Answers to:

Mathematics
IEB 2016 Paper 1

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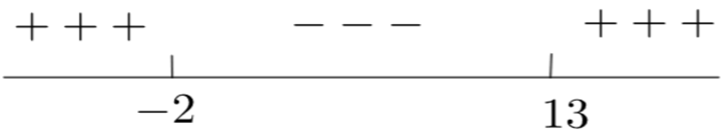
SECTION A

QUESTION 1

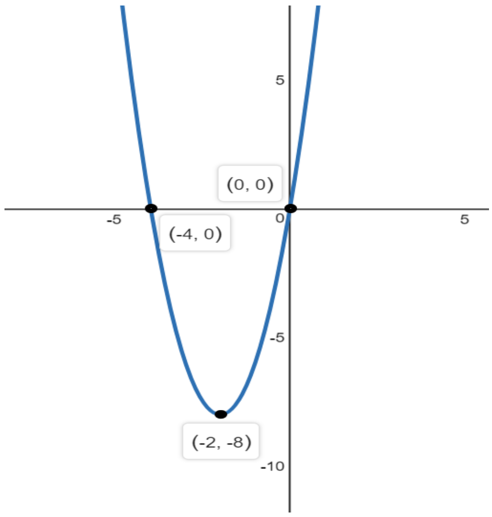
a.

1. $$\frac{4x}{2} - \frac{2x+1}{3} = 5$$
$$\therefore 12x - 2(2x+1) = 30$$
$$\therefore 8x = 32$$
$$\therefore x = 4$$

2. $(x-5)(x-6) \leq 56$
$x^2 - 11x + 30 \leq 56$
$x^2 - 11x - 26 \leq 0$
$(x-13)(x+2) \leq 0$
Critical values: $x = 13$ or $x = -2$
$\therefore -2 \leq x \leq 13$



b. $TP(-2;-8)$
Y-int: $(0;0)$
X-int: Let $y = 0$
$\therefore 2(x+2)^2 - 8 = 0$
$\therefore (x+2)^2 = 4$
$\therefore x + 2 = -2$ or $x + 2 = 2$
$\therefore x = 0$ or $x = -4$
$\therefore x$-int: $(0;0)$ and $(-4;0)$



c.

1. $x = -1$ and $y = 2$

2. $$\frac{4}{x+1} + 2 = x$$
$$\therefore 4 + 2(x+1) = x(x+1)$$
$$\therefore 4 + 2x + 2 = x^2 + x$$
$$\therefore x^2 - x - 6 = 0$$
$$\therefore (x-3)(x+2) = 0$$
$\therefore (3;3)$ and $(2;2)$

d. $c = -1$ or $-\frac{1}{4}$ or $-\frac{1}{9}$ or $-\frac{1}{16}$ or $-\frac{1}{25}$, ..., etc.

e. Remember that $b^2 - 4ac < 0$, so $3 - k < 0$, $\therefore k > 3$

QUESTION 2

a.

1. LHS $= 3\left(\frac{1}{3}\right) = 1$

   RHS $= \sqrt{6\left(\frac{1}{3}\right) - 1} = -1$

   LHS $\neq$ RHS $\therefore\ x = \frac{1}{3}$ is incorrect

2. $$3x = -\sqrt{6x-1}$$
   $$(3x)^2 = 6x - 1$$
   $$9x^2 - 6x + 1 = 0$$
   $$\therefore\ x = \frac{1}{3}$$
   From part (1), there is no solution.

b. $7^{x+a}(1+3) = 28(7^{a^2})$

$$7^{x+a} = \frac{28(7^{a^2})}{4}$$
$$7^{x+a} = 7(7^{a^2})$$
$$7^{x+a} = 7^{1+a^2}$$
$$\therefore\ x = a^2 - a + 1$$

QUESTION 3

a. $4800 - \left(4800 \times \frac{13{,}5}{100}\right) = \text{R } 4152$

b. $415200 = x\left[\frac{1-\left(1+\frac{7}{1200}\right)^{(-5\times 12)}}{\frac{7}{1200}}\right]$

$$\therefore\ x \approx \text{R } 8221{,}46$$

QUESTION 4

a. Amount paid for all 110 laptops: $6000 \times 110 = 660000$

Depreciation over 5 years: $A = 660000\left(1 - \frac{15}{100}\right)^5 \approx 292845{,}51$

Inflation: $A = P(1+i)^n$

$\therefore A = 660000\left(1 + \frac{6}{100}\right)^5$

$\therefore A = 883228{,}88$

Hence the amount required in 5 years less “buy-back” $= 883228{,}88 - 292845{,}51$

$= \text{R } 590383{,}37$

b. Sinking Fund:

$$F = x\left[\frac{(1+i)^n - 1}{i}\right]$$

$$590383{,}37 = x\left[\frac{\left(1+\frac{12}{1200}\right)^{(5\times 12)} - 1}{\frac{12}{1200}}\right]$$

$$\therefore x \approx \text{R } 7228{,}92$$

QUESTION 5

a. $7 + 12 + 17 + \cdots + (5y + 2) = 36$, this is an arithmetic sequence.

$$\therefore S_n = \frac{n}{2}(a + l)$$

$$\therefore \frac{y}{2}(7 + 5y + 2) = 36$$

$$\therefore 5y^2 + 9y = 72$$

$$\therefore 5y^2 + 9y - 72 = 0$$

$$\therefore (5y + 24)(y - 3) = 0$$

$$\therefore y = 3$$

b.

1. $3p - (2p + 14) = (p + 7) - 3p$

   $3p - 2p - 14 = p + 7 - 3p$

   $3p = 21$

   $p = 7$

2. $a = 28$ and $d = -7$

   $\therefore S_{38} = \frac{38}{2}[2(28) + (38 - 1)(-7)]$

   $\therefore S_{38} = -3857$

c. $T_n = an^2 + bn + c$

$7; 13; 21; 31; \ldots$

$\therefore$ First differences: $6; 8; 10; \ldots$

$\therefore$ Second differences: $2; 2; 2; \ldots$

$\therefore$ Quadratic sequence

$\therefore 2a = 2$ and so $a = 1$

$\therefore 3a + b = 6$ and so $b = 3$

$\therefore a + b + c = 7$ and so $c = 3$

$\therefore T_n = n^2 + 3n + 3$

d. $9; 6; 4; \ldots$ Geometric sequence $\therefore r = \frac{2}{3}$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$25 = \frac{9\left[\left(\frac{2}{3}\right)^n - 1\right]}{\frac{2}{3} - 1}$$

$$\left(\frac{2}{3}\right)^n = \frac{2}{27}$$

$$n = \log_{\frac{2}{3}} \frac{2}{27}$$

$$n = 6{,}41$$

$\therefore n = 7$ is the smallest

e. Volume of pyramid 1 $= \frac{1}{3} \times (9 \times 9) \times 27 = 729\ cm^3$

Volume of pyramid 2 $= \frac{1}{3} \times \left(\frac{81}{3}\right) \times \frac{27}{3} = 81\ cm^3$

Volume of pyramid 3 $= \frac{1}{3} \times \left(\frac{27}{3}\right) \times \frac{9}{3} = 9\ cm^3$

The sequence is geometric with $a = 729$ and $r = \frac{1}{9}$

$\therefore S_\infty = \frac{a}{1 - r} = 820\frac{1}{8}\ cm^3$

QUESTION 6

a. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

Working out:

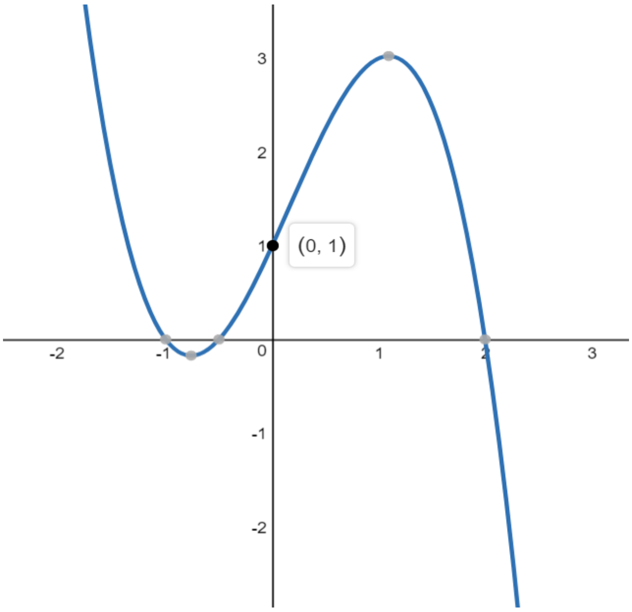
$$f(x) = 3x^2 + 2x$$
$$f(x+h) = 3(x+h)^2 + 2(x+h)$$
$$f(x+h) = 3x^2 + 6xh + 3h^2 + 2x + 2h$$

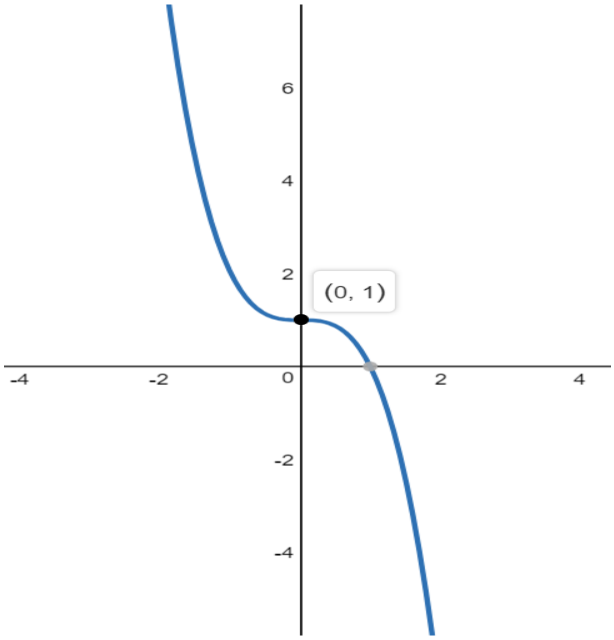
$$\therefore f'(x) = \lim_{\mathrm{h} \to 0} \frac{3x^2 + 6xh + 3h^2 + 2x + 2h - (3x^2 + 2x)}{h}$$
$$f'(x) = \lim_{h \to 0} \frac{6xh + 3h^2 + 2}{h}$$
$$f'(x) = \lim_{h \to 0} (6x + 3h + 2)$$
$$f'(x) = 6x + 2$$

b. $y = -x^{-1} + x^{\frac{1}{2}}$

$\therefore \frac{dy}{dx} = +x^{-2} + \frac{1}{2} x^{-\frac{1}{2}}$

QUESTION 7

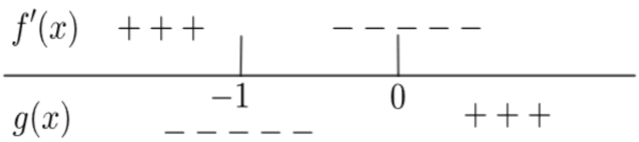




Above are two possible graphs, where we have concave down for $x > 0$ and inflection point at the point $(0; 1)$.

QUESTION 8

a. $f'(x).g(x) < 0$, Find critical values:
$f'(x) = 0$ at $x = -1$ (TP)
$g(x) = 0$ at $x = 0$
$\therefore x < -1$ or $x > 0$



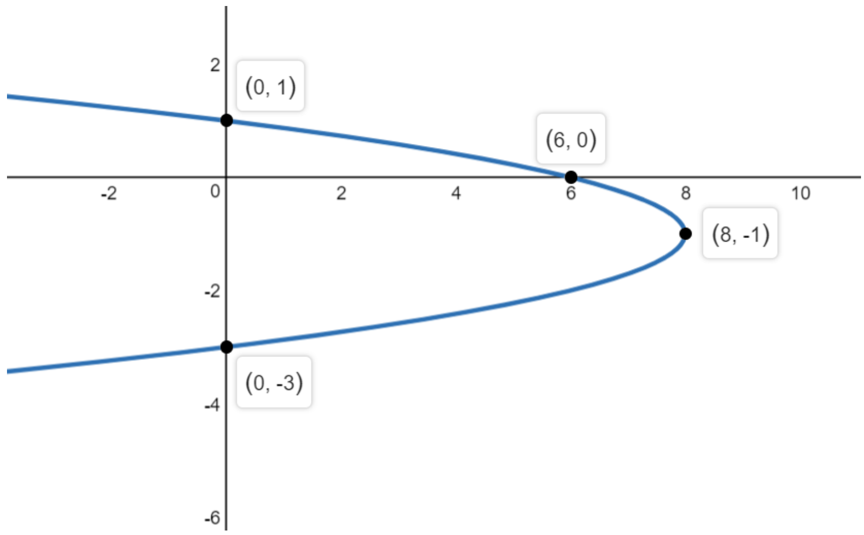
b. $g(x) = d^x + q$, sub. $(0; 0)$
$\therefore 0 = d^0 + q$
$\therefore q = -1$, sub. $(1; 2)$
$\therefore 2 = d^1 - 1$
$\therefore d = 3$
$\therefore g(x) = 3^x - 1$

c. Inverse of g:
$x = 3^y - 1$
$3^y = x + 1$
$\therefore y = \log_3(x + 1)$

d. Domain of g^{-1} is when $x + 1 > 0$, $\therefore x > -1$

e. $f(x) = a(x + 3)(x - 1)$, sub. $(0; 6)$
$\therefore 6 = a(3)(-1)$
$\therefore a = -2$
Hence $f(x) = -2(x + 3)(x - 1)$
$= -2x^2 - 4x + 6$
$\therefore a = -2, b = -4, c = 6$

f.



g. $k > -6$

QUESTION 9

a. $f(1) = a(1)^3 + b(1)^2 = a + b$

$f(2) = a(2)^3 + b(2)^2 = 8a + 4b$

$\therefore \frac{y_B - y_A}{x_B - x_A} = 5,5$

$\therefore \frac{8a + 4b - (a+b)}{2-1} = 5,5$

$\therefore 7a + 3b = 5,5$ … Eq(1)

$f'(x) = 3ax^2 + 2bx$

$-18 = 3a(6)^2 + 2b(6)$

$-18 = 108a + 12b$ … Eq(2)

Now, $4 \times$Eq(1) – Eq(2): $\begin{cases} 28a + 12b = 22 \\ 108a + 12b = -18 \end{cases}$

$\therefore -80a = 40$

$\therefore a = -\frac{1}{2}$ and $b = 3$

b. $f(x)$ increases when $f'(x) \geq 0$

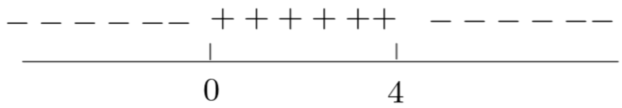
$\therefore -\frac{3}{2}x^2 + 6x \geq 0$

$\therefore -3x^2 + 12x \geq 0$

$-3x(x - 4) \geq 0$

Critical values: $x = 0$ or $x = 4$

$\therefore 0 \leq x \leq 4$



c. f is concave down when:

$f''(x) < 0$

$\therefore -6x + 12 < 0$

$\therefore x > 2$

QUESTION 10

We are given that:

$h + r = 9$

$\therefore h = 9 - r$

Now,

$V = \pi r^2 h$

$\therefore V = \pi r^2(9 - r)$

$\therefore V = 9\pi r^2 - \pi r^3$

Hence,

$\frac{dV}{dr} = 18\pi r - 3\pi r^2$, set $\frac{dV}{dr} = 0$. Then,

$0 = 18\pi r - 3\pi r^2$

$0 = 3\pi r(6 - r)$

But $r \neq 0$, $\therefore r = 6$ units

QUESTION 11

a.

1. $\frac{46}{80} \times \frac{45}{79} = 0{,}3$

2. $\left(\frac{9}{80} \times \frac{25}{79}\right) + \left(\frac{25}{80} \times \frac{9}{79}\right)$
$= \frac{45}{1264} + \frac{45}{1264}$
$= \frac{45}{632} \approx 0{,}07$

b. $\frac{8!}{2!2!} = 10080$

c. We have: $P(\text{Khanya will win}) = P(RB) + P(RRRB) + P(RRRRRB) + \cdots$
Note: Busi and Khanya alternate, so the first probability is given by:
$P(RB) =$ Probability that Busi chooses R first, then Khanya chooses B second;
$P(RRRB) =$ Probability that Busi chooses R first, then Khanya chooses R second, then Busi chooses R third, then Khanya chooses B fourth, etc.

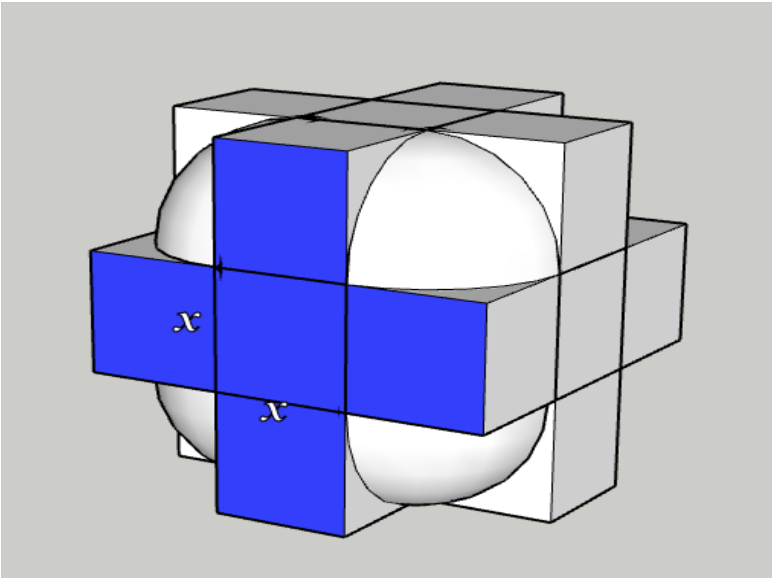
Hence: $P(\text{Khanya will win}) = \left(\frac{6}{7} \times \frac{1}{7}\right) + \left[\left(\frac{6}{7}\right)^3 \times \frac{1}{7}\right] + \left[\left(\frac{6}{7}\right)^5 \times \frac{1}{7}\right] + \cdots$
This is an infinite geometric series and we have that:

$a = \frac{6}{7^2}$ and $r = \left(\frac{6}{7}\right)^2 = \frac{36}{49}$. Thus $-1 < r < 1$.
$\therefore P(\text{Khanya will win}) = \frac{a}{1-r}$
$= \frac{\frac{6}{7^2}}{1-\left(\frac{6}{7}\right)^2}$
$= \frac{6}{13} \approx 0{,}46$

QUESTION 12

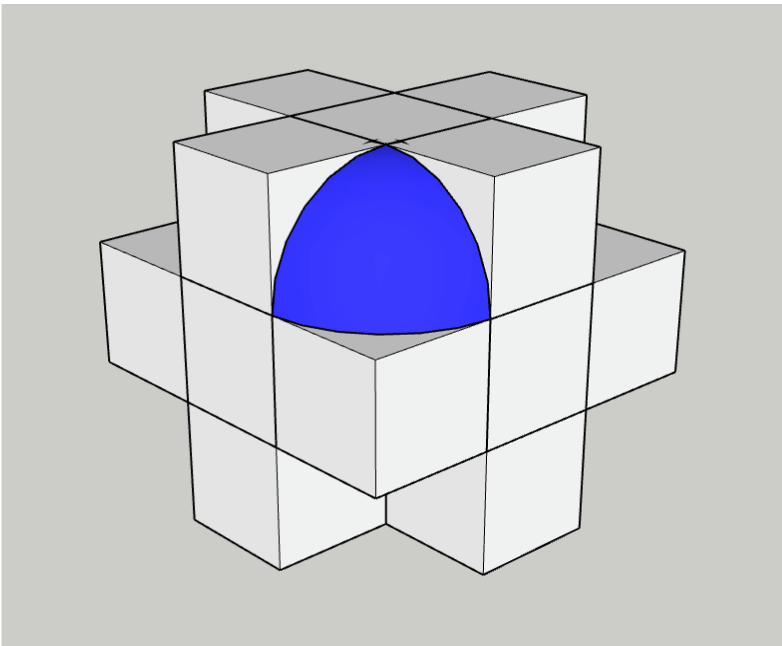
We are given that the total surface area of the paperweight $= 28\ mm^2$. To find the total surface area, we need to identify the three major components that make up the paperweight. These are:

(1) The "cube-part" of the paperweight, i.e. there are 6 parts of the five faced section:



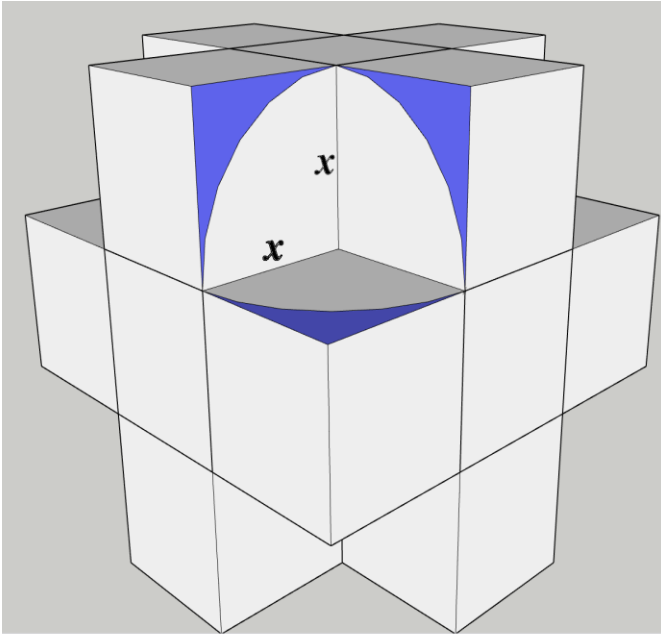
$\therefore 6 \times 5x^2 = 30x^2$

(2) The "sphere part" of the paperweight, i.e. there are 8 equal parts:



$\therefore 8 \times \left(\frac{1}{8} \times 4\pi x^2\right) = 4\pi x^2$

(3) The "corner pieces" of the sphere in the paperweight: There are 3 corners and 8 pieces



Note: We want the blue area, which is Area of square – Area of quarter circle.
$\therefore x^2 - \frac{1}{4}\pi x^2$. There are 3 blue pieces in one corner and there are 8 corners.
$\therefore 8 \times 3 \times (x^2 - \frac{1}{4}\pi x^2)$.

Hence the total surface area of the paperweight is given by adding the three major components areas:

$$30x^2 + 8 \times \left(\frac{1}{8} \times 4\pi x^2\right) + 8 \times 3 \times \left(x^2 - \frac{1}{4}\pi x^2\right) = 28$$
$$\therefore 30x^2 + 4\pi x^2 + 24x^2 - 6\pi x^2 = 28$$
$$\therefore x^2(54 - 2\pi) = 28$$
$$\therefore x^2 = \frac{28}{54-2\pi}$$
$$\therefore x = 0{,}766$$
$$\therefore x \approx 0{,}8$$