23000 \). So this would not be considered a successful day.

NB: It is important that you are familiar with using your calculator for statistics questions such as regression modelling, finding means/variances, etc. We have attached a step-by-step instruction guide on how to use your Casio calculator to compute these statistical operations.
How to use a Casio calculator for Regression modelling
Press:
MODE → 3:STAT → 2: A + Bx
Enter data into the x and y columns
Press: AC
To find A:
SHIFT → 1 → 5:Reg → 1:A → =
To find B:
SHIFT → 1 → 5:Reg → 2:B → =
To find r (correlation coefficient)
SHIFT → 1 → 5:Reg → 3:r → =
To find ŷ given ŷ:
Enter ŷ - value → SHIFT → 1 → 5:Reg → 5: ŷ → =
To find the mean point (x̅; y̅)
SHIFT → 1 → 4:Var → 2: x̅ → =
SHIFT → 1 → 4:Var → 5: y̅ → =

How to use a Casio calculator to find Mean and Standard Deviation
Press:
MODE → 3:STAT → 1: 1 - VAR
Enter data into the x and FREQ columns

If no FREQ column then PRESS:
SHIFT → SET UP → page down → 4: STAT → 1: ON

Press: AC: →
To find the mean:
SHIFT → 1 → 4: Var → 2: x̅

To find the standard deviation:
SHIFT → 1 → 4: Var → 3: σx
Remember: variance = (σx)^2
SECTION B

QUESTION 7

a. $A = 250$ and $B = 502$

b. 
1. $\bar{x} \approx 47,14$ (Use calculator)

2. $65 < x \leq 75$

c.

d. 
1. No, the data is skewed to the left since the mean is less than the median.

2. No, the mean is not a good indicator since it’s affected by the extremes. The median will be a better measure.
QUESTION 8

a. We are given $TP = 3$, then $TR = TP$ (radii are equal) and so $TR = 3$.
Also, we have $TP \perp OP$ ($OP$ is a tangent to circle $T$). Hence we have that:

\[ OP^2 = OT^2 - TP^2 \]
\[ \therefore OP = 4 \]
\[ \therefore OR = OP = 4 \] (tangents drawn from the same point $O$)
\[ \therefore x_T = 4 \]
\[ \therefore T(4; 3). \]

b. We have: $\tan T\hat{O}R = \frac{3}{4}$
\[ \therefore T\hat{O}R = 36,87° \]

c. From the diagram, we have:

\[ P\hat{O}R = 2 \times 36,87° \]
\[ = 71,74° \] (properties of kite $OPTR$)
\[ \therefore \sin P\hat{O}R = \frac{y_p}{4} \]
\[ \therefore y_p = 4 \sin 71,74° \]
\[ = 3,84 \text{ units}. \]

QUESTION 9

a. We have: $OC^2 = OB^2 + BC^2$
\[ \therefore OC^2 = 20 + 80 \]
\[ \therefore OC = \sqrt{100} = 10 \text{ units}. \]

b. From our diagram, we have:

\[ \tan O\hat{C}B = \frac{\sqrt{20}}{\sqrt{80}} = \frac{1}{2} \]
\[ \therefore m_{AC} = \tan(180° - O\hat{C}B) \]
\[ \therefore m_{AC} = -\tan O\hat{C}B \]
\[ \therefore m_{AC} = -\frac{1}{2} \]
c.

From part b. we know that \( m_{AC} = -\frac{1}{2} \). Then, \( m_{OB} = -\frac{1}{m_{AC}} = 2 \) (since \( OB \perp AC \)).

Then, in our diagram, we have that \( B(k; 2k) \). Now, using Pythag on \( \triangle OMB \), we have:

\[
OM^2 + BM^2 = OB^2
\]
\[
\therefore k^2 + (2k)^2 = (\sqrt{20})^2
\]
\[
\therefore 5k^2 = 20
\]
\[
\therefore k = 2
\]
\[
\therefore B(2; 4).
\]

d. Proof:

Let \( C\hat{O}B = \theta \).

\[
\therefore A\hat{O}B = 90^\circ - \theta
\]
\[
\therefore O\hat{A}B = 90^\circ - A\hat{O}B
\]
\[
= 90^\circ - (90^\circ - \theta)
\]
\[
= \theta
\]
\[
\therefore O\hat{A}B = C\hat{O}B
\]
\[
\therefore \triangle ABO \parallel \triangle OBC \text{ (AAA)}
\]
\[
\therefore \frac{AB}{OB} = \frac{BO}{BC}
\]
\[
\therefore AB = \frac{OB^2}{BC} \blacksquare
\]
QUESTION 10

a. Let $AB = 4k$ and $BC = 7k$. Then $AC = AB + BC = 11k$.
   \[\frac{FE}{FC} = \frac{AB}{AC} = \frac{4}{11}\] (By the Proportionality theorem)

b. Let $AG = 9m$ and $AF = 17m$. Then $GF = AF - AG = 8m$.
   \[\frac{CD}{DF} = \frac{AG}{GF} = \frac{9}{8}\] (By the Proportionality theorem)

C.

Let $FE = 4p$ and $EC = 7p$. By part b. we also have $FD = 8m$ and $DC = 9m$.
We are given that $FC = 374$. Hence we have:

\[FC = FE + EC = 11p = 374. \quad \text{Then} \quad p = 34. \quad \text{Similarly, we have} \]
\[FC = FD + DC = 17m = 374. \quad \text{Then} \quad m = 22. \]
Hence $ED = 374 - FE - DC = 374 - 4p - 9m = 40$ km.
Therefore it will take $40 \times 50 = 2000$ hours to build the section from E to D.
QUESTION 11

a. Proof:

We have $\hat{C}_2 = \hat{D}$ (angles in the same segment $FE$)

$\hat{C}_1 + \hat{C}_2 = 90^\circ$ (angle in a semi-circle)

$\therefore \hat{C}_1 + \hat{D} = 90^\circ$ (since $\hat{C}_2 = \hat{D}$)

$b. We are given that $\hat{D} = 38^\circ$. Let us make the following construction:

Construct chord $BF$.

Then, we have: $\hat{C}_1 = 90^\circ - \hat{C}_2$

$= 90^\circ - 38^\circ$ (since $\hat{C}_2 = \hat{D}$)

$= 52^\circ$

Now, $\hat{A}FB = \hat{C}_1 = 52^\circ$ (By the tan-chord thm.)

$\hat{ABF} = 52^\circ$ (By the tan-chord thm.)

$\therefore \hat{BAF} = 180^\circ - (\hat{ABF} + \hat{AFB})$

$= 180^\circ - (52^\circ + 52^\circ)$

$= 76^\circ$ (angles in a $\Delta$)
QUESTION 12

a. We have: Area of $\Delta ADC = \frac{1}{2} \times AD \times DC \times \sin \widehat{ADC}$
   
   $= \frac{1}{2} \times 6 \times 6 \times \sin 130^\circ$
   
   $= 13.8 \text{ units}^2$

b. Proof:
   Since DABC is a cyclic quad (see diagram), we have:
   
   $\widehat{ABC} + \widehat{ADC} = 180^\circ$
   
   $\therefore \widehat{ADC} = 180^\circ - 130^\circ = 50^\circ$

   Now, $\widehat{ADB} = \widehat{DBC}$ ($AD = DC$, $\therefore$ equal chords subtend equal angles)

   Hence $\widehat{DBC} = \frac{1}{2}(50^\circ) = 25^\circ$. $\blacksquare$

c. Firstly, we have that: $BC = 6 + 6 = 12$ units (line from the centre bisects chord)

   Now, using the sine rule:
   
   $\frac{12}{\sin \widehat{BCD}} = \frac{6}{\sin 25^\circ}$

   $\therefore \sin \widehat{BCD} = 2 \sin 25^\circ$

   $\therefore \sin \widehat{BCD} = 0.845 \ldots$

   $\therefore$ Key angle $= 57.7^\circ$

   $\therefore \widehat{BCD} = 180^\circ - 57.7^\circ$

   $= 122.3^\circ$

   $\therefore \theta = 180^\circ - (25^\circ + 122.3^\circ)$ (Angles in $\Delta DBC$)

   $\therefore \theta = 32.7^\circ$
d. When we lift point B vertically 9 units above point A, we get the following triangle:

We want to find angle \( T \hat{C} A \). Using the cosine rule on \( \Delta ADC \), we get:

\[
AC^2 = 6^2 + 6^2 - 2(6)(6) \cos 130^\circ
\]
\[
\therefore AC^2 = 118,28 \ldots
\]
\[
\therefore AC = 10,875 \ldots
\]
\[
\therefore \tan T \hat{C} A = \frac{TA}{AC}
\]
\[
\therefore \tan T \hat{C} A = \frac{9}{10,875} \ldots
\]
\[
\therefore T \hat{C} A = 39,6^\circ.
\]