Answers to:

Mathematics
IEB 2015 Paper 2

A+

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SECTION A

QUESTION 1

a. Since M is the midpoint of line AB, we have:

$$M = \left(\frac{x_A + x_B}{2}; \frac{y_A + y_B}{2}\right)$$
$$= \left(\frac{0+12}{2}; \frac{6+0}{2}\right)$$
$$= (6; 3)$$

b. We are given that $AMCO$ is a cyclic quad, hence:

$C\widehat{M}A + A\widehat{O}C = 180°$ (Opp. angles of a cyclic quad are supp)

$\therefore C\widehat{M}A + 90° = 180°$ (Since $AOC = 90°$)

$\therefore C\widehat{M}A = 90°$

Now, we can say that $m_{AB} \times m_{MC} = -1$ (Since $MC \perp AB$)

Therefore $m_{AB} = \frac{y_B - y_A}{x_B - x_A}$

$= \frac{0-6}{12-0}$

$= -\frac{1}{2}$

Hence $m_{MC} = -\frac{1}{m_{MC}} = 2$ and line $MC: y = 2x + c$. Sub point $M(6; 3)$:

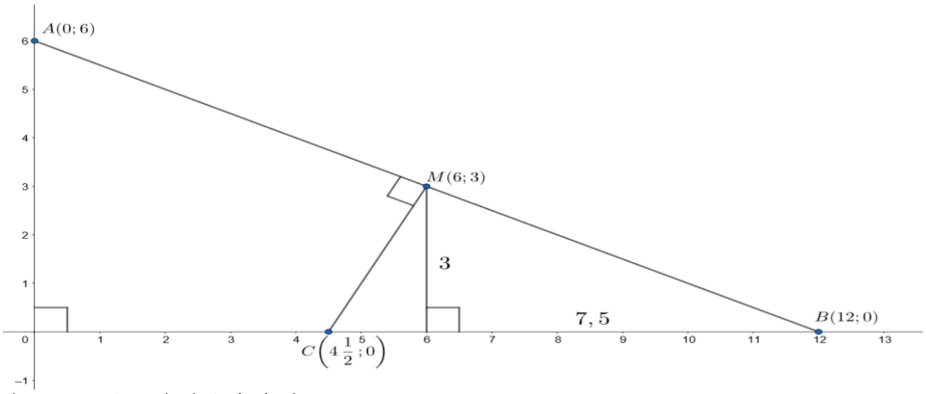
Thus $3 = 2(6) + c \therefore c = -9$. Hence $y = 2x - 9$.

c.

1. R.T.P: Area of $\Delta MCB = 11{,}25$ units^2

   Proof:

   We have that Area of $\Delta MCB = \frac{1}{2}bh$, where $b = BC$ and h is the perpendicular height from the base b. To get the length of BC, we first need the co-ordinates of point C. Since C is the x-int of line MC, sub $y = 0$ into $y = 2x - 9$, then $2x - 9 = 0 \therefore x = \frac{9}{2} = 4\frac{1}{2}$. Hence $C\left(4\frac{1}{2}; 0\right)$. Now, we have that:
   $BC = OB - OC = 12 - 4\frac{1}{2} = 7\frac{1}{2}$. Also, the perpendicular height is given by $y_M = 3 = h$. Thus Area of $\Delta MCB = \frac{1}{2}bh = \frac{1}{2}\left(7\frac{1}{2}\right)(3) = 11{,}25$ units^2.



2. We can see from our diagram that:
   Area of $AMCO$ = Area of ΔAOC − Area of ΔMCB
   $= \frac{1}{2}(12)(6) - 11{,}25$
   $= 36 - 11{,}25$
   $= 24{,}75$ units^2

QUESTION 2

a. We can see that N is a point of intersection of the lines $y = x$ and $y = \frac{1}{2}x + 4$.
   Hence: $\frac{1}{2}x + 4 = x$
   $\therefore \frac{1}{2}x = 4$
   $\therefore x = 8$. Sub back into $y = x$
   Therefore $N(8; 8)$.

b. We note that N is the centre of the circle and point B is perpendicularly above a point on the circle, so the radius of the circle is 8 and hence $B(16; 16)$.

c. We can see that D has the same y co-ordinate as point $B(16; 16)$. So we have: $D(x, 16)$. Point D also lies on the line $7y = 10x$, hence $7(16) = 10x \therefore x = 11{,}2$. Thus $D(11{,}2; 16)$. So we have $DB = 16 - 11{,}2 = 4{,}8$ units.

QUESTION 3

a.

1. $\frac{\sin(180° - \theta)\cos(90° - \theta) - 1}{\cos(-\theta)} = \frac{\sin\theta\sin\theta - 1}{\cos\theta}$
   $= \frac{\sin^2\theta - 1}{\cos\theta}$
   $= \frac{-\cos^2\theta}{\cos\theta}$ $(1 - \sin^2\theta = \cos^2\theta \therefore \sin^2\theta - 1 = -\cos^2\theta)$
   $= -\cos\theta$

2. We can see that under the square root, we have the same expression as in a. 1 Hence it simplifies to $\sqrt{-\cos\theta}$. Now, this will only be real when $-\cos\theta > 0$, i.e. when $\cos\theta < 0$. This happens in the second and third quadrants. Hence we have real solutions when $\theta \in (90°; 270°)$.

b.

1. R.T.P: $\tan\theta\sin\theta+\cos\theta=\frac{1}{\cos\theta}$

   Proof:

   $$\begin{aligned}\text{L.H.S} &= \tan\theta\sin\theta+\cos\theta\\ &= \frac{\sin\theta}{\cos\theta}\sin\theta+\cos\theta\\ &= \frac{\sin^2\theta+\cos^2\theta}{\cos\theta}\\ &= \frac{1}{\cos\theta}\ \blacksquare\end{aligned}$$

2. We have by part b. 1 that $\tan\theta\sin\theta+\cos\theta=\frac{1}{\cos\theta}$. Hence we have:

   $$\frac{1}{\cos\theta}=\frac{3}{\sin\theta}$$
   $$\therefore\frac{\sin\theta}{\cos\theta}=3$$
   $\therefore\tan\theta=3$

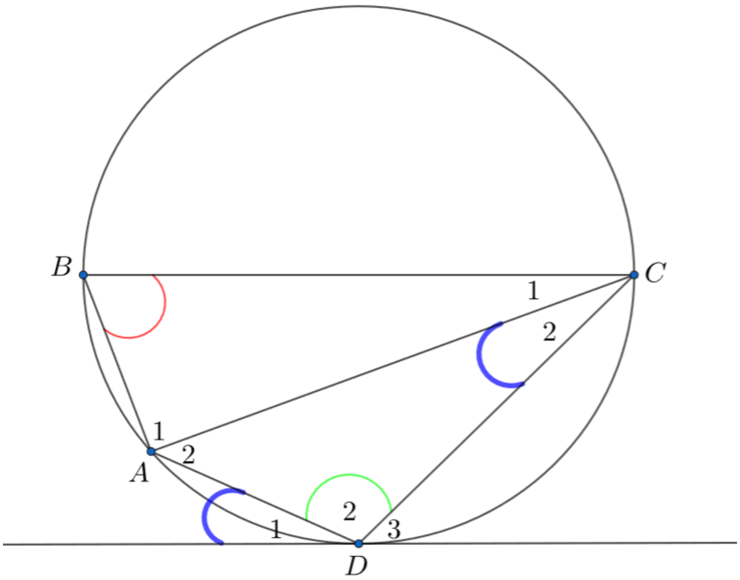
   $\therefore$ Key angle $=71{,}57°$.

   Hence the general solution is given by: $\theta=71{,}57°+k.180°, k\in\mathbb{Z}$.

QUESTION 4

R.T.P: $\hat{C}_2+\widehat{D}_3=\hat{B}$

Proof:



We have: $\hat{B}+\widehat{D}_2=180°$ (Opp. angles of a cyclic quad are supp.)

$\widehat{D}_1+\widehat{D}_2+\widehat{D}_3=180°$ (Angles on a straight line)

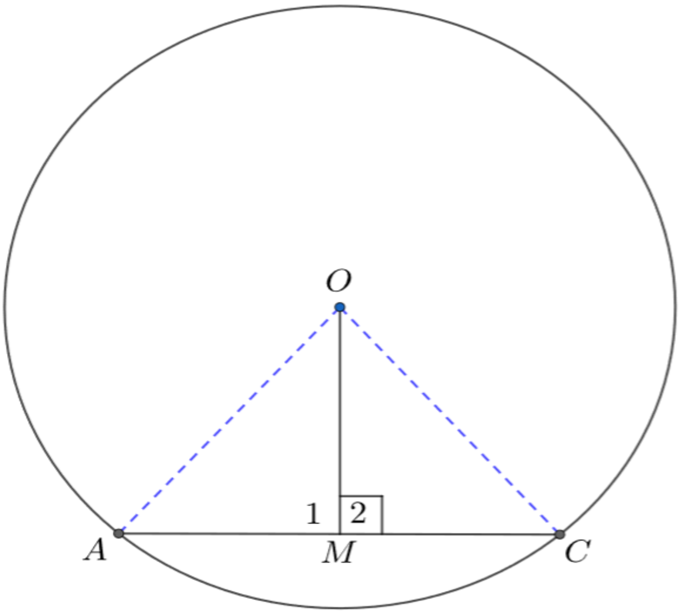
$\widehat{D}_1=\hat{C}_2$ (tan-chord theorem)

Hence $\hat{B}+\widehat{D}_2=\widehat{D}_1+\widehat{D}_2+\widehat{D}_3$ and $\widehat{D}_1=\hat{C}_2$, therefore

$$\begin{aligned}\hat{B}&=\widehat{D}_1+\widehat{D}_3\\ &=\hat{C}_2+\widehat{D}_3.\end{aligned}$$

QUESTION 5

a.



R.T.P: $AM = MC$

Proof:

We make the following constructions: Draw in lines AO and OC.

Now, in ΔOAM and ΔOCM, OM is a common side.

Also, $OA = OC$ (radii) and $\widehat{M}_1 = \widehat{M}_2 = 90°$ (given).

Therefore $\Delta AOM \equiv \Delta COM$ (by R.H.S).

Hence $AM = MC$ ■

b.

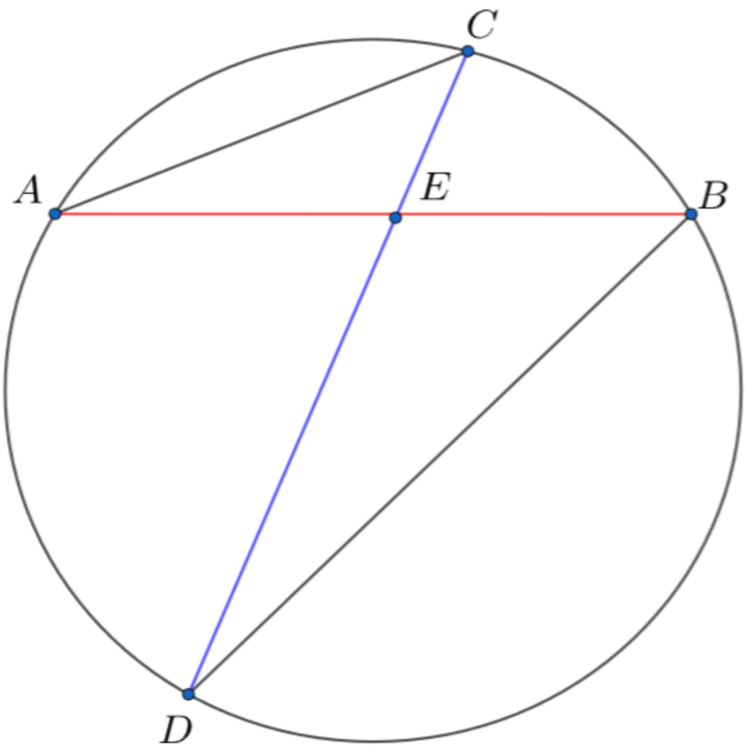
1. We have that: $\hat{B} = 90°$ (Angles in a semi-circle)
   Then, ΔABE is a right-angled triangle with lengths $AB = 12$ and $AE = 20$.
   Hence: $AB^2 + BE^2 = AE^2$
   $\therefore BE = \sqrt{20^2 - 12^2} = 16$ units.

2. In ΔGBE, we have that: $\frac{BC}{CE} = \frac{GO}{OE}$ (Proportionality theorem) and $GO = 4$ (radius of smaller circle) and $OE = 10$ (radius of bigger circle). Hence $\frac{BC}{CE} = \frac{4}{10} = \frac{2}{5}$.

3. From part b. 1 we know that $BE = 16$ units, therefore:
$BD = \frac{1}{2}(16) = 8$ units (Line from the centre ⊥ chord bisects the chord)
We also have that: $\frac{BC}{BE} = \frac{GO}{GE}$ (Proportionality theorem). We are given that $GO = 4$ and $GE = GO + OE = 4 + 10 = 14$. Hence we have:
$\frac{BC}{16} = \frac{4}{14}$, therefore $BC = \frac{32}{7}$. Thus $CD = BD - BC = 8 - \frac{32}{7} = \frac{24}{7}$.

QUESTION 6

a.



b. R.T.P: $\Delta AEC ||| \Delta DEB$

Proof:

From our diagram in part a. we can see that:

$\hat{A} = \hat{D}$ (Angles in the same segment CB)

$\hat{C} = \hat{B}$ (Angles in the same segment AD)

$A\hat{E}C = D\hat{E}B$ (Vertically opp. angles)

Hence $\Delta AEC ||| \Delta DEB$ (A.A.A) ■

c. R.T.P: $AE.EB = CE.ED$

Proof:

We have that: $\frac{AE}{EC} = \frac{DE}{EB}$ (since $\Delta AEC ||| \Delta DEB$)

Hence we get $AE.EB = CE.ED$ ■

QUESTION 7

How to use a Casio calculator for Regression modelling

Press:

MODE → 3:STAT → 2: A + Bx

Enter data into the x and y columns

Press: AC

To find A:

SHIFT → 1 → 5:Reg → 1:A → =

To find B:

SHIFT → 1 → 5:Reg → 2:B → =

To find r (correlation coefficient)

SHIFT → 1 → 5:Reg → 3:r → =

To find $\hat{y}$ given $\hat{x}$:

Enter $\hat{x}$ - value → SHIFT → 1 → 5:Reg → 5: $\hat{y}$ → =

To find the mean point $(\bar{x};\bar{y})$

SHIFT → 1 → 4:Var → 2: $\bar{x}$ → =

SHIFT → 1 → 4:Var → 5: $\bar{y}$ → =

How to use a Casio calculator to find Mean and Standard Deviation

Press:

MODE → 3:STAT → 1: 1 - VAR

Enter data into the x and FREQ columns

If **no** FREQ column then PRESS:

SHIFT → SET UP → page down → 4: STAT → 1: ON

Press: AC: →

To find the mean:

SHIFT → 1 → 4: Var → 2: $\bar{x}$

To find the standard deviation:

SHIFT → 1 → 4: Var → 3: σx

Remember: $variance = (\sigma x)^2$

NB: It is important that you are familiar with using your calculator for statistics questions such as regression modelling, finding means/variances, etc. We have attached above a step-by-step instruction guide on how to use your Casio calculator to compute these statistical operations.

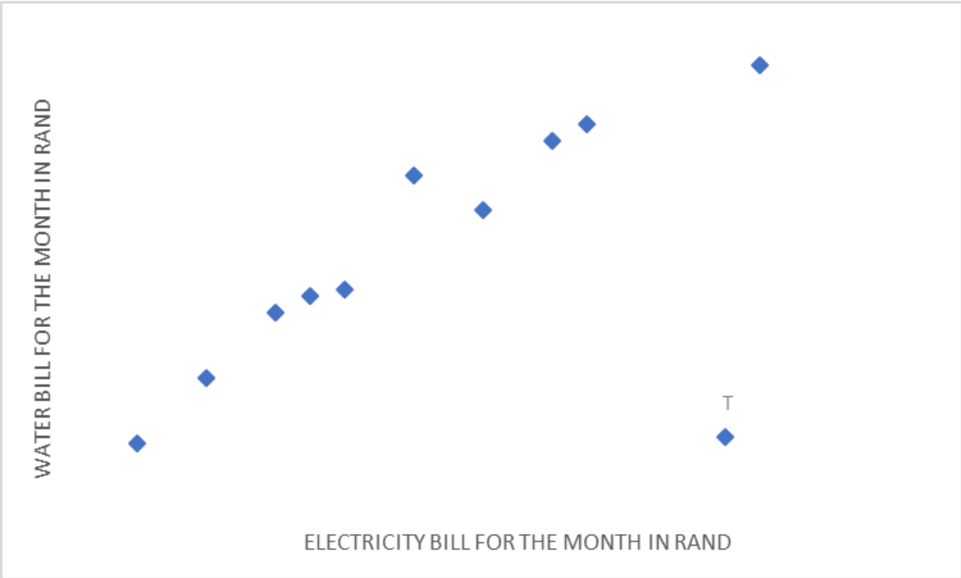
a.

1. $y = 4x - 2$

2. $r = 1$, so we have a perfect correlation.

b.

1.



2. There is a positive relationship since as the water bill increases, so does the electricity bill.

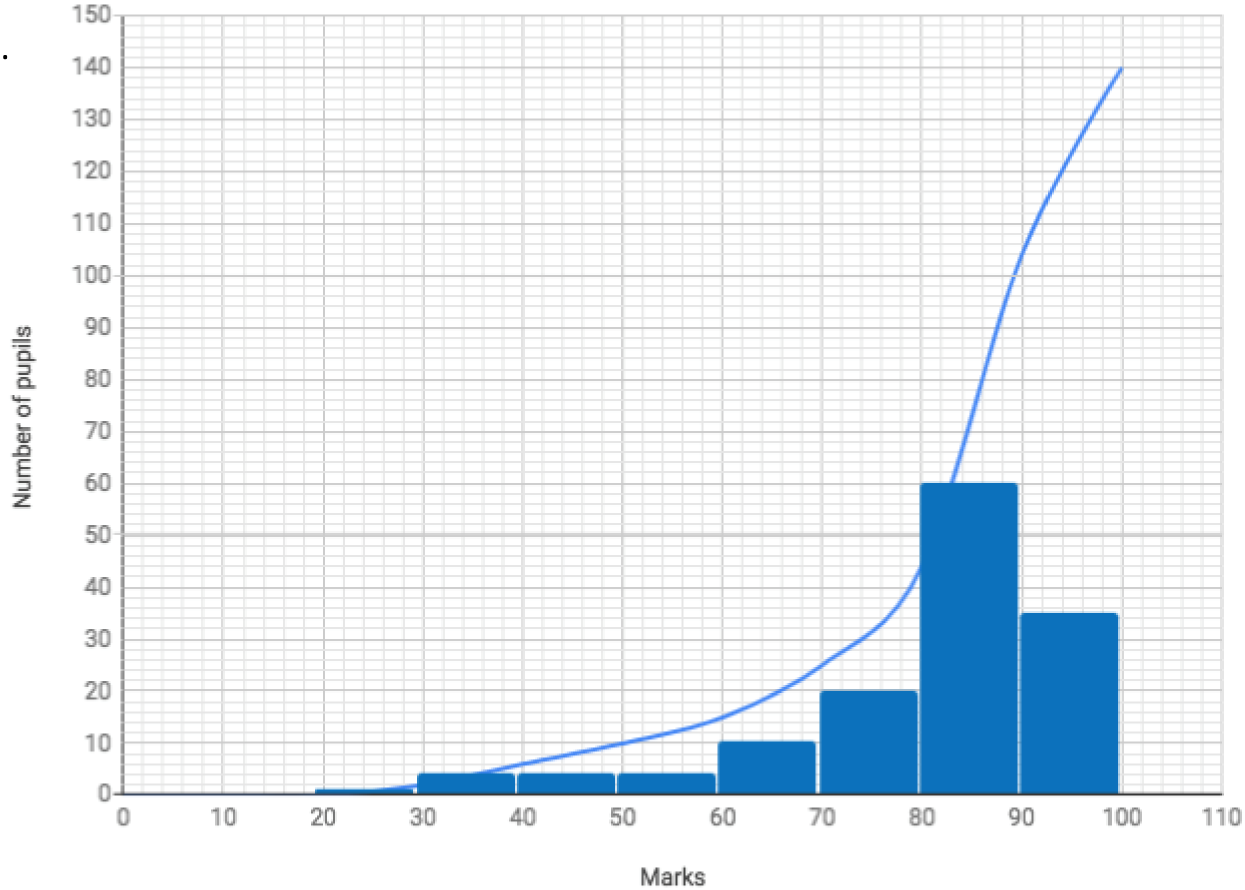
3. It would increase

4. B would increase

5. No, you cannot get an estimate as you would be extrapolating data, since the equation is only valid for values between 100 and 1000.

QUESTION 8

a.



b. The mark distribution is skewed to the left

c. The statement is TRUE since the data is skewed to the left

SECTION B

QUESTION 9

a. We can see from the first two equations we have: $\hat{A} + \hat{B} + \hat{C} = \hat{D} + \hat{E} + \hat{F}$ and from the third equation we have $\hat{A} = \hat{E}$. Thus we have: $\hat{B} + \hat{C} = \hat{D} + \hat{F}$

b. R.T.P: $ABCD$ is a parallelogram

Proof:

From the given information, we have:

$\hat{B}_1 = \hat{D}_2$ (Alternate angles are equal, since $AB \parallel DC$)

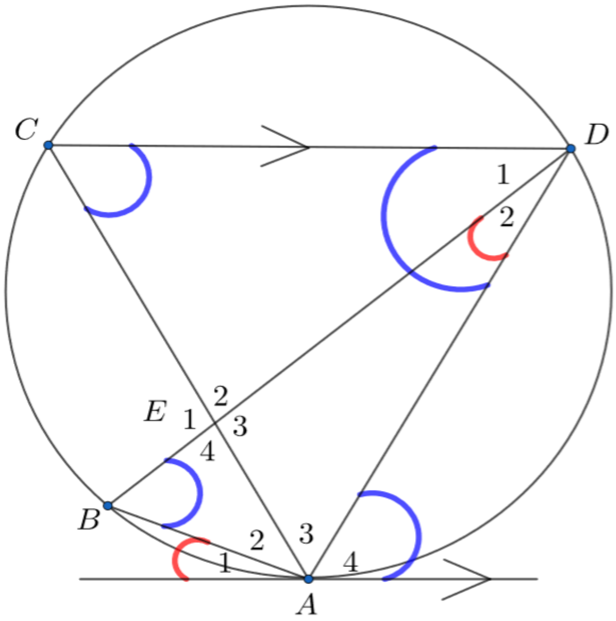
$\hat{C} = \hat{A}$ (Angles subtended from an equal chord BD from equal circles)

$\therefore \hat{B}_2 = \hat{D}_1$ (Angles in a triangle)

However, $\hat{B}_2$ and $\hat{D}$_1 are alternate angles, hence $BC \parallel AD$ and so $ABCD$ is a parallelogram ■

QUESTION 10

a.



We have that $\hat{C} = \hat{B} = 55°$ (Angles in the same segment AD)
Now, $\widehat{D}_2 = \hat{A}_1 = 18°$ (tan-chord theorem)
Also, we have $\hat{A}_4 = \hat{C} = 55°$ (tan-chord theorem)
Then, $\widehat{D} = \widehat{D}_1 + \widehat{D}_2 = \hat{A}_4$ (Alternate angles are equal, CD ∥ tangent)
Hence we get $\widehat{D}_1 = \hat{A}_4 - \widehat{D}_2$
$\therefore \widehat{D}_1 = 55° - 18°$
$= 37°$
Lastly, we have: $\hat{E}_2 = 180° - \left(\hat{C} + \widehat{D}_1\right)$
$= 180° - (55° + 37°)$
$= 88°$

b.

1. R.T.P: $\hat{O}_1 + 2\hat{A} = 180°$
   Proof:
   We have that: $P\widehat{D}O = 90°$ (line from centre is ⊥ to the tangent)
   $P\hat{E}O = 90°$ (line from centre is ⊥ to the tangent)
   $\hat{P} = 2\hat{A}$ (Angle at centre $\hat{P} = 2 \times$ Angle at circum)
   $\therefore \hat{O}_1 + \hat{P} = 180°$ (Opp. angles of a cyclic quad)
   Hence $\hat{O}_1 + 2\hat{A} = 180°$ ■

2. R.T.P: $\hat{C}_3 + \hat{E}_1 = 90° + \hat{A}$

Proof:

We have that: $\hat{O}_2 = 360° - \hat{O}_1$

$= 360° - (180° - 2\hat{A})$

$= 180° + 2\hat{A}$

Now, $2\widehat{K}_2 = \hat{O}_2$ (Angle at centre = 2 × angle at circum)

$\therefore \widehat{K}_2 = 90° + \hat{A}$

Then, $\widehat{K}_2 = \hat{C}_3 + \hat{E}_1$ (Ext. angle of Δ = sum of int. opp. angles)

$\therefore \hat{C}_3 + \hat{E}_1 = 90° + \hat{A}$ ■

QUESTION 11

a. Point A is a point of intersection, so we have:

$$2\sin 2x + 2 = 2\cos x + 2$$
$$\therefore \sin 2x = \cos x$$
$$\therefore 2\sin x \cos x = \cos x$$
$$\therefore \cos x\,(2\sin x - 1) = 0$$
$$\therefore \cos x = 0 \text{ or } 2\sin x - 1 = 0$$

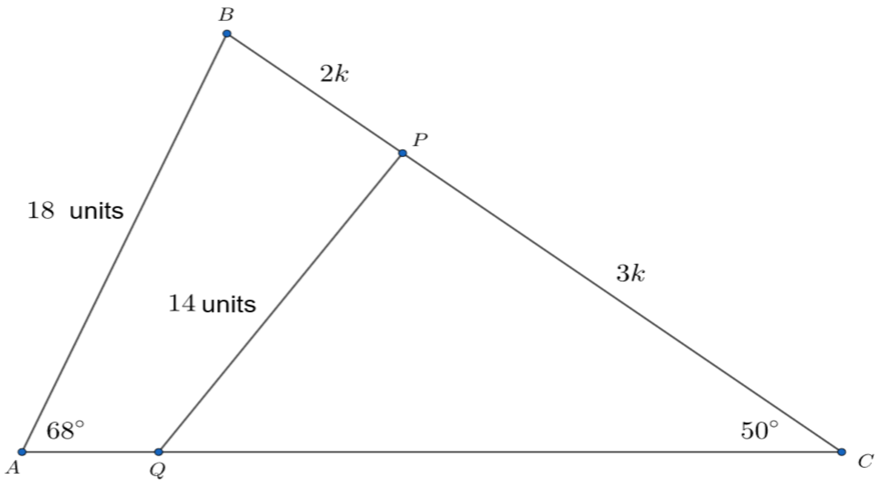
For $\cos x = 0$: $x = 90° + k.360°, k \in \mathbb{Z}$ or $x = 270° + k.360°, k \in \mathbb{Z}$

For $2\sin x - 1 = 0$: $\sin x = \frac{1}{2}$

Therefore $x = 30° + k.360°, k \in \mathbb{Z}$ or $x = 150° + k.360°, k \in \mathbb{Z}$.

Using this information, we can deduce that point A is closer to the middle and so $A(150°; -\sqrt{3} + 2)$

b.



We are given that the ratio $BP:PC$ is $2:3$. So, let $BP = 2k$ and $PC = 3k$, for some positive integer k. Note that $BC = BP + PC = 5k$.

Now, we have by the sine rule: $\frac{BC}{\sin 68°} = \frac{18}{\sin 50°}$
Therefore $BC = \frac{18 \sin 68°}{\sin 50°}$. Then, since PC makes up $\frac{3}{5}$ of BC, we get: $PC = BC \times \frac{3}{5}$, i.e. $PC = \frac{3 \times 18 \sin 68°}{5 \times \sin 50°}$. Lastly, we use the sine rule again in ΔPQC, i.e., $\frac{\sin P\hat{Q}C}{PC} = \frac{\sin 50°}{14}$ $\therefore$ $\sin P\hat{Q}C = \frac{PC \sin 50°}{14} = \frac{3 \times 18 \sin 68° \times \sin 50°}{5 \times 14 \sin 50°} = \frac{54 \sin 68°}{70}$.
Hence $P\hat{Q}C = \sin^{-1}\left(\frac{54 \sin 68°}{70}\right) = 45{,}66°$.

c. R.T.P: $\frac{\cos(A-45°)}{\cos(A+45°)} = \frac{1+\sin 2A}{\cos 2A}$
Proof:
We will simplify both the L.H.S and R.H.S

$$\begin{aligned} \text{L.H.S} &= \frac{\cos(A-45°)}{\cos(A+45°)} \\ &= \frac{\cos A \cos 45° + \sin A \sin 45°}{\cos A \cos 45° - \sin A \sin 45°} \\ &= \frac{\frac{\sqrt{2}}{2}(\cos A + \sin A)}{\frac{\sqrt{2}}{2}(\cos A - \sin A)} \qquad \left(\text{Since } \sin 45° = \cos 45° = \frac{\sqrt{2}}{2}\right) \\ &= \frac{\cos A + \sin A}{\cos A - \sin A} \end{aligned}$$

$$\begin{aligned} \text{R.H.S} &= \frac{1+\sin 2A}{\cos 2A} \\ &= \frac{\cos^2 A + 2 \sin A \cos A + \sin^2 A}{\cos^2 A - \sin^2 A} \qquad (\text{Since } 1 = \cos^2 A + \sin^2 A) \\ &= \frac{(\cos A + \sin A)(\cos A + \sin A)}{(\cos A + \sin A)(\cos A - \sin A)} \\ &= \frac{\cos A + \sin A}{\cos A - \sin A} \end{aligned}$$

Therefore L.H.S = R.H.S

QUESTION 12

a. We complete the square:

$$\therefore x^2 + y^2 + 10x - 6y = 30$$

$$\therefore x^2 + 10x + (5)^2 + y^2 - 6y + (3)^2 = 30 + 5^2 + 3^2$$

$$\therefore (x+5)^2 + (y-3)^2 = 64$$

$\therefore$ Radius of circle Q is 8 units

b. From the given information, we have that: $P(7;-2)$ and $Q(-5;3)$. Hence

$$\begin{aligned} dist(PQ) &= \sqrt{\left(x_P - x_Q\right)^2 + \left(y_P - y_Q\right)^2} \\ &= \sqrt{(7-(-5))^2 + (-2-3)^2} \\ &= \sqrt{12^2 + 5^2} \\ &= \sqrt{169} \\ &= 13 \text{ units} \end{aligned}$$

c. We first calculate the gradient of line PQ:

$$\begin{aligned} M_{PQ} &= \frac{y_P - y_Q}{x_P - x_Q} \\ &= \frac{-2-(3)}{7-(-5)} \\ &= -\frac{5}{12} \end{aligned}$$

Now, we have: $y = -\frac{5}{12}x + c$. Sub point $P(7;-2)$:

Therefore $-2 = -\frac{5}{12}(7) + c$, then $c = \frac{35}{12} - \frac{24}{12} = \frac{11}{12}$.

Hence $y = -\frac{5}{12}x + \frac{11}{12}$, then $12y = -5x + 11$.

Thus $5x + 12y = 11$.

d. We can see that point A is a point of intersection between the straight line PQ and circle with centre P. We have:

Circle P: $(x-7)^2+(y+2)^2=49$
Line PQ: $5x+12y=11$, make x the subject of the formula and the sub into circle P. So, $x=\frac{11}{5}-\frac{12}{5}y$, then

$$\therefore \left(\frac{11}{5}-\frac{12}{5}y-7\right)^2+(y+2)^2=49$$
$$\therefore \left(\frac{11-12y-35}{5}\right)^2+y^2+4y+4=49$$
$$\therefore \left(\frac{-12y-24}{5}\right)^2+y^2+4y+4=49$$
$$\therefore \left(\frac{144y^2+576y+576}{25}\right)+y^2+4y+4=1225$$
$$\therefore 144y^2+576y+576+25y^2+100y+100-1225=0$$
$$\therefore 169y^2+676y-549=0$$
$$\therefore y=\frac{-676\pm\sqrt{(676)^2-4(169)(-549)}}{2(169)}$$
$$\therefore y=\frac{9}{13} \text{ or } y=-\frac{61}{13}$$

However, we can see that the y-co-ordinate of A is positive, hence we use $y=\frac{9}{13}$. Now sub this into $x=\frac{11}{5}-\frac{12}{5}y$, therefore $x=\frac{7}{13}$. Hence $A\left(\frac{7}{13};\frac{9}{13}\right)$.

e. We can see that chord CD is where both circles intersect each other. Hence:
Circle P: $(x-7)^2+(y+2)^2-49=0$
Circle Q: $x^2+y^2+10x-6y-30=0$
Therefore we get:

$$(x-7)^2+(y+2)^2-49=x^2+y^2+10x-6y-30$$
$$\therefore x^2-14x+49+y^2+4y+4-49=x^2+y^2+10x-6y-30$$
$$\therefore -14x+4y+4=10x-6y-30$$
$$\therefore -24x+10y=-34$$
$$\therefore y=\frac{24}{10}x-\frac{34}{10}$$
$$\therefore y=\frac{12}{5}x-\frac{17}{5}$$

f. R.T.P: $CD\perp PQ$
Proof: We have the equations of lines PQ and CD and we note that:
$m_{CD}\times m_{PQ}=\frac{12}{5}\times -\frac{5}{12}=-1$. Hence $PQ\perp CD$ ■

QUESTION 13

Let us first calculate the length of BC using the cosine rule:

So, we have: $BC^2 = AB^2 + AC^2 - 2 \times AB \times AC \times \cos 30°$

$= 1^2 + (1{,}5)^2 - 2(1)(1{,}5)\cos 30°$ (Note: 100cm = 1m)

$= 0{,}651 \ldots$ (and 150cm= 1,5m)

$\therefore BC = \sqrt{0{,}651 \ldots}$

$= 0{,}8074179764$

Now, using the sine area rule, we have that:

Area of triangle $ABC = \frac{1}{2}(AB)(AC)\sin 30°$

$= 0{,}375 \text{ m}^2$

However, we can also calculate the area of triangle using: $\frac{1}{2} \times BC \times h$, where h is the perpendicular height of triangle ABC (with base BC). This h is also the height of our cylindrical container. Hence:

$\frac{1}{2}(0{,}8074179764)h = 0{,}375$

$\therefore h = 0{,}9288869234$

Lastly, we have that the volume of the water tank is:

$V = \pi r^2 h$

$= \pi(3)^2(0{,}9288869234)$

$= 26{,}26 \text{ m}^2$