GRADE 12 EXAMINATION
NOVEMBER 2019

ADVANCED PROGRAMME MATHEMATICS: PAPER I
MODULE 1: CALCULUS AND ALGEBRA

Time: 2 hours 200 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 8 pages and an Information Booklet of 4 pages (i–iv). Please check that your question paper is complete.
2. Non-programmable and non-graphical calculators may be used, unless otherwise indicated.
3. All necessary calculations must be clearly shown and writing must be legible.
4. Diagrams have not been drawn to scale.
5. Round off your answers to two decimal digits, unless otherwise indicated.

QUESTION 1

1.1 Solve for $x \in \mathbb{R}$ without using a calculator and showing all working:

(a) $\left|x^2 - 12\right| = x$ (6)

(b) $e^x + 12e^{-x} = 8$ (8)

1.2 If $z = a + bi$ and $z^2 = 23 - 6z$ then find all possible real values of a and b. (10)

1.3 Solve $f(x) = x^4 + x^3 - 2x^2 + 2x + 4 = 0$ in $\mathbb{C}$, if it is given that $f(1-i) = 0$. (8)

[32]

QUESTION 2

Use Mathematical Induction to prove that

$$\sum_{i=1}^{n} 2^i = 2^{n+1} - 2$$ **[12]**

QUESTION 3

Determine $f'(x)$ by first principles if $f(x) = \sqrt{x+3}$.

[8]

QUESTION 4

4.1 Consider the function: $f(x) = \dfrac{x^2 + bx - 6}{2x - a}$

Determine the real values of a and b if the function has a vertical asymptote at $x = 4$ and an oblique asymptote of $y = \dfrac{1}{2}x + 4$ (8)

4.2 Determine the real values of a and b if the function $f(x) = \dfrac{x^2 + ax + b}{2x - 3}$ has a stationary point at $(1; 2)$ (11)

[19]

QUESTION 5

Consider the function *f,* defined as follows:

$$f(x)=\begin{cases} 0,5x+4 & x<-4 \\ 3 & -4\le x<-2 \\ 2 & x=-2 \\ 0,5x^2+1 & -2<x<2 \\ g(x) & x\ge 2 \end{cases}$$

Answer the following questions paying careful attention to the **notation** you use:

5.1 Determine $\lim_{x\to -4} f(x)$ if it exists. If not, explain why. (4)

5.2 Why is *f* discontinuous at $x=-2$? (5)

5.3 What type of discontinuity occurs at $x=-2$? (2)

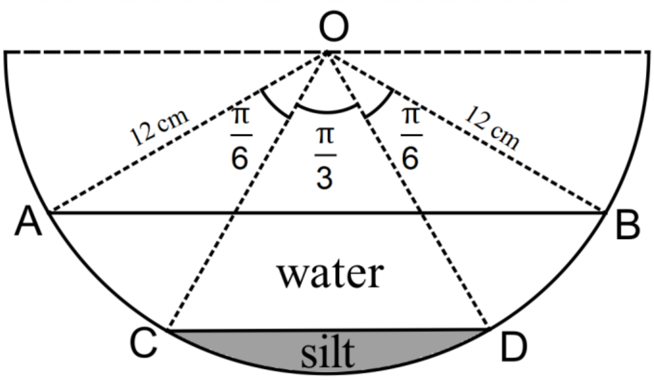
5.4 Determine $g(x)$, if $g(x)$ is a **linear function** and f must be differentiable at $x=2$. (8)

[19]

QUESTION 6

Consider the diagram below. It represents the cross-section of a semi-circular gutter with O the centre of the semi-circle. There is silt at the bottom of the gutter. The surface of the silt, CD, is parallel to the surface of the water, AB. Important angles, in radians, are as shown in the diagram. If the radius of the gutter is 12 cm and the gutter is 2 m long, then calculate the volume of water in the gutter, to the nearest litre.

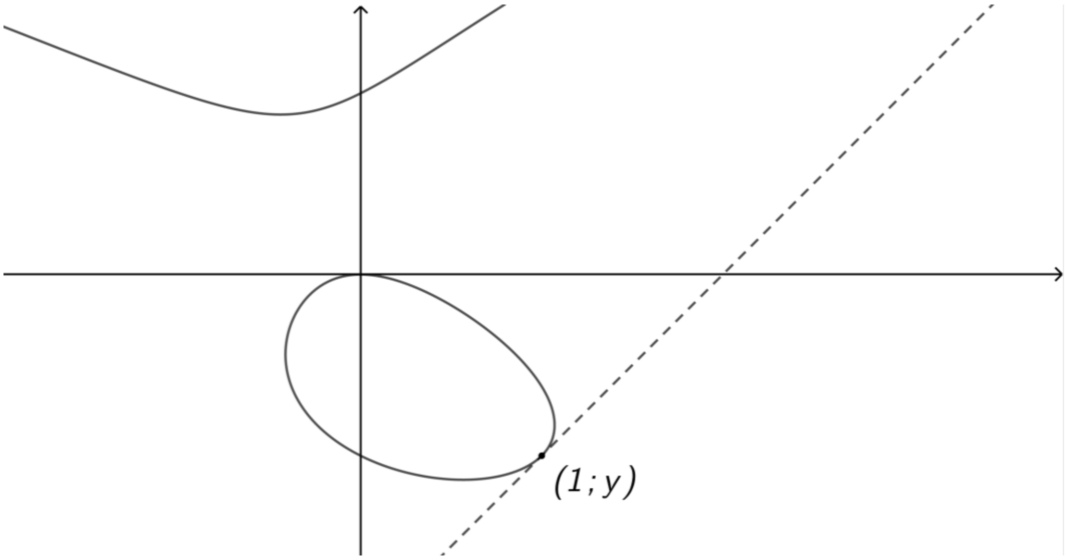
Remember: 1 cm^3 = 1 ml and 1 litre = 1 000 ml.



[10]

QUESTION 7

Below is the graph of the relationship: $y^3 - xy = y + x^2$.

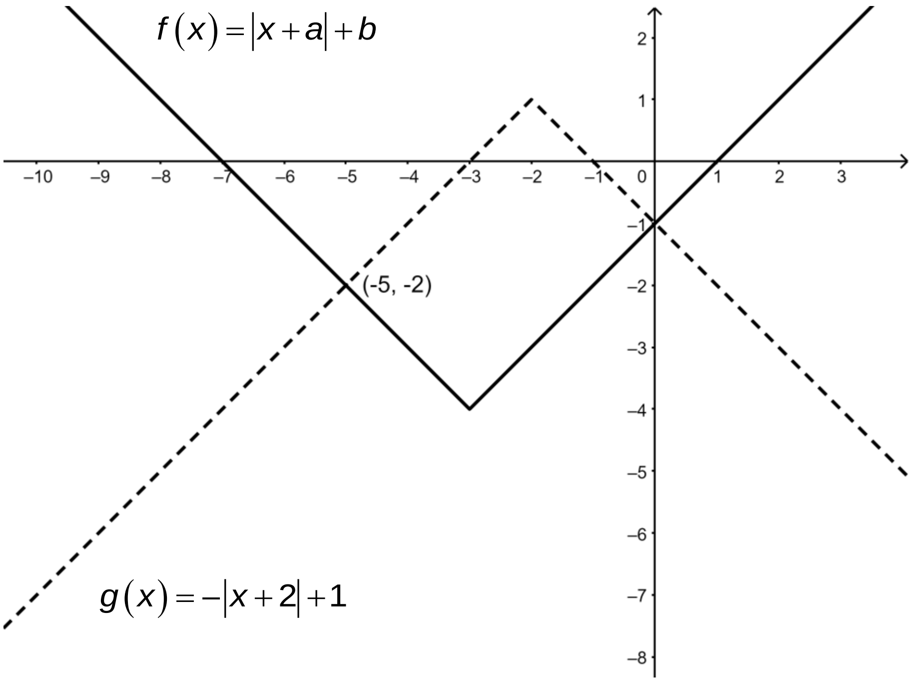


Determine the equation of the tangent (indicated with a dotted line) if it is known that the *x*-coordinate of the point of contact is 1.

[10]

QUESTION 8

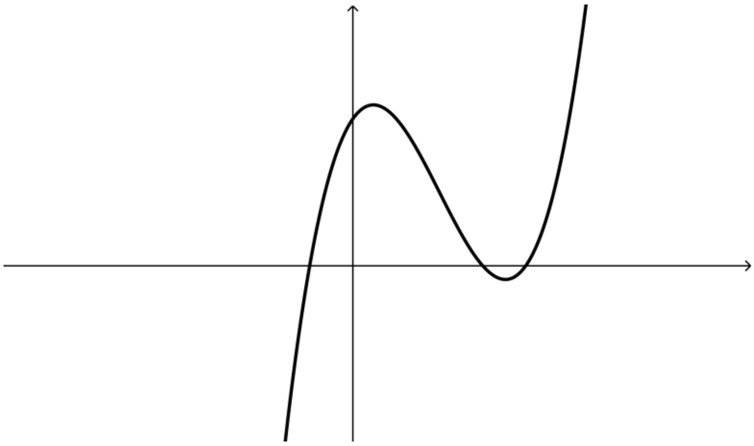
8.1 Consider the functions $f(x) = |x + a| + b$ and $g(x) = -|x + 2| + 1$ drawn on a scaled set of axes.



(a) Determine the values of *a* and *b.* (4)

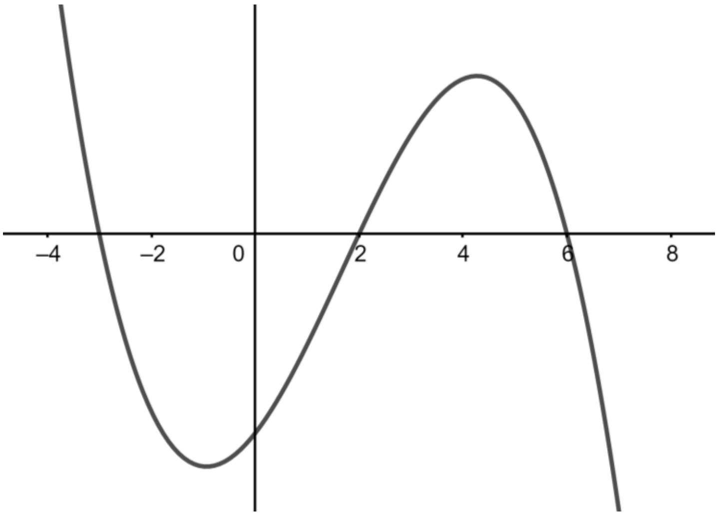
(b) Hence, or otherwise, solve for *x* in: $|x + 3| + |x + 2| \leq 5$. (8)

8.2 Given the graph of $y = f(x)$ in the diagram below, draw on your own set of axes in your Answer Book a rough sketch of $y = f(|x|)$.



(4)

8.3 Consider the function, *f*, drawn below.



Given that $\int_0^2 f(x)\,dx = -38{,}7$ and $\int_2^6 f(x)\,dx = 74{,}7$ determine:

(a) $\int_0^6 f(x)\,dx$ (2)

(b) $\int_0^6 |f(x)|\,dx$ (2)

[20]

QUESTION 9

Consider the function $f(x) = x\ln(x) - \sqrt{x^2+4},\ x > 0$

9.1 Given that *f* is continuous at every value in its domain, justify why *f* has at least one root on the interval $x \in [1; 5]$. (4)

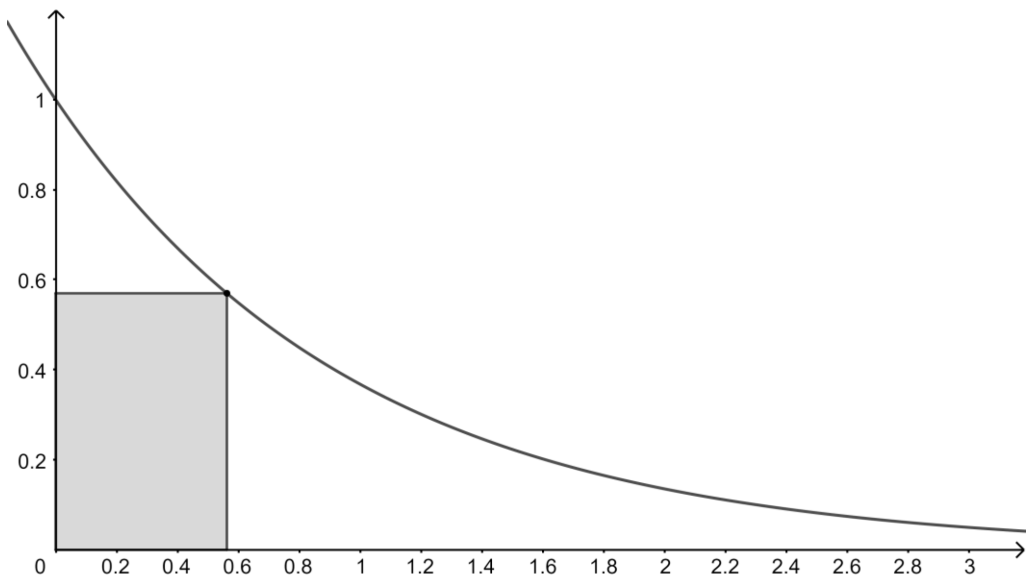
9.2 Use Newton-Raphson iteration to find this root. You should:

- use an initial guess of $x = 1$
- show the iterative formula you use
- show your first two approximations
- give your answer to 5 decimal places (10)

[14]

QUESTION 10

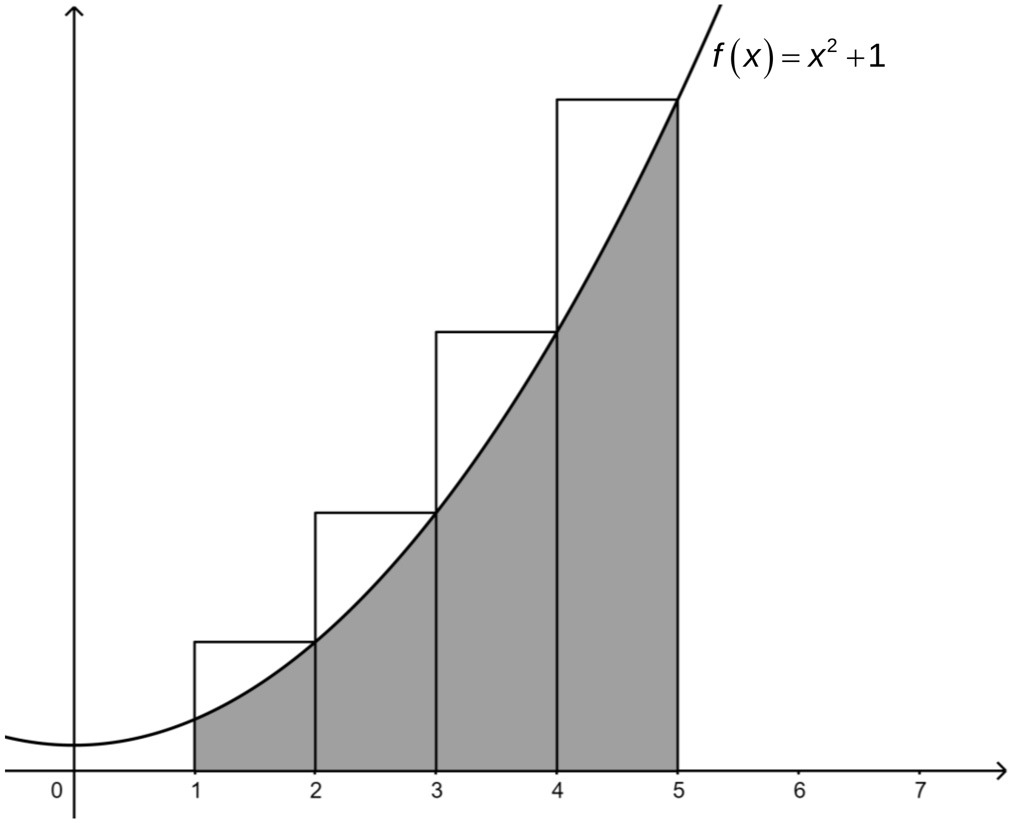
Consider the diagram below where a rectangle is formed in the first quadrant. The bottom left corner is placed on the origin, while the top right corner is placed on the curve of $y = e^{-x}$. Calculate, to 3 decimal places, the maximum area of the rectangle that can be achieved by placing it in this way.



[6]

QUESTION 11

11.1 Robyn is using rectangles to estimate the shaded area. Calculate her percentage error to one decimal place.



(6)

11.2 (a) Resolve $\dfrac{3x^2 + 11x - 5}{x^3 + 3x^2 - 4}$ into partial fractions. (10)

(b) Hence, or otherwise, determine $\displaystyle\int \frac{3x^2 + 11x - 5}{x^3 + 3x^2 - 4}\, dx$ (6)

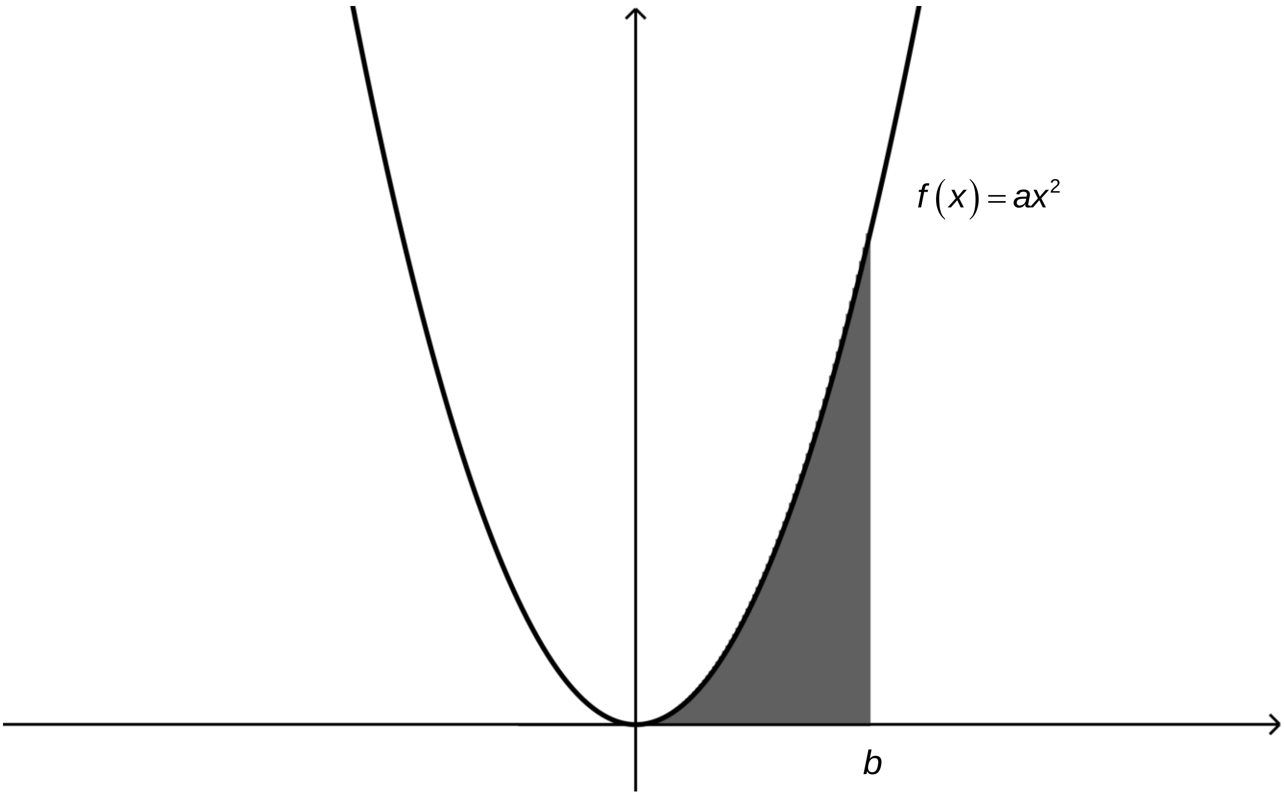
11.3 Determine $\int xe^{2x}\, dx$ (8)

11.4 Determine $\int \operatorname{cosec}^2 x \cot^2 x\, dx$ (6)

[36]

QUESTION 12

Consider the diagram below.



The area bounded by the function *f*, the *x*-axis, the lines $x = 0$ and $x = b$ is $\frac{160}{3}$ *units*^{2}.
When this area is rotated around the *x*-axis the resulting volume is 1280π *units*^{3}.

Determine the values of *a* and *b*.

[14]

Total: 200 marks