Answers to:

Mathematics
IEB 2015 Paper 1

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QUESTION 1

a.

1. $(x-3)(x+1)=5$
$\therefore x^2-2x-8=0$
$\therefore (x-4)(x+2)=0$
$\therefore x=4$ or $x=-2$

2. $9^{2x-1}=\frac{3^x}{3}$
$\therefore 3^{2(2x-1)}=3^{x-1}$
$\therefore 3^{4x-2}=3^{x-1}$
$\therefore 4x-2=x-1$
$\therefore 3x=1$
$\therefore x=\frac{1}{3}$

3. $2\sqrt{2-7x}=\sqrt{-36x}$
$\therefore \left(2\sqrt{2-7x}\right)^2=\left(\sqrt{-36x}\right)^2$
$\therefore 4(2-7x)=-36x$
$\therefore 8-28x=-36x$
$\therefore -8x=8$
$\therefore x=-1$

b. $x^2+2kx+k=kx+k$
$\therefore x^2+kx=0$
$\therefore x(x+k)=0$
$\therefore x=0$ or $x=-k$.
Hence the points of intersection are given by: Sub $x=0$ and $x=-k$ into $y=kx+k$
$\therefore (0,k)$ and $(-k;-k^2+k)$

c.

1. To get the roots, we use the quadratic equation:
$$\therefore x=\frac{-n\pm\sqrt{n^2-4(9)(49)}}{2(9)}$$
$$=\frac{-n\pm\sqrt{n^2-1764}}{18}$$

2. For equal roots, we have: $\Delta=b^2-4ac=0$.
Hence $n^2-1764=0$
$\therefore n=\pm\sqrt{1764}=\pm 42$

QUESTION 2

a.

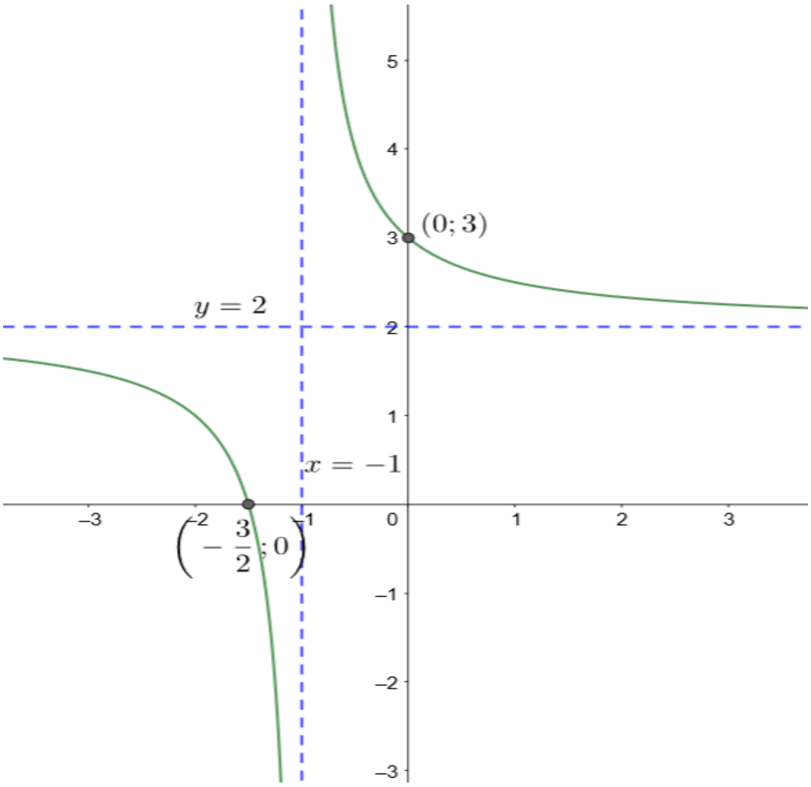
1. $x = -1$ and $y = 2$

2. x-int: $y = 0$: $\frac{1}{x+1} + 2 = 0$

$$\therefore 1 + 2(x+1) = 0$$
$$\therefore 2x + 3 = 0$$
$$\therefore x = -\frac{3}{2}$$

y-int: $x = 0$: $f(0) = \frac{1}{0+1} + 2 = 3$

3.



b.

1. x-int: $y = 0$: $2.3^x - 1 = 0$

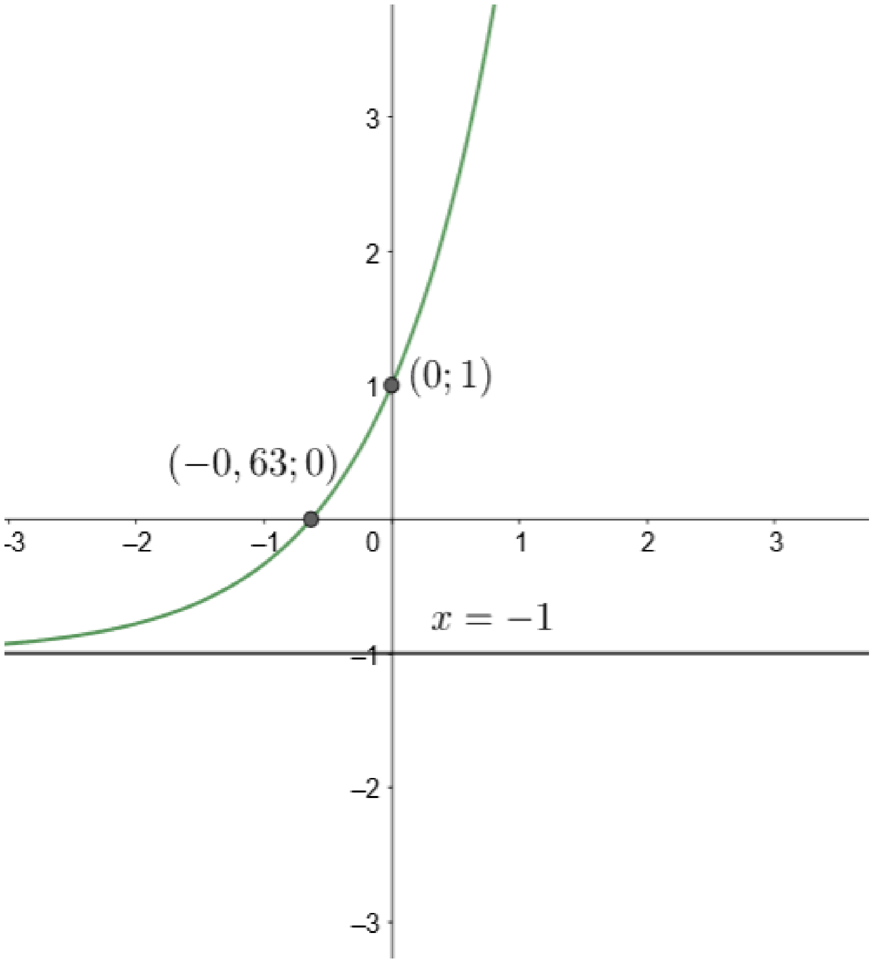
$$\therefore 3^x = \frac{1}{2}$$

$$\therefore x = \log_3\left(\frac{1}{2}\right) = -0{,}63$$

y-int: $x = 0$: $g(0) = 2.3^0 - 1 = 1$

Hence, our intercepts are: $(-0{,}63; 0)$ and $(0; 1)$

2.



QUESTION 3

a. We are told that the bank finances the remaining 40%. Hence the bank is financing $40\% \times 1800000 = 720000$. To get the monthly instalments we use:

$$\mathrm{R}\ 720000 = x\left[\frac{1-\left(1+\frac{8}{12\times100}\right)^{(-12\times10)}}{\frac{8}{12\times100}}\right]$$

$$\therefore x = \mathrm{R}\ 8736 \text{ (To the nearest rand)}$$

b. We first calculate what the loan will be in 3 years' time:

$$A = P\left(1+\frac{i^{(n)}}{12}\right)^{n}$$

$$\therefore A = 720000\left(1+\frac{8}{12\times100}\right)^{(3\times12)}$$
$$= 914570{,}67$$
$$= \mathrm{R}\ 914571 \text{ (To the nearest rand)}$$

Now, we calculate how much we pay in 3 years' time:

$$F = 8736\left[\frac{\left(1+\frac{8}{1200}\right)^{36}-1}{\frac{8}{12\times100}}\right]$$
$$= 354118{,}63$$
$$= \mathrm{R}\ 354119 \text{ (To the nearest rand)}$$

Hence the balance outstanding after 3 years is given by:
$\mathrm{R}\ 914571 - \mathrm{R}\ 354119 = \mathrm{R}\ 560452$

c. We have: Amount paid towards loan after 3 years is:
$\mathrm{R}\ 720000 - \mathrm{R}\ 560452 = \mathrm{R}\ 159548$
Also: Amount paid through the monthly instalments after 3 years is:
$\mathrm{R}\ 8736 \times 36 = \mathrm{R}\ 314496$
Hence the interest paid is:
$\mathrm{R}\ 314496 - \mathrm{R}\ 159548 = \mathrm{R}\ 154948$
Therefore the percentage of the total paid to the bank as interest charges over 3 years is:
$\frac{154948}{314496} \times 100 = 49\%$

QUESTION 4

a. We have $f(x) = \frac{7}{x}$. Therefore:

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0} \frac{\frac{7}{x+h}-\frac{7}{x}}{h} \qquad \text{(L.C.D in numerator terms is } x(x+h)\text{)} \\
&= \lim_{h\to 0} \frac{\frac{7x-7(x+h)}{x(x+h)}}{h} \\
&= \lim_{h\to 0} \frac{7x-7x-7h}{x(x+h)h} \\
&= \lim_{h\to 0} -\frac{7h}{x(x+h)h} \\
&= \lim_{h\to 0} -\frac{7}{x(x+h)} \\
&= -\frac{7}{x^2}
\end{aligned}
$$

b.
$$
\begin{aligned}
D_x\left(\frac{14\pi}{x^{-1}} - 3\sqrt[3]{x^2}\right) &= D_x\left(14\pi x - 3x^{\frac{2}{3}}\right) \\
&= 14\pi - 3\left(\frac{2}{3}\right)x^{-\frac{1}{3}} \\
&= 14\pi - 2x^{-\frac{1}{3}} \\
&= 14\pi - \frac{2}{x^{\frac{1}{3}}} \\
&= 14\pi - \frac{2}{\sqrt[3]{x}}
\end{aligned}
$$

QUESTION 5

a. We are given an arithmetic sequence, so $T_n = a + (n-1)d$. We are also given that $T_1 = a = 6$ and that $T_5 = 18$. Hence, we have:

$$\begin{aligned} T_5 &= a + 4d \\ &= 6 + 4d \\ &= 18 \end{aligned}$$

$\therefore 4d = 12$

$\therefore d = 3$

Hence: $S_n = \frac{n}{2}(2a + (n-1)d)$

$$\begin{aligned} S_{38} &= \frac{38}{2}(2(6) + (37)(3)) \\ &= 2237 \end{aligned}$$

b.

1. Initially our screen has an area of 1000 units^2. So, we have:

   First day: $\frac{1}{3} \times 1000$ of the screen is blocked out $= \frac{1000}{3}$

   Second day: $\frac{1}{3} \times$ First day blocked out screen $+$ First day

   $= \frac{1000}{3} + \frac{1}{3}\left(\frac{1000}{3}\right) = \frac{4000}{9}$

   Third day: $\frac{1}{3} \times$ Second day blocked out screen $+$ Second day

   $= \frac{4000}{9} + \frac{1}{3}\left(\frac{4000}{9}\right) = \frac{16000}{27}$

   Hence the total area blocked is: $\frac{16000}{27} \approx 592{,}6$ units^2

2. We have a geometric sequence with $a = \frac{1000}{3}$ and $r = \frac{4}{3}$. Since $r > 1$, the infinite series S_∞ will diverge. Thus, eventually the whole screen will be blocked out.

QUESTION 6

a.

1. (iii) Roots are real and equal

2. (i) Roots are non-real

3. (ii) Roots are real and unequal

b.

1. We are given the two roots, hence we can use:
   $y = a(x-3)(x-7)$. Now sub point $E(6;6)$:
   $\therefore 6 = a(6-3)(6-7)$
   $\therefore 6 = -3a$
   $\therefore a = -2$
   $\therefore y = -2(x-3)(x-7)$
   $\therefore y = -2x^2 + 20x - 42$

2. We are given: $m_{AD} = -2$. Hence $y = -2x + c$. Sub the point $A(3;0)$:
   $\therefore 0 = -2(3) + c$
   $\therefore c = 6$
   $\therefore y = -2x + 6$

3. We note that point C and D have the same x co-ordinate. Since C is the T.P of the parabola, it must be halfway between the intercepts, hence $x = 5$. Now, we calculate the co-ordinates of points C and D.

   For point C: Sub $x = 5$ into parabola $\therefore y = -2(5)^2 + 20(5) - 42 = 8$.
   $\therefore C(5;8)$
   For point D: Sub $x = 5$ into $y = -2x + 6 \therefore y = -2(5) + 6 = -4$.
   $\therefore D(5;-4)$
   Hence the length of CD is given by $8 - (-4) = 12$ units.

c. We are given: $f(x) = -\frac{1}{50}(x^2 - 100); 0 \leq x \leq 10$. Before we find the inverse function, we need to determine the range of f
(since this will be the domain of f^{-1}). Hence the range of f is given by $[0; 2]$
(We find this by sub. $x = 0$ and $x = 10$ into f)
Now, to find the inverse function, we interchange the roles of x and y:

$$y = -\frac{1}{50}(x^2 - 100)$$
$$\therefore x = -\frac{1}{50}(y^2 - 100)$$
$$\therefore y^2 - 100 = -50x$$
$$\therefore y^2 = 100 - 50x$$
$$\therefore y = \sqrt{100 - 50x}, 0 \leq x \leq 2$$

SECTION B

QUESTION 7

a. We are given that: $f'(1) = 12$ and $f''(1) = -24$. Now,

$f'(x) = 9x^2 + 2bx + c$

$\therefore f'(1) = 9 + 2b + c$

$\therefore 12 = 9 + 2b + c$ … Eq(1)

$f''(x) = 18x + 2b$

$\therefore f''(1) = 18 + 2b$

$\therefore -24 = 18 + 2b$

$\therefore 2b = -42$

$\therefore b = -21$. Now sub. into Eq(1)

$\therefore 9 + 2(-21) + c = 12$

$\therefore c = 45$

Hence we have: $f(x) = 3x^3 - 21x^2 + 45x - 27$

b. The graph is concave down when $f''(x) < 0$, so:

$f''(x) = 18x - 42 < 0$

$\therefore x < \frac{42}{18} = \frac{7}{3}$

c. We are given that point $L(2; 3)$ and $N(p; 0)$. Since $MN \perp x$-axis, we have that $x_M = x_N = p$. So we have $M(p; y_M)$. We need to determine y_M, which lies on the tangent line at point L. Now, the equation of the tangent line at point L is given by $y = mx + c$, where:

$m = f'(2) = 9(2)^2 - 42(2) + 45 = -3$

Therefore $y = -3x + c$. Sub point $L(2; 3)$. Then $3 = -3(2) + c \therefore c = 9$.
Hence $y = -3x + 9$. Lastly, we have: $y_M = -3(p) + 9 = -3p + 9$.
Thus the length of line MN is given by: $-3p + 9 - (0) = -3p + 9$

d. We have: $\frac{f'(x)}{f(x)} = \frac{9x^2-42x+45}{3x^3-21x^2+45x-27}$

We are going to try and factorise our numerator and denominator to see if there are any common factors.

Numerator: $f'(x) = 9x^2 - 42x + 45$
$= 3(3x^2 - 14x + 15)$
$= 3(3x - 5)(x - 3)$

Denominator: $f(x) = 3x^3 - 21x^2 + 45x - 27$
$= 3(x^3 - 7x^2 + 15x - 9)$

We will factorise the cubic expression in the brackets:

Using the Factor-Remainder Theorem, the factors of -9 are $\{\pm1; \pm3; \pm9\}$
Let us try $x = 1$. Then $(1)^3 - 7(1)^2 + 15(1) - 9 = 1 - 7 + 15 - 9 = 0$.
Hence $x - 1$ is a factor of $x^3 - 7x^2 + 15x - 9$.

Now: $x^3 - 7x^2 + 15x - 9 = (x - 1)(x^2 + bx + 9)$. We use method by inspection to determine the value of b. F.O.I.L-ing out gives us: $x^3 + bx^2 + 9x - x^2 - bx - 9$
So, $x^3 + (b - 1)x^2 + (9 - b)x - 9 = x^3 - 7x^2 + 15x - 9$. So we must have that $b - 1 = -7 \therefore b = -6$ and $9 - b = 15 \therefore b = -6$. This works out and so we have:

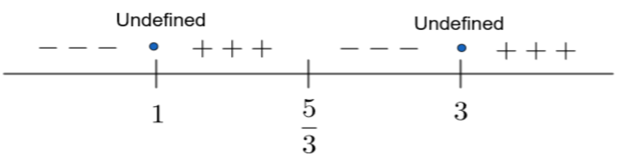
$f(x) = 3(x - 1)(x^2 - 6x + 9)$
$= 3(x - 1)(x - 3)^2$

Hence we have: $\frac{f'(x)}{f(x)} = \frac{3(3x-5)(x-3)}{3(x-1)(x-3)^2} = \frac{3x-5}{(x-1)(x-3)} \leq 0$

The critical values are when:

$3x - 5 = 0 \therefore x = \frac{5}{3}$

$(x - 1)(x - 3) = 0 \therefore x = 1$ or $x = 3$. We now draw our number line:

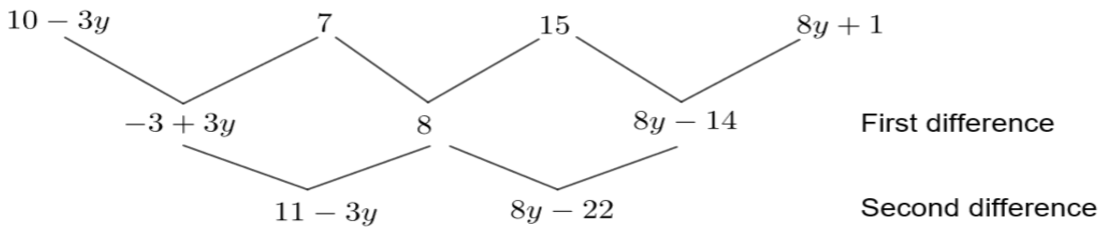


We can see that at $x = 1$ or $x = 3$ our expression will be undefined, since the the denominator will be 0. Hence we have:

$x < 1$ or $\frac{5}{3} \leq x < 3$.

QUESTION 8

a.



We are given that the above sequence is quadratic. Then the sequence must have a common second difference. Hence:

$$11-3y = 8y-22$$
$$\therefore 11y = 33$$
$$\therefore y = 3$$

b.

1. We are given that: $T_1 = 1$
$T_2 = 4$
$T_3 = 16$
$T_4 = 64$

Therefore we have a geometric sequence with $a = 1$ and $r = 4$. Hence
$$T_n = ar^{n-1}$$
$$= 4^{n-1}$$

Now, each investor invests $\mathrm{R}\ 250$, hence to calculate what the n^{th} investor invests, we have to multiply our T_n by 250. So, we get:

$T_n = 250(4^{n-1})$. So the $a = 250$ and $r = 4$. Thus to get $\mathrm{R}\ 21845250$, we use the sum formula:
$$S_n = \frac{a(r^n-1)}{r-1}$$
$$= \frac{250(4^n-1)}{4-1}$$
$$\therefore 21845250 = \frac{250(4^n-1)}{4-1}$$
$$\therefore 262143 = 4^n - 1$$
$$\therefore 4^n = 262144$$
$$\therefore n = \log_4 262144$$
$$= 9$$

Hence at level 9 the investors total contributions will be $\mathrm{R}\ 21845250$.

2. We return to our original geometric sequence which counts the number of investors, i.e., $T_n = 4^{n-1}$, with $a = 1$ and $r = 4$. We want to determine between which **two consecutive levels, the difference is 6×2^{17}.**

So, the **difference between two consecutive terms (levels)** is given by:

$$T_{n+1} - T_n = 4^{(n+1)-1} - 4^{n-1}$$
$$= 4^n - 4^{n-1}$$

$$\therefore 4^n - 4^{n-1} = 6 \times 2^{17}$$
$$\therefore 4^n(1 - 4^{-1}) = 6 \times 2^{17}$$
$$\therefore 4^n\left(1 - \frac{1}{4}\right) = 6 \times 2^{17}$$
$$\therefore 4^n\left(\frac{3}{4}\right) = 6 \times 2^{17}$$
$$\therefore 4^n = 8 \times 2^{17}$$
$$\therefore 4^n = 2^3 \times 2^{17}$$
$$\therefore 4^n = 2^{20}$$
$$\therefore 4^n = 4^{10} \qquad \text{(Since } 4^{10} = (2^2)^{10} = 2^{20}\text{)}$$
$$\therefore n = 10$$

Hence we have that between the 10^{th} and 11^{th} levels the difference between two consecutive terms is 6×2^{17}.

QUESTION 9

a. Initially we start out with 7 cards in the box, i.e., $\{1,2,3,4,5,6,7\}$. Now, before we begin we have to quickly **check when will the product of two numbers be odd.** This can only be true when we multiply two odd numbers together, since **even $\times$ even $=$ even and odd $\times$ even $=$ even. So only odd $\times$ odd $=$ odd.**

Now, let $A = \{First\ card\ chosen\ is\ odd\}$
$B = \{Second\ card\ chosen\ is\ odd\}$

To help us visualise this better, when choosing an odd numbered card, we have:

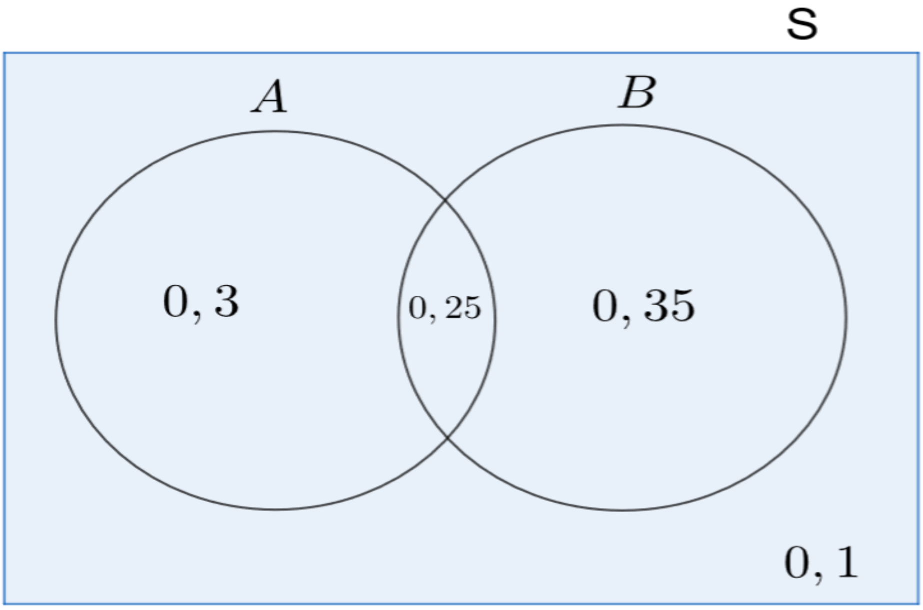
So we have $P(A) = \frac{4}{7}$. Now, after we choose an odd card from the 7, we have 6 left, since there is no replacement. Then, we have the following case:

So we have $P(B) = \frac{3}{6}$. Thus to get the probability that the product of these two cards being odd, we have:

$$\begin{aligned} P(\text{Odd product}) &= P(A \cap B) \\ &= P(A) \times P(B) \quad \text{(These events are independent)} \\ &= \frac{4}{7} \times \frac{3}{6} \\ &= \frac{2}{7} \approx 0{,}3 \end{aligned}$$

b.

1.



2. $P(A \text{ and } B') = P(A) - P(A \text{ and } B) = 0{,}55 - 0{,}25 = 0{,}3$

3. $$\begin{aligned} P(A \text{ or } B') &= P(A) + P(B') - P(A \text{ and } B') \\ &= P(A) + (1 - P(B)) - P(A \text{ and } B') \\ &= 0{,}55 + (1 - 0{,}6) - (0{,}3) \\ &= 1{,}55 - 0{,}9 \\ &= 0{,}65 \end{aligned}$$

c.

1. We have 6 different cities, so the total number of different visits is $6! = 720$

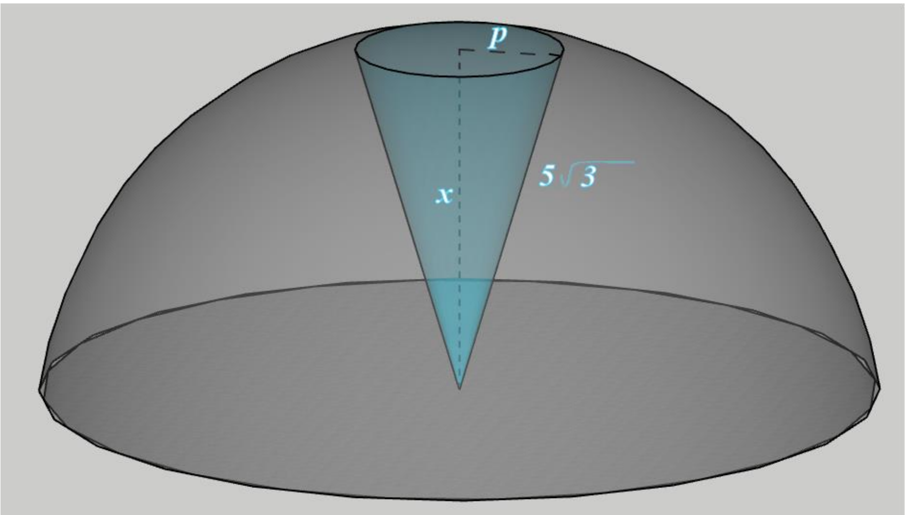
2. If we group (Rome, Madrid, Florence) together in that order, then there are 3 different cities left which we can visit in any order. So there are 3 cities and one grouped arrangement to visit, hence we have $4! = 24$ different ways.

3. We know that there are $4!$ ways to visit Rome, Madrid and Florence and there is 6! Total ways to visit. However, we can visit Rome, Madrid and Florence in any order and since there are 3 cities we have 3! ways to visit these 3 cities.

   Therefore we have: $P(\text{visit Rome, Madrid and Florence grouped}) = \frac{3! \times 4!}{6!}$
   $= \frac{144}{720}$
   $= 0{,}2$

QUESTION 10

a. Using the given information and drawing it on our diagram yields a right angled triangle with dimensions x, p and $5\sqrt{3}$ cm. The volume of a right circular cone is given by: $V = \frac{1}{3}\pi r^3 H$, where H is the perpendicular height.



So, we have: $x^2 + p^2 = \left(5\sqrt{3}\right)^2$

$\therefore p^2 = 75 - x^2$ (Note: p is the radius of the cone)

Now, $V = \frac{1}{3}\pi(75 - x^2)(x)$ (Note: x is the height of the cone)

$= 25\pi x - \frac{1}{3}\pi x^3$

Then $\frac{dV}{dx} = 25\pi - \pi x^2$. We set $\frac{dV}{dx} = 0$ to find the value of x yielding the greatest volume for the cone.

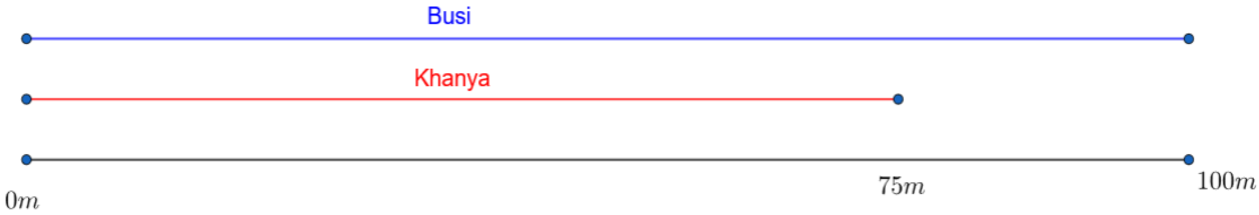
Hence: $25\pi - \pi x^2 = 0$

$\therefore x^2 = 25$

$\therefore x = \pm 5$. However, we cannot have a negative length.

Thus $x = 5$

b. Initially we have that Busi will finish the 100m race and Khanya will only have ran 75m in the same time as Busi crossed the finish line.



If Busi decides to start the 100m race, 25m behind the starting line, then we will have that once Busi has covered 100m, it will only be 75m of the actual track line and so she will be at the same place as Khanya is at the same time (we are assuming they run at the same speeds as the previous race). Now, we see the following relationship between the two runners:

Busi covers 100m in a certain time, say x seconds. Then Khanya covers 75m in the same time x seconds. **So the relationship is that whatever distance Busi covers, Khanya will only cover $\frac{3}{4}$ of that distance, in the same period of time.**

Hence, since Busi and Khanya are both at 75m at time x, Busi will run the 25m (to complete the full 100m) in a certain time and Khanya will only cover $\frac{3}{4}(25)$ in that same amount of time. Thus Busi will still finish the race first and Khanya will have covered $75 + \frac{3}{4}(25) = 93{,}75$m. So Khanya would have been $100 - 93{,}75 = 6{,}25$m away from the finish line when Busi finishes first.