Answers to:

Mathematics
IEB 2016 Paper 2

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QUESTION 1

a. We are given $O\hat{A}B = 135°$ and $O\hat{C}B = 90°$, so $O\hat{A}B + O\hat{C}B = 225°$.
Hence opp. angles do not add up to $180°$.

b. Using tan-gradient, we have $m_{AB} = \tan 45° = 1$. Since $OA = 8$ units, then $A(0; 8)$.
Substitute point A into $y = mx + c$, then $y = x + 8$.

c.

1. Since $OC = 6$ units, then $C(6; 0)$. Hence $x = 6$ is the equation of BC.

2. $OCBA$ is a parallelogram with base lengths of OA and BC. The perpendicular height is given by OC. Note $B(6; 14)$, hence:
Area = $\frac{1}{2}(OA + BC)(OC) = \frac{1}{2}(8 + 14)(6) = 66\, units^2$

QUESTION 2

a.

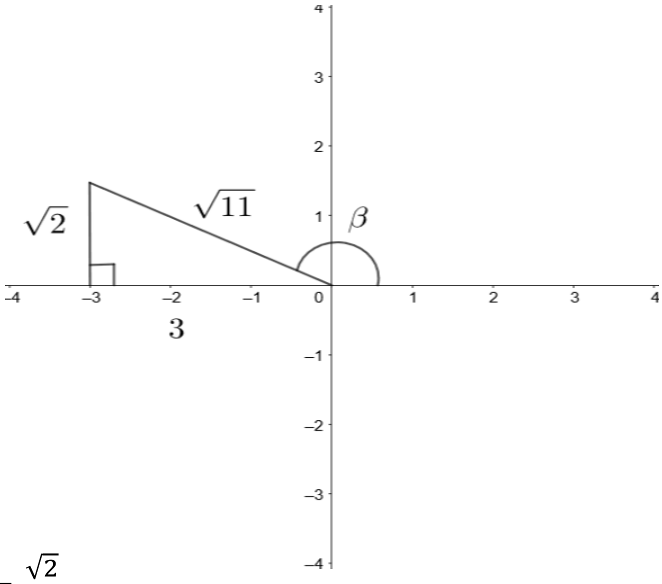
1. $M = \frac{2\sin^2\theta + 2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$
$\therefore M = \frac{2\sin\theta(\sin\theta + \cos\theta)}{(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)}$
$\therefore M = \frac{2\sin\theta}{(\cos\theta - \sin\theta)}$
$\therefore M = P$

2. P is undefined when: $\cos\theta - \sin\theta = 0$
$\therefore \cos\theta = \sin\theta$
$\therefore 1 = \tan\theta$
$\therefore$ Reference angle $= 45°$
$\therefore \theta = -135°$ or $45°$ or $225°$

b.

1. Quadrant 2

2.



$\therefore \tan\beta = \frac{y}{x} = \frac{\sqrt{2}}{-3}$

c.

1. $\cos(\alpha - 30°) - \cos(\alpha + 30°)$

$= \cos\alpha\cos 30° + sin\alpha \sin 30° - (\cos\alpha\cos 30° - \sin\alpha\sin 30°)$

$= \cos\alpha\cos 30° + \sin\alpha\sin 30° - \cos\alpha\cos 30° + \sin\alpha\sin 30°$

$= 2\sin\alpha\sin 30° = 2\sin\alpha \times \left(\frac{1}{2}\right)$

$= \sin\alpha$

2. $\sin\alpha = 2\sin^2\alpha$

$\therefore 0 = \sin\alpha(2\sin\alpha - 1)$

$\therefore \sin\alpha = 0$ or $\sin\alpha = \frac{1}{2}$

$\therefore \alpha = 0° + k.180°, k \in \mathbb{Z}$

$\therefore \alpha = 30° + k.360°, k \in \mathbb{Z}$

or $\alpha = 150° + k.360°, k \in \mathbb{Z}$

QUESTION 3

a. We have: Radius of circle Q is $9 - 5 = 4$ units. Now,

x_Q of the centre of circle Q is $9 + 5 = 14$ units,

y_Q of the centre of circle Q is 5 units.

Hence the equation of circle Q is given by: $(x - 14)^2 + (y - 5)^2 = 16$

b. We are given: $(x - p)^2 + y^2 - 22y = -117$. We complete the square on the LHS:

$\therefore (x - p)^2 + y^2 - 22y + 121 = -117 + 121$

$\therefore (x - p)^2 + (y - 11)^2 = 4$

Hence the length of RQ is $4 + 2 = 6$ units.

c. To get the length of AB, we first calculate:

Length of: $PR^2 = PQ^2 + QR^2$

$\therefore PR = \sqrt{(14-5)^2 + (11-5)^2}$

$\therefore PR = \sqrt{117}$

Length of: $PA = 5$

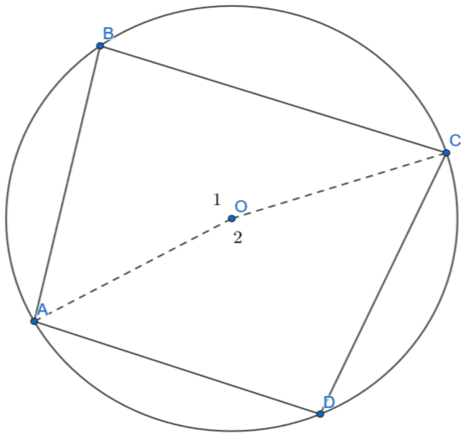
Length of: $BR = 2$

Therefore the length of line $AB = PR - PA - BR$

$= \sqrt{117} - 5 - 2$

$= 3{,}82$ units

QUESTION 4



a. Draw AO and OC. Then,

R.T.P: $\hat{B} + \widehat{D} = 180°$

Proof:

$\hat{O}_2 = 2 \times \hat{B}$ (Angle at centre)

$\hat{O}_1 = 2 \times \widehat{D}$ (Angle at centre)

$\hat{O}_1 + \hat{O}_2 = 360°$ (Angle around a pt.)

$\therefore 2\widehat{D} + 2\hat{B} = 360°$

$\therefore \hat{B} + \widehat{D} = 180°.$ ■

b. We have: $A\hat{B}C = 62°$ (tan-chord thm.)

$A\hat{O}C = 124°$ (Angle at centre = 2×Angle at circum.)

$\hat{C}_2 = \hat{A}_3 = 28°$ ($OC = OA$, radii are equal)

$\hat{A}_2 = 25°$ (Given)

$\therefore \hat{C}_1 = 180° - (\hat{A}_2 + \hat{A}_3 + \hat{B} + \hat{C}_2)$ (Angles in a Δ)

$\therefore \hat{C}_1 = 180° - (25° + 28° + 62° + 28°)$

$= 37°$

c.

1. We can conclude that $N = Q$.

2. Proof: $\widehat{D}_1 = \hat{B}$ (ext. angle of a cyclic quad)

$\widehat{D}_1 = \hat{A}_1 + \hat{C}_2$ (ext. angle of Δ = sum of two int. opp. angles)

$\therefore \hat{B} = \hat{A}_1 + \hat{C}_2.$ ■

QUESTION 5

a. Sub. $x = 360°$ into $y = 3\sin x + 1$:
$\therefore y = 3\sin 360° + 1$
$\therefore y = 1$
$\therefore B(360°; 1)$

b. $3\sin x + 1 = -1$
$\therefore 3\sin x = -2$
$\therefore \sin x = -\frac{2}{3}$
Key angle $= 41{,}81°$

Hence $x = 221{,}81°$ or $x = 318{,}19°$.

c. From the graph, we can see that any straight line $g(x) = k$ will cut through $f(x)$ in the interval $[0°; 180°]$ in between the values of $1 \leq k \leq 4$. Hence there will be no solutions when $k > 4$ or $k < 1$.

QUESTION 6

a. Using you calculator, we get: $r = 0.9755$, therefore a very strong relationship.

b. Remember that: $y = a + bx$, where: $A = 2788{,}26$ and $B = 1658{,}39$. Hence $y = 2788{,}26 + 1658{,}39x$.

c. $y = \text{R } 34297{,}67$.
Hence the managers projected income based on the line of best fit is R 34297,67 and the actual sales was $\text{R } 23000$. So this would not be considered a successful day.

NB: It is important that you are familiar with using your calculator for statistics questions such as regression modelling, finding means/variances, etc. We have attached a step-by-step instruction guide on how to use your Casio calculator to compute these statistical operations.

How to use a Casio calculator for Regression modelling

Press:
MODE → 3:STAT → 2: A + Bx
Enter data into the x and y columns
Press: AC
To find A:
SHIFT → 1 → 5:Reg → 1:A → =
To find B:
SHIFT → 1 → 5:Reg → 2:B → =
To find r (correlation coefficient)
SHIFT → 1 → 5:Reg → 3:r → =
To find $\hat{y}$ given $\hat{x}$:
Enter $\hat{x}$ - value → SHIFT → 1 → 5:Reg → 5: $\hat{y}$ → =
To find the mean point $(\bar{x}; \bar{y})$
SHIFT → 1 → 4:Var → 2: $\bar{x}$ → =
SHIFT → 1 → 4:Var → 5: $\bar{y}$ → =

How to use a Casio calculator to find Mean and Standard Deviation

Press:
MODE → 3:STAT → 1: 1 - VAR
Enter data into the x and FREQ columns

If **no** FREQ column then PRESS:
SHIFT → SET UP → page down → 4: STAT → 1: ON

Press: AC: →
To find the mean:
SHIFT → 1 → 4: Var → 2: $\bar{x}$

To find the standard deviation:
SHIFT → 1 → 4: Var → 3: σx
Remember: $variance = (\sigma x)^2$

SECTION B

QUESTION 7

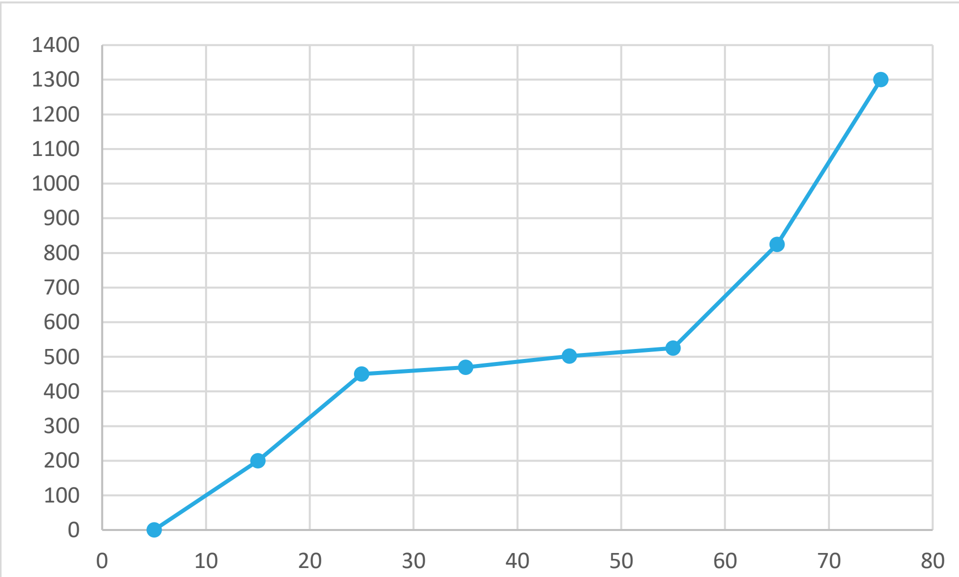
a. $A = 250$ and $B = 502$

b.

1. $\bar{x} \approx 47{,}14$ (Use calculator)

2. $65 < x \leq 75$

c.



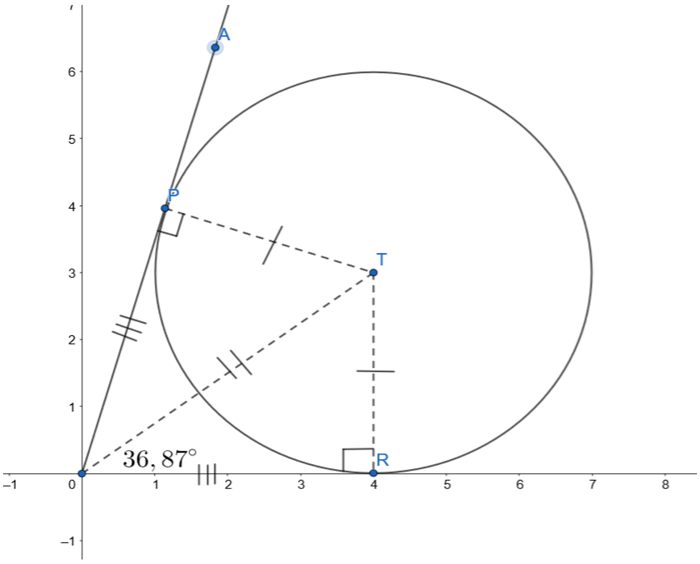
d.

1. No, the data is skewed to the left since the mean is less than the median.

2. No, the mean is not a good indicator since it's affected by the extremes. The median will be a better measure.

QUESTION 8

a. We are given $TP = 3$, then $TR = TP$ (radii are equal) and so $TR = 3$.
Also, we have $TP \perp OP$ (OP is a tangent to circle T). Hence we have that:
$OP^2 = OT^2 - TP^2$
$\therefore OP = 4$
$\therefore OR = OP = 4$ (tangents drawn from the same point O)
$\therefore x_T = 4$
$\therefore T(4; 3)$.



b. We have: $\tan T\hat{O}R = \frac{3}{4}$
$\therefore T\hat{O}R = 36{,}87°$

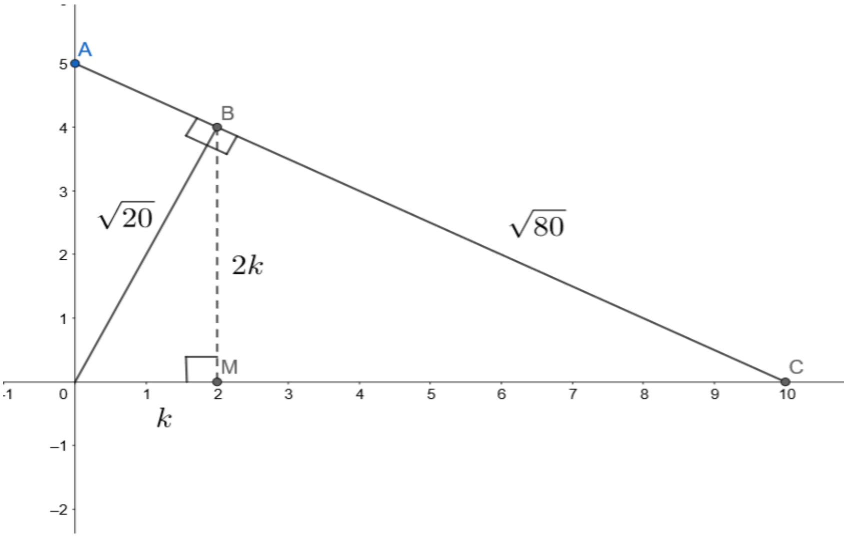
c. From the diagram, we have:
$P\hat{O}R = 2 \times 36{,}87°$
$= 71{,}74°$ (properties of kite OPTR)
$\therefore \sin P\hat{O}R = \frac{y_p}{4}$
$\therefore y_p = 4 \sin 71{,}74°$
$= 3{,}84$ units.

QUESTION 9

a. We have: $OC^2 = OB^2 + BC^2$
$\therefore OC^2 = 20 + 80$
$\therefore OC = \sqrt{100} = 10$ units.

b. From our diagram, we have:
$\tan O\hat{C}B = \frac{\sqrt{20}}{\sqrt{80}} = \frac{1}{2}$
$\therefore m_{AC} = \tan(180° - O\hat{C}B)$
$\therefore m_{AC} = -\tan O\hat{C}B$
$\therefore m_{AC} = -\frac{1}{2}$

c.



From part b. we know that $m_{AC} = -\frac{1}{2}$. Then, $m_{OB} = -\frac{1}{m_{AC}} = 2$ (since $OB \perp AC$). Then, in our diagram, we have that $B(k; 2k)$. Now, using Pythag on ΔOMB, we have:

$$OM^2 + BM^2 = OB^2$$
$$\therefore k^2 + (2k)^2 = \left(\sqrt{20}\right)^2$$
$$\therefore 5k^2 = 20$$
$$\therefore k = 2$$
$$\therefore B(2; 4).$$

d. Proof:

Let $C\hat{O}B = \theta$.

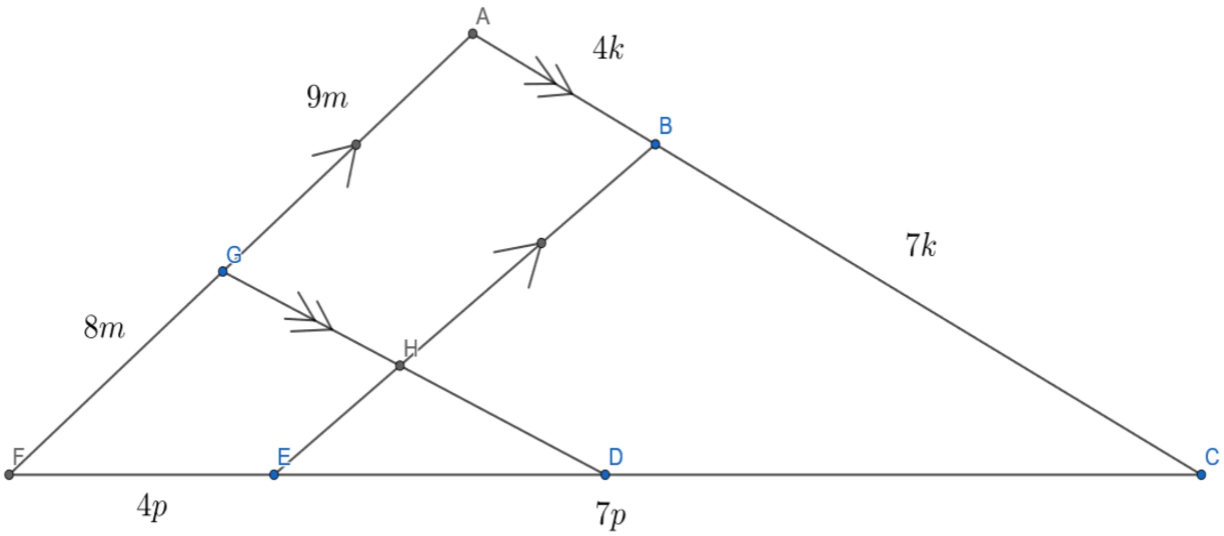
$$\therefore A\hat{O}B = 90° - \theta$$
$$\therefore O\hat{A}B = 90° - A\hat{O}B$$
$$= 90° - (90° - \theta)$$
$$= \theta$$
$$\therefore O\hat{A}B = C\hat{O}B$$
$$\therefore \Delta ABO \ ||| \ \Delta OBC \text{ (AAA)}$$
$$\therefore \frac{AB}{OB} = \frac{BO}{BC}$$
$$\therefore AB = \frac{OB^2}{BC} \ \blacksquare$$

QUESTION 10

a. Let $AB = 4k$ and $BC = 7k$. Then $AC = AB + BC = 11k$.
$\therefore \frac{FE}{FC} = \frac{AB}{AC} = \frac{4}{11}$. (By the Proportionality theorem)

b. Let $AG = 9m$ and $AF = 17m$. Then $GF = AF - AG = 8m$.
$\therefore \frac{CD}{DF} = \frac{AG}{GF} = \frac{9}{8}$. (By the Proportionality theorem)

c.



Let $FE = 4p$ and $EC = 7p$. By part b. we also have $FD = 8m$ and $DC = 9m$.
We are given that $FC = 374$. Hence we have:

$FC = FE + EC = 11p = 374$. Then $p = 34$. Similarly, we have
$FC = FD + DC = 17m = 374$. Then $m = 22$.
Hence $ED = 374 - FE - DC = 374 - 4p - 9m = 40$ km.
Therefore it will take $40 \times 50 = 2000$ hours to build the section from E to D.

QUESTION 11

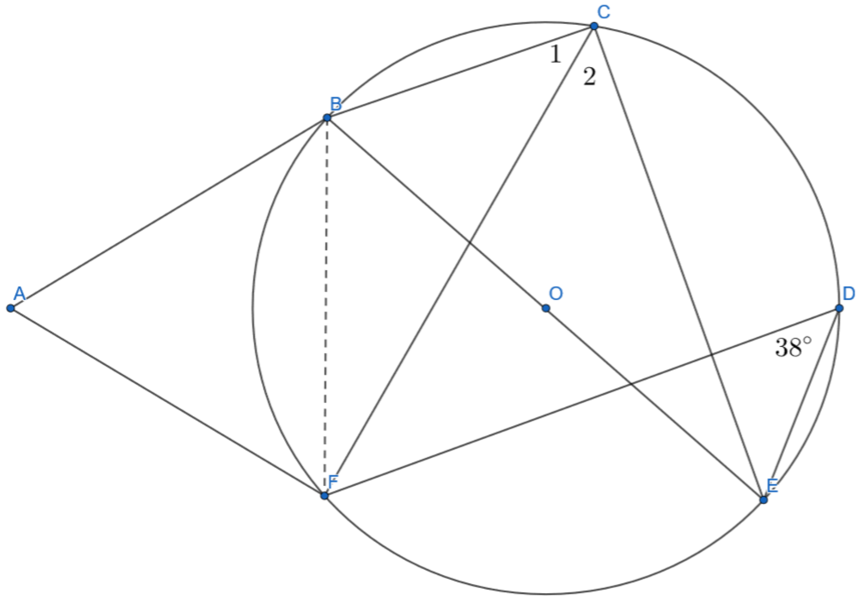
a. Proof:

We have $\hat{C}_2 = \widehat{D}$ (angles in the same segment FE)

$\hat{C}_1 + \hat{C}_2 = 90°$ (angle in a semi-circle)

$\therefore \hat{C}_1 + \widehat{D} = 90°$ (since $\hat{C}_2 = \widehat{D}$) ■

b. We are given that $\widehat{D} = 38°$. Let us make the following construction:
Construct chord BF.



Then, we have: $\hat{C}_1 = 90° - \hat{C}_2$

$= 90° - 38°$ (since $\hat{C}_2 = \widehat{D}$)

$= 52°$

Now, $A\hat{F}B = \hat{C}_1 = 52°$ (By the tan-chord thm.)

$A\hat{B}F = 52°$ (By the tan-chord thm.)

$\therefore B\hat{A}F = 180° - (A\hat{B}F + A\hat{F}B)$

$= 180° - (52° + 52°)$

$= 76°$ (angles in a Δ)

QUESTION 12

a. We have: Area of $\Delta ADC = \frac{1}{2} \times AD \times DC \times \sin A\widehat{D}C$

$= \frac{1}{2} \times 6 \times 6 \times \sin 130°$

$= 13{,}8$ units^2

b. Proof:

Since DABC is a cyclic quad (see diagram), we have:

$A\widehat{B}C + A\widehat{D}C = 180°$

$\therefore A\widehat{B}C = 180° - 130° = 50°$

Now, $A\widehat{B}D = D\widehat{B}C$ ($AD = DC$, $\therefore$ equal chords subtend equal angles)

Hence $D\widehat{B}C = \frac{1}{2}(50°) = 25°$. ■

c. Firstly, we have that: $BC = 6 + 6 = 12$ units (line from the centre bisects chord)

Now, using the sine rule:

$$\frac{\sin B\widehat{D}C}{12} = \frac{\sin 25°}{6}$$

$\therefore \sin B\widehat{D}C = 2 \sin 25°$

$\therefore \sin B\widehat{D}C = 0.845 \ldots$

$\therefore$ Key angle $= 57{,}7°$

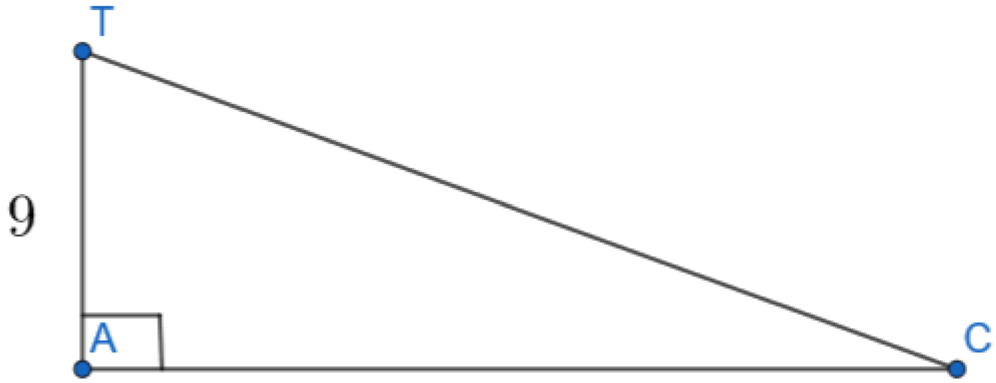
$\therefore B\widehat{D}C = 180° - 57{,}7°$

$= 122{,}3°$

$\therefore \theta = 180° - (25° + 122{,}3°)$ (Angles in ΔDBC)

$\therefore \theta = 32{,}7°$

d. When we lift point B vertically 9 units above point A, we get the following triangle:



We want to find angle $T\hat{C}A$. Using the cosine rule on ΔADC, we get:

$$AC^2 = 6^2 + 6^2 - 2(6)(6)\cos 130°$$
$$\therefore AC^2 = 118{,}28\ldots$$
$$\therefore AC = 10{,}875\ldots$$
$$\therefore \tan T\hat{C}A = \frac{TA}{AC}$$
$$\therefore \tan T\hat{C}A = \frac{9}{10{,}875\ldots}$$
$$\therefore T\hat{C}A = 39{,}6°.$$