Answers to:

Mathematics
IEB 2015 Paper 2

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SECTION A

QUESTION 1

a. Since \( M \) is the midpoint of line \( AB \), we have:
\[
M = \left( \frac{x_A + x_B}{2}, \frac{y_A + y_B}{2} \right) = \left( \frac{0 + 12}{2}, \frac{6 + 0}{2} \right) = (6; 3)
\]

b. We are given that \( AMCO \) is a cyclic quad, hence:
\[
\measuredangle CAMO = 180^\circ \text{ (Opp. angles of a cyclic quad are supp)}
\]
\[
\therefore \measuredangle CAMO = 180^\circ \text{ (Since } AOC = 90^\circ \text{)}
\]
\[
\quad \therefore CM \parallel AB
\]

Now, we can say that \( m_{AB} \times m_{MC} = -1 \) (Since \( MC \perp AB \))

Therefore \( m_{AB} = \frac{y_B - y_A}{x_B - x_A} = \frac{6 - 0}{12 - 0} = \frac{1}{2} \)

Hence \( m_{MC} = -\frac{1}{m_{MC}} = 2 \) and line \( MC: y = 2x + c \).
Sub point \( M(6; 3) \):
Thus \( 3 = 2(6) + c \therefore c = -9 \).
Hence \( y = 2x - 9 \).

c.

1. R.T.P: Area of \( \Delta MCB = 11,25 \text{ units}^2 \)

Proof:
We have that Area of \( \Delta MCB = \frac{1}{2}bh \), where \( b = BC \) and \( h \) is the perpendicular height from the base \( b \). To get the length of \( BC \), we first need the co-ordinates of point \( C \). Since \( C \) is the \( x \)-int of line \( MC \), sub \( y = 0 \) into \( y = 2x - 9 \), then \( 2x - 9 = 0 \therefore x = \frac{9}{2} = 4 \frac{1}{2} \).

Hence \( C \left( \frac{9}{2}, 0 \right) \).

Now, we have that:
\[
BC = OB - OC = 12 - 4 \frac{1}{2} = 7 \frac{1}{2}
\]
Also, the perpendicular height is given by \( y_M = 3 = h \).
Thus Area of \( \Delta MCB = \frac{1}{2}bh = \frac{1}{2} \left( 7 \frac{1}{2} \right) (3) = 11,25 \text{ units}^2 \).
2. We can see from our diagram that:
   \[
   \text{Area of } AMCO = \text{Area of } \triangle AOC - \text{Area of } \triangle MCB \\
   = \frac{1}{2} (12)(6) - 11,25 \\
   = 36 - 11,25 \\
   = 24,75 \text{ units}^2
   \]

**QUESTION 2**

a. We can see that \( N \) is a point of intersection of the lines \( y = x \) and \( y = \frac{1}{2} x + 4 \).
   
   Hence: \( \frac{1}{2} x + 4 = x \)  
   \[\therefore \frac{1}{2} x = 4\]  
   \[\therefore x = 8\]  
   Sub back into \( y = x \)  
   Therefore \( N(8;8) \).

b. We note that \( N \) is the centre of the circle and point \( B \) is perpendicularly above a point on the circle, so the radius of the circle is 8 and hence \( B(16;16) \).

c. We can see that \( D \) has the same \( y \) co-ordinate as point \( B(16;16) \). So we have:  
   \( D(x,16) \). Point \( D \) also lies on the line \( 7y = 10x \), hence \( 7(16) = 10x \therefore x = 11,2 \).  
   Thus \( D(11,2;16) \). So we have \( DB = 16 - 11,2 = 4,8 \text{ units} \).

**QUESTION 3**

a.  
   \[1. \frac{\sin(180^\circ - \theta) \cos(90^\circ - \theta) - 1}{\cos(-\theta)} = \frac{\sin \theta \sin \theta - 1}{\cos \theta} \]
   \[= \frac{\sin^2 \theta - 1}{\cos \theta} \]
   \[= - \frac{\cos^2 \theta}{\cos \theta} \]
   \[= - \cos \theta \]

   2. We can see that under the square root, we have the same expression as in a. 1  
   Hence it simplifies to \( \sqrt{-\cos \theta} \). Now, this will only be real when \( -\cos \theta > 0 \), i.e. when \( \cos \theta < 0 \). This happens in the second and third quadrants. Hence we have real solutions when \( \theta \in (90^\circ; 270^\circ) \).
b.  

1. R.T.P: \( \tan \theta \sin \theta + \cos \theta = \frac{1}{\cos \theta} \)

Proof:
L.H.S = \( \tan \theta \sin \theta + \cos \theta \)
= \( \frac{\sin \theta}{\cos \theta} \cdot \sin \theta + \cos \theta \)
= \( \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta} \)
= \( \frac{1}{\cos \theta} \) \[\Box\]

2. We have by part b. 1 that \( \tan \theta \sin \theta + \cos \theta = \frac{1}{\cos \theta} \). Hence we have:

\( \frac{1}{\cos \theta} = \frac{3}{\sin \theta} \)
\[\therefore \frac{\cos \theta}{\sin \theta} = 3 \]
\[\therefore \tan \theta = 3 \]
\[\therefore \text{Key angle} = 71,57^\circ. \]
Hence the general solution is given by: \( \theta = 71,57^\circ + k \cdot 180^\circ, k \in \mathbb{Z} \).

QUESTION 4

R.T.P: \( \hat{C}_2 + \hat{D}_3 = \hat{B} \)

Proof:

We have: \( \hat{B} + \hat{D}_2 = 180^\circ \) (Opp. angles of a cyclic quad are supp.)
\( \hat{D}_1 + \hat{D}_2 + \hat{D}_3 = 180^\circ \) (Angles on a straight line)
\( \hat{D}_1 = \hat{C}_2 \) (tan-chord theorem)
Hence \( \hat{B} + \hat{D}_2 = \hat{D}_1 + \hat{D}_2 + \hat{D}_3 \) and \( \hat{D}_1 = \hat{C}_2 \), therefore
\( \hat{B} = \hat{D}_1 + \hat{D}_3 \)
\[ = \hat{C}_2 + \hat{D}_3. \]
QUESTION 5

a.

R.T.P: \( AM = MC \)
Proof:
We make the following constructions: Draw in lines \( AO \) and \( OC \).
Now, in \( \triangle OAM \) and \( \triangle OCM \), \( OM \) is a common side.
Also, \( OA = OC \) (radii) and \( \hat{M_1} = \hat{M_2} = 90° \) (given).
Therefore \( \triangle AOM \equiv \triangle COM \) (by R.H.S).
Hence \( AM = MC \). □

b.

1. We have that: \( \hat{B} = 90° \) (Angles in a semi-circle)
   Then, \( \triangle ABE \) is a right-angled triangle with lengths \( AB = 12 \) and \( AE = 20 \).
   Hence: \( AB^2 + BE^2 = AE^2 \)
   \[ \therefore BE = \sqrt{20^2 - 12^2} = 16 \text{ units.} \]

2. In \( \triangle GBE \), we have that: \( \frac{BC}{CE} = \frac{GO}{OE} \) (Proportionality theorem) and \( GO = 4 \)
   (radius of smaller circle) and \( OE = 10 \) (radius of bigger circle). Hence
   \( \frac{BC}{CE} = \frac{4}{10} = \frac{2}{5}, \)
3. From part b. 1 we know that $BE = 16$ units, therefore:

$$BD = \frac{1}{2} (16) = 8$$

units (Line from the centre \perp chord bisects the chord)

We also have that: $\frac{BC}{BE} = \frac{GO}{GE}$ (Proportionality theorem). We are given that $GO = 4$ and $GE = GO + OE = 4 + 10 = 14$. Hence we have:

$$\frac{BC}{16} = \frac{4}{14},$$

therefore $BC = \frac{32}{7}$. Thus $CD = BD - BC = 8 - \frac{32}{7} = \frac{24}{7}$.

**QUESTION 6**

a. 

b. R.T.P: $\triangle AEC \parallel \parallel \triangle DEB$

Proof:

From our diagram in part a. we can see that:

$\hat{A} = \hat{D}$ (Angles in the same segment $CB$)

$\hat{C} = \hat{B}$ (Angles in the same segment $AD$)

$A\hat{E}C = D\hat{E}B$ (Vertically opp. angles)

Hence $\triangle AEC \parallel \parallel \triangle DEB$ (A.A.A) ■

c. R.T.P: $AE \cdot EB = CE \cdot ED$

Proof:

We have that: $\frac{AE}{EC} = \frac{DE}{EB}$ (since $\triangle AEC \parallel \parallel \triangle DEB$)

Hence we get $AE \cdot EB = CE \cdot ED$ ■
QUESTION 7

How to use a Casio calculator for Regression modelling
Press:
MODE → 3:STAT → 2: A + Bx
Enter data into the x and y columns
Press: AC
To find A:
SHIFT → 1 → 5:Reg → 1:A → =
To find B:
SHIFT → 1 → 5:Reg → 2:B → =
To find r (correlation coefficient)
SHIFT → 1 → 5:Reg → 3:r → =
To find $\hat{y}$ given $\hat{x}$:
Enter $\hat{x}$ - value → SHIFT → 1 → 5:Reg → 5: $\hat{y}$ → =
To find the mean point ($\bar{x}; \bar{y}$)
SHIFT → 1 → 4:Var → 2: $\bar{x}$ → =
SHIFT → 1 → 4:Var → 5: $\bar{y}$ → =

How to use a Casio calculator to find Mean and Standard Deviation
Press:
MODE → 3:STAT → 1: 1 - VAR
Enter data into the x and FREQ columns
If no FREQ column then PRESS:
SHIFT → SET UP → page down → 4: STAT → 1: ON
Press: AC: →
To find the mean:
SHIFT → 1 → 4: Var → 2: $\bar{x}$
To find the standard deviation:
SHIFT → 1 → 4: Var → 3: $\sigma_x$
Remember: variance = $(\sigma_x)^2$

NB: It is important that you are familiar with using your calculator for statistics questions such as regression modelling, finding means/variances, etc. We have attached above a step-by-step instruction guide on how to use your Casio calculator to compute these statistical operations.
a.
1. \( y = 4x - 2 \)
2. \( r = 1 \), so we have a perfect correlation.

b.
1. 

2. There is a positive relationship since as the water bill increases, so does the electricity bill.

3. It would increase

4. B would increase

5. No, you cannot get an estimate as you would be extrapolating data, since the equation is only valid for values between 100 and 1000.
QUESTION 8

b. The mark distribution is skewed to the left

c. The statement is TRUE since the data is skewed to the left
QUESTION 9

a. We can see from the first two equations we have: \( \hat{A} + \hat{B} + \hat{C} = \hat{D} + \hat{E} + \hat{F} \) and from the third equation we have \( \hat{A} = \hat{E} \). Thus we have: \( \hat{B} + \hat{C} = \hat{D} + \hat{F} \)

b. R.T.P: \( ABCD \) is a parallelogram
   
   Proof:
   
   From the given information, we have:
   
   \( \hat{B}_1 = \hat{D}_2 \) (Alternate angles are equal, since \( AB \parallel DC \))
   
   \( \hat{C} = \hat{A} \) (Angles subtended from an equal chord \( BD \) from equal circles)
   
   \( \therefore \hat{B}_2 = \hat{D}_1 \) (Angles in a triangle)

   However, \( \hat{B}_2 \) and \( \hat{D}_1 \) are alternate angles, hence \( BC \parallel AD \) and so \( ABCD \) is a parallelogram \( \Box \)
QUESTION 10

a.

We have that \( \hat{C} = \hat{B} = 55\degree \) (Angles in the same segment \( AD \))

Now, \( \hat{D}_2 = \hat{A}_1 = 18\degree \) (tan-chord theorem)

Also, we have \( \hat{A}_4 = \hat{C} = 55\degree \) (tan-chord theorem)

Then, \( \hat{D} = \hat{D}_1 + \hat{D}_2 = \hat{A}_4 \) (Alternate angles are equal, \( CD \parallel \text{tangent} \))

Hence we get \( \hat{D}_1 = \hat{A}_4 - \hat{D}_2 \)

\[ \therefore \hat{D}_1 = 55\degree - 18\degree = 37\degree \]

Lastly, we have: \( \hat{E}_2 = 180\degree - (\hat{C} + \hat{D}_1) \)

\[ = 180\degree - (55\degree + 37\degree) \]

\[ = 88\degree \]

b.

1. R.T.P: \( \hat{O}_1 + 2\hat{A} = 180\degree \)

   Proof:

   We have that: \( P\hat{D}O = 90\degree \) (line from centre is \( \perp \) to the tangent)

   \( P\hat{E}O = 90\degree \) (line from centre is \( \perp \) to the tangent)

   \( \hat{P} = 2\hat{A} \) (Angle at centre \( \hat{P} = 2 \times \text{Angle at circum} \))

   \[ \therefore \hat{O}_1 + \hat{P} = 180\degree \] (Opp. angles of a cyclic quad)

   Hence \( \hat{O}_1 + 2\hat{A} = 180\degree \) ■
2. R.T.P: \( \hat{C}_3 + \hat{E}_1 = 90^\circ + \hat{A} \)

Proof:

We have that: \( \hat{O}_2 = 360^\circ - \hat{O}_1 \)
\[ = 360^\circ - (180^\circ - 2\hat{A}) \]
\[ = 180^\circ + 2\hat{A} \]

Now, \( 2\hat{R}_2 = \hat{O}_2 \) (Angle at centre = 2 \times angle at circum)
\[ \therefore \hat{R}_2 = 90^\circ + \hat{A} \]

Then, \( \hat{R}_2 = \hat{C}_3 + \hat{E}_1 \) (Ext. angle of \( \Delta \) = sum of int. opp. angles)
\[ \therefore \hat{C}_3 + \hat{E}_1 = 90^\circ + \hat{A} \]

QUESTION 11

a. Point \( A \) is a point of intersection, so we have:

\[ 2 \sin 2x + 2 = 2 \cos x + 2 \]
\[ \therefore \sin 2x = \cos x \]
\[ \therefore 2 \sin x \cos x = \cos x \]
\[ \therefore \cos x (2 \sin x - 1) = 0 \]
\[ \therefore \cos x = 0 \text{ or } 2 \sin x - 1 = 0 \]

For \( \cos x = 0 \): \( x = 90^\circ + k.360^\circ, k \in \mathbb{Z} \) or \( x = 270^\circ + k.360^\circ, k \in \mathbb{Z} \)

For \( 2 \sin x - 1 = 0 \): \( \sin x = \frac{1}{2} \)

Therefore \( x = 30^\circ + k.360^\circ, k \in \mathbb{Z} \) or \( x = 150^\circ + k.360^\circ, k \in \mathbb{Z} \).

Using this information, we can deduce that point \( A \) is closer to the middle and so \( A(150^\circ; -\sqrt{3} + 2) \)
b. We are given that the ratio \( BP:PC \) is 2:3. So, let \( BP = 2k \) and \( PC = 3k \), for some positive integer \( k \). Note that \( BC = BP + PC = 5k \).

Now, we have by the sine rule:
\[
\frac{BC}{\sin 68^\circ} = \frac{18}{\sin 50^\circ}
\]
Therefore \( BC = \frac{18 \sin 68^\circ}{\sin 50^\circ} \). Then, since \( PC \) makes up \( \frac{3}{5} \) of \( BC \), we get:
\[
PC = BC \times \frac{3}{5}
\]
\[
= \frac{3 \times 18 \sin 68^\circ}{5 \sin 50^\circ}
\]
Lastly, we use the sine rule again in \( \triangle PQC \), i.e.,
\[
\sin \frac{PQ}{PC} = \frac{\sin 50^\circ}{14} \implies \sin \frac{PQ}{PC} = \frac{PC \sin 50^\circ}{14} = \frac{3 \times 18 \sin 68^\circ \times \sin 50^\circ}{5 \times 14 \sin 50^\circ} = \frac{54 \sin 68^\circ}{70}
\]
Hence \( PQ = \sin^{-1} \left( \frac{54 \sin 68^\circ}{70} \right) = 45.66^\circ \).

c. R.T.P: \( \frac{\cos(A-45^\circ)}{\cos(A+45^\circ)} = \frac{1+\sin 2A}{\cos 2A} \)

Proof:
We will simplify both the L.H.S and R.H.S

L.H.S = \[
\frac{\cos(A-45^\circ)}{\cos(A+45^\circ)} = \frac{\cos A \cos 45^\circ + \sin A \sin 45^\circ}{\cos A \cos 45^\circ - \sin A \sin 45^\circ}
\]
\[
= \frac{\frac{1}{2}(\cos A + \sin A)}{\frac{1}{2}(\cos A - \sin A)}
\]
\[
= \frac{\cos A + \sin A}{\cos A - \sin A}
\]
(Since \( \sin 45^\circ = \cos 45^\circ = \frac{\sqrt{2}}{2} \))

R.H.S = \[
\frac{1+\sin 2A}{\cos 2A}
\]
\[
= \frac{\cos^2 A + 2 \sin A \cos A + \sin^2 A}{\cos^2 A - \sin^2 A}
\]
\[
= \frac{(\cos A + \sin A)(\cos A + \sin A)}{(\cos A + \sin A)(\cos A - \sin A)}
\]
\[
= \frac{\cos A + \sin A}{\cos A - \sin A}
\]
(Since \( 1 = \cos^2 A + \sin^2 A \))

Therefore L.H.S = R.H.S
QUESTION 12

a. We complete the square:
\[ \therefore x^2 + y^2 + 10x - 6y = 30 \]
\[ \therefore x^2 + 10x + (5)^2 + y^2 - 6y + (3)^2 = 30 + 5^2 + 3^2 \]
\[ \therefore (x + 5)^2 + (y - 3)^2 = 64 \]
\[ \therefore \text{Radius of circle } Q \text{ is } 8 \text{ units} \]

b. From the given information, we have that: \( P(7; -2) \) and \( Q(-5; 3) \). Hence
\[ \text{dist}(PQ) = \sqrt{(x_p - x_Q)^2 + (y_p - y_Q)^2} \]
\[ = \sqrt{(7 - (-5))^2 + (-2 - 3)^2} \]
\[ = \sqrt{12^2 + 5^2} \]
\[ = \sqrt{169} \]
\[ = 13 \text{ units} \]

c. We first calculate the gradient of line \( PQ \):
\[ M_{PQ} = \frac{y_p - y_Q}{x_p - x_Q} \]
\[ = \frac{-2 - (3)}{7 - (-5)} \]
\[ = \frac{-5}{12} \]

Now, we have: \( y = -\frac{5}{12}x + c \). Sub point \( P(7; -2) \):

Therefore \( -2 = -\frac{5}{12}(7) + c \), then \( c = \frac{35}{12} - \frac{24}{12} = \frac{11}{12} \).

Hence \( y = -\frac{5}{12}x + \frac{11}{12} \), then \( 12y = -5x + 11 \).
Thus \( 5x + 12y = 11 \).
d. We can see that point $A$ is a point of intersection between the straight line $PQ$ and circle with centre $P$. We have:

Circle $P$: $(x - 7)^2 + (y + 2)^2 = 49$
Line $PQ$: $5x + 12y = 11$, make $x$ the subject of the formula and the sub into circle $P$. So, $x = \frac{11}{5} - \frac{12}{5}y$, then

\[
\therefore \left(\frac{11}{5} - \frac{12}{5}y - 7\right)^2 + (y + 2)^2 = 49
\]
\[
\therefore \left(\frac{11 - 12y - 35}{5}\right)^2 + y^2 + 4y + 4 = 49
\]
\[
\therefore \left(\frac{-12y - 24}{5}\right)^2 + y^2 + 4y + 4 = 49
\]
\[
\therefore \left(\frac{144y^2 + 576y + 576}{25}\right) + y^2 + 4y + 4 = 1225
\]
\[
\therefore 144y^2 + 576y + 576 + 25y^2 + 100y + 100 - 1225 = 0
\]
\[
\therefore 169y^2 + 676y - 549 = 0
\]
\[
\therefore y = \frac{-676 \pm \sqrt{676^2 - 4(169)(-549)}}{2(169)}
\]
\[
\therefore y = \frac{9}{13} \text{ or } y = -\frac{61}{13}
\]

However, we can see that the $y$-co-ordinate of $A$ is positive, hence we use $y = \frac{9}{13}$. Now sub this into $x = \frac{11}{5} - \frac{12}{5}y$, therefore $x = \frac{7}{13}$. Hence $A\left(\frac{7}{13}, \frac{9}{13}\right)$.

e. We can see that chord $CD$ is where both circles intersect each other. Hence:

Circle $P$: $(x - 7)^2 + (y + 2)^2 = 49$
Circle $Q$: $x^2 + y^2 + 10x - 6y - 30 = 0$

Therefore we get:

\[
(x - 7)^2 + (y + 2)^2 - 49 = x^2 + y^2 + 10x - 6y - 30
\]
\[
\therefore x^2 - 14x + 49 + y^2 + 4y + 4 - 49 = x^2 + y^2 + 10x - 6y - 30
\]
\[
\therefore -14x + 4y + 4 = 10x - 6y - 30
\]
\[
\therefore -24x + 10y = -34
\]
\[
\therefore y = \frac{24}{10}x - \frac{34}{10}
\]
\[
\therefore y = \frac{12}{5}x - \frac{17}{5}
\]

f. R.T.P: $CD \perp PQ$

Proof: We have the equations of lines $PQ$ and $CD$ and we note that:

$m_{CD} \times m_{PQ} = \frac{12}{5} \times -\frac{5}{12} = -1$. Hence $PQ \perp CD$
QUESTION 13

Let us first calculate the length of $BC$ using the cosine rule:

So, we have:

$$BC^2 = AB^2 + AC^2 - 2 \times AB \times AC \times \cos 30^\circ$$

$$= 1^2 + (1,5)^2 - 2(1)(1,5) \cos 30^\circ$$

(Note: 100cm = 1m)

$$= 0,651 \ldots$$

$$\therefore BC = \sqrt{0,651} \ldots$$

$$= 0,8074179764$$

Now, using the sine area rule, we have that:

Area of triangle $ABC = \frac{1}{2}(AB)(AC) \sin 30^\circ$

$$= 0,375 \text{ m}^2$$

However, we can also calculate the area of triangle using: $\frac{1}{2} \times BC \times h$, where $h$ is the perpendicular height of triangle $ABC$ (with base $BC$). This $h$ is also the height of our cylindrical container. Hence:

$$\frac{1}{2}(0,8074179764)h = 0,375$$

$$\therefore h = 0,9288869234$$

Lastly, we have that the volume of the water tank is:

$$V = \pi r^2 h$$

$$= \pi (3)^2(0,9288869234)$$

$$= 26,26 \text{ m}^2$$