basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

TECHNICAL MATHEMATICS P2

MARKING GUIDELINES

EXEMPLAR 2018

MARKS: 150

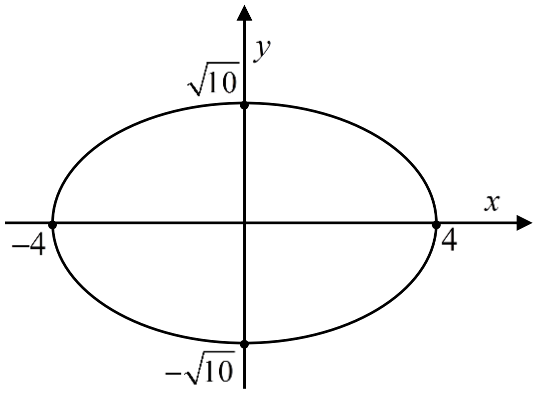
These marking guidelines consist of 14 pages

QUESTION 1

1.1	$\text{AD} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(5-1)^2 + (-2-4)^2}$ $= \sqrt{52}$ $= 2\sqrt{13}$	✓Substitution in correct formula ✓Simplification ✓ $2\sqrt{13}$ simplified surd (3)
1.2	$\text{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$ $\text{M}\left(\frac{1+5}{2}; \frac{4-2}{2}\right)$ $\text{M}(3; 1)$	✓Substitution ✓$\text{M}(3; 1)$ (2)
1.3	$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{1-4}{-3-1}$ $= \frac{3}{4}$ AB ‖ MC $m_{MC} = \frac{3}{4}$ $y - y_1 = m(x - x_1)$ *or* $y = mx + c$ $y - 1 = \frac{3}{4}(x-3)$ $1 = \frac{3}{4}(3) + c$ $-3x + 4y + 5 = 0$ $c = \frac{4-9}{4} = -\frac{5}{4}$ *or* $3x + -4y - 5 = 0$ $y = \frac{3}{4}x - \frac{5}{4}$ $-3x + 4y + 5 = 0$	✓Substitution in correct formula ✓Gradient of MC ✓Substitution in correct formula ✓Simplification ✓Correct answer in correct form (5)
1.4	$\tan\alpha = \frac{3}{4}$ $\alpha \approx 36{,}87°$	✓ $\tan\alpha = \frac{3}{4}$ ✓ $36{,}87°$ (2)

1.5	$m_{AD}=\frac{y_2-y_1}{x_2-x_1}$ $=\frac{-2-4}{5-1}$ $=\frac{-3}{2}$ $\tan\beta=\frac{-3}{2}$ $\beta\approx 123,69°$ $\hat{BAD}=123,69°-36,87°$ $\quad$ *ext ∠ of Δ* $=86,82°$	✓ $\frac{-3}{2}$ ✓ $\beta=123,69°$ ✓ $123,69°-36,87°$ ✓ $86,82°$ (4)
		[16]

QUESTION 2

2.1.1	$x^2+y^2=r^2$ $r^2=(10)^2+(-4)^2$ $\therefore r=\sqrt{116}=2\sqrt{29}$	✓Substitution. ✓ $\sqrt{116}=2\sqrt{29}$ (2)
2.1.2	$m_{OH}=m_{OF}=\dfrac{0-(-4)}{0-10}$ $=-\dfrac{4}{10}$ $=-\dfrac{2}{5}$	✓Substitution. ✓ $-\dfrac{2}{5}$ (2)
2.1.3	$m_{\text{GH}}=\dfrac{5}{2}$ $(m_{\text{GH}}\times m_{\text{OH}}=-1)$ By symmetry H$(-10;4)$ $y-y_1=m(x-x_1)$ *or* $y=mx+c$ $y-4=\dfrac{5}{2}(x+10)$ $4=\dfrac{5}{2}(-10)+c$ $y=\dfrac{5}{2}x+29$ $c=29$ $y=\dfrac{5}{2}x+29$	✓ m_{GH} ✓ H$(-10;4)$ ✓Substitution of H ✓Equation (4)
2.2	 	✓Both *x*-intercepts ✓Both *y*-intercepts ✓Shape (3)
		[11]

QUESTION 3

3.1.1	$OP^2 = (12)^2 + (-5)^2$ $\therefore OP = 13$ units	✓ $OP = 13$ units (1)
3.1.2	$5\cot\theta - 13\cos\theta$ $= 5\left(\frac{12}{-5}\right) - 13\left(\frac{12}{13}\right)$ $= -24$	✓ $\frac{12}{-5}$ ✓ $\frac{12}{13}$ ✓Simplification (3)
3.1.3	$\text{cosec}^2 x - 1$ $= \left(\frac{13}{-5}\right)^2 - 1$ $= \frac{144}{25}$ *or* $\text{cosec}^2 x - 1 = \cot^2 x$ $= \left(\frac{12}{-5}\right)^2$ $= \frac{144}{25}$	✓ $\left(\frac{13}{-5}\right)^2$ *or* $\left(\frac{12}{-5}\right)^2$ ✓Simplification (2)
3.2	$\sec(a-b)$ $= \sec(2,659 - 1,112)$ $= \sec 1,547$ $= \frac{1}{\cos 1,547}$ $\approx 42,03$	✓Substitution ✓Reciprocal ✓Simplification using radians (3)
		[**9**]

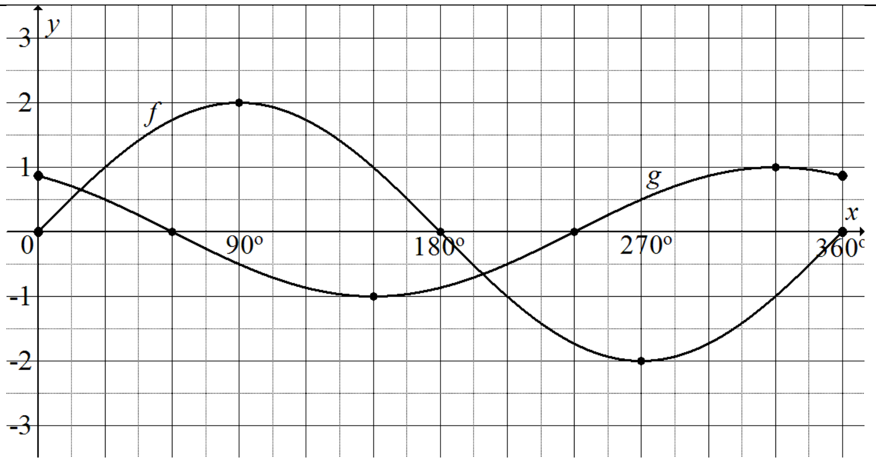
QUESTION 4

4.1	$\frac{\sin(360°-x).\cos(180°-x).\tan 120°}{\cos^2 x.\sin\frac{5\pi}{6}}$ $=\frac{(-\sin x).(-\cos x).(-\tan 60°)}{\cos^2 x.\sin\frac{\pi}{6}}$ $=\frac{-\sin x.\cos x.-\sqrt{3}}{\cos^2 x\cdot\frac{1}{2}}$ $=\frac{2\sqrt{3}\sin x}{\cos x}$ $=2\sqrt{3}\tan x$	✓ $-\sin x$ ✓ $-\cos x$ ✓ $-\tan 60^{o}$ ✓$\sin\frac{\pi}{6}$ ✓ $-\sqrt{3}$ and $\frac{1}{2}$ ✓simplification ✓ $\tan x$ (7)
4.2	$\cos^2 3x$	✓ $\cos^2 3x$ (1)
4.3	$LHS=\frac{\sin x}{\cos x}(\sin x)$ $=\frac{\sin^2 x}{\cos x}$ $RHS=\sec x-\cos x$ $=\frac{1}{\cos x}-\cos x$ $=\frac{1-\cos^2 x}{\cos x}$ $=\frac{\sin^2 x}{\cos x}$ $=LHS$	✓ $\frac{\sin x}{\cos x}$ ✓ $\frac{1}{\cos x}$ ✓ $\frac{1-\cos^2 x}{\cos x}$ ✓ $\sin^2 x$ (4)
4.4	$\text{cosec } 2x=2{,}114$ $\sin 2x=\frac{1}{2{,}114}$ $2x=\sin^{-1}\left(\frac{1}{2{,}114}\right)$ $2x=\sin^{-1}\left(\frac{1}{2{,}114}\right)$ $2x\approx 28{,}23^{o}$ *or* $2x\approx 180°-28{,}23°$ $x\approx 14{,}12°$ *or* $x\approx 75{,}89°$	✓using reciprocal ✓inverse ✓ value of x ✓ another value of x (4)
		[16]

QUESTION 5

5.1	$\sin 55^\circ = \frac{50}{AC}$ $AC = \frac{50}{\sin 55^\circ}$ $\approx 61\,\text{m}$	✓Definition ✓AC subject ✓ $61\,\text{m}$ (3)
5.2	$AD \approx 61\text{ m}$ $DC^2 = AC^2 + AD^2 - 2AC.AD\cos 65^\circ$ $DC^2 = (61)^2 + (61)^2 - 2(61)(61)\cos 65^\circ$ $DC \approx 66\,\text{m}$	✓AD ≈ 61 m ✓Using cosine rule ✓Substitution ✓ $66\,\text{m}$ (4)
5.3	$BD = \sqrt{AD^2 - AB^2}$ $= \sqrt{(61)^2 - (50)^2}$ $\approx 35\,\text{m}$ OR $\tan 55^\circ = \frac{50}{BD}$ $BD = \frac{50}{\tan 55^\circ}$ $= 35\,m$ $\therefore \textit{area of } \Delta BDC = \frac{1}{2}(35)(66)\sin B\hat{D}C = 563$ $\sin B\hat{D}C = \frac{563}{\frac{1}{2}(35)(66)}$ $\therefore \quad B\hat{D}C = \sin^{-1}\left(\frac{563}{\frac{1}{2}(35)(66)}\right)$ $= 29{,}17^\circ$	✓Using Pythagoras theorem or tan ✓BD = 35 m ✓Substitution in area formula ✓Area = 563 ✓ $\sin B\hat{D}C = \frac{563}{\frac{1}{2}(35)(66)}$ ✓Simplification (6)
		[13]

QUESTION 6

6.1		$f(x)$ ✓x-intercepts ✓y-intercept ✓Shape $g(x)$ ✓y-intercept ✓Turning points ✓Shape (6)
6.2	2	✓2 (1)
6.3	360°	✓360^{o} (1)
6.4	$60° < x < 240°$	✓✓Answer with correct notation (2)
		[10]

QUESTION 7

7.1	the angle in the alternate segment	✓Correct statement (1)
7.2.1	$\hat{R}_2 = \hat{S}_1 = 38°$ tan – chord theorem	✓Statement ✓Reason (2)
7.2.2	$\hat{M}_1 = 2\hat{R}_2 = 76°$ $\angle$at center = 2$\angle$ at circum	✓Statement ✓Reason (2)
7.2.3.	$\hat{S}_2 = 90° - 38° = 52°$ tan $\perp$ radius	✓Statement ✓Reason (2)
7.2.4	$\hat{R}_1 = 90^o$ $\angle$ in a semi – circle $\hat{Q}_1 = \hat{S}_1 = 38^o$ tan– chord theorem $\therefore \hat{Q}_2 = 180^o - (17^o + 90^o + 38^o)$ sum of $\angle$'s of a Δ $= 35^o$	✓Statement ✓Reason ✓Reason ✓Statement ✓Statement (5)
7.2.5	$\hat{R}_2 = 38°$ and $\hat{P}_1 = 17°$ $\therefore \hat{R}_2 \neq \hat{P}_1$ (alternate angles are not equal)	✓Alternate angles are not equal *or* $\hat{R}_2 \neq \hat{P}_1$ *or* $\hat{R}_2 = 38°$ and $\hat{P}_1 = 17°$ (1)
		[13]

QUESTION 8

8.1	divides the other two sides proportionally	✓Answer (1)
8.2.1	$\frac{x}{8}=\frac{4}{10}$ $10x=32$ $x=3,2$	✓Prop. ✓Simplification ✓Value of *x* (3)
8.2.2	RTSP is a parallelogram (both pairs of opposite sides of a quadrilateral are parallel)	✓Statement ✓Reason (2)
8.2.3	$\frac{y}{9}=\frac{3,2}{8}$ Prop. Theorem ; TS||MP $y = 3,6$	✓Statement ✓Reason ✓Value of *y* (3)
8.2.4	$\frac{\text{MR}}{\text{TS}}=\frac{10}{4}=2,5$ $\frac{\text{RT}}{\text{SN}}=\frac{9}{3,6}=2,5$ $\frac{\text{MT}}{\text{TN}}=\frac{8}{3,2}=2,5$ $\Delta\text{MRT}\,\vert\vert\vert\,\Delta\text{TSN}$ sides of triangle are in the same proportion	✓Ratio ✓Ratio ✓Ratio ✓Reason (4)
		[13]

QUESTION 9

9.1	$\hat{K}_1 = L\hat{H}F$ ext∠ of cyclic quad $= G\hat{F}K$ corres. angles LK // GF	✓Statement ✓Reason ✓Statement/Reason (3)
9.2	$M\hat{F}G = K_1 = 104°$ *corres∠s*; FG KL $\hat{G} + 104° + 20° = 180°$ *angles of* Δ $\hat{G} = 56°$ $M\hat{F}G = \hat{K}_1 = 104°$ *corres.∠s*; FG // KL $\hat{G} + 104° + 20° = 180°$ *angles of* Δ $\hat{G} = 56°$ **OR** $M\hat{L}K + 104° + 20° = 180°$ *angles of* Δ *∠s*; FG // KL $M\hat{L}K = 56°$ $M\hat{L}K = \hat{G}$ *corresp ∠s*; FG // KL $\hat{G} = 56°$	✓Statement ✓Statement ✓Reason ✓Statement ✓Statement ✓Reason (3)
9.3.1	$\frac{10}{30} = \frac{12}{MG}$ (Prop. Theorem; KL // FG) $MG = \frac{360}{10} = 36$ units	✓Statement ✓Reason ✓36 units (3)
9.3.2	$M\hat{H}F = G\hat{F}M = 104°$ proved in question 9.1. $\hat{M}$ is common $\hat{G} = \hat{F}_2$ sum of angles of a Δ ΔMFH ||| ΔMGF ∠;∠;∠	✓Statement ✓Statement and reason ✓Reason (3)
9.3.3	ΔMFH ||| ΔMGF ||| ΔMLK	✓ ΔMLK (1)
		[13]

QUESTION 10

10.1	$x^2 - 4dh + 4h^2 = 0$ $x^2 - (4 \times 220\,mm \times 60\,mm) + 4(60\,mm)^2 = 0$ $x^2 = \sqrt{67200\,mm^2}$ $\therefore\ x = 259,23\,mm$	✓Formula ✓Substitution. ✓Simplifying ✓Length (4)
10.2.1	$v = \pi Dn$ $= \pi(18\,m)(\frac{225}{60\,s})$ $\approx 212,06\ m/s$	✓Correct formula ✓Correct diameter ✓Substitution ✓212,06 m/s (4)
10.2.2	$\omega = 2\pi n$ $= 2\pi(\frac{225}{60\,s})$ $\approx 23,56\ rad/s$	✓Correct formula ✓Substitution ✓23,56 rad/s (3)
10.3.1	$L\hat{B}A = 180° - 70° = 110°$ co-int.angles; AK // BL	✓Statement ✓Reason (2)
10.3.2	$K\hat{A}D = 360^O - 140^O = 220^O$ $\approx 3,84\text{rad}$ $s_1 = r\theta = 50(3,84)$ $\approx 192\,cm$ $s_2 = 48,8\,cm$ Length of belt $= 110 + 192 + 110 + 48,80$ $= 460,80\,cm$	✓Size of $K\hat{A}D$ ✓Conversion to radians ✓Correct substitution in arc length formula ✓Arc length s_1 ✓Length of DE = 110 ✓Length of belt (6)
		[19]

QUESTION 11

11.1.1	Volume of pyramid $=\frac{1}{3}(area\ of\ base)(height)$ $=\frac{1}{3}(4m\times 4m)(1{,}1m)$ $=5{,}87m^3$ Volume of Cube $=(l)(b)(h)$ $=(4m)(4m)(4m)$ $=64m^3$ Total Volume $=5{,}87\ m^3+64\ m^3$ $\approx 69{,}87m^3$	✓Substitution in correct formula ✓Height ✓ $5{,}87\text{m}^3$ ✓Substitution in correct formula ✓ 64m^3 ✓ $69{,}87\text{m}^3$ (6)
11.1.2	total surface area = surface area of cube base + surface area of pyramid $= 4\ (\text{side}\times\text{side}) + 4\left(\frac{1}{2}\times\text{base}\times\text{slant height}\right)$ $=4(4m\times 4m)\ +\ 4(\frac{1}{2}\times 4m\ \times\sqrt{1{,}1+2^2}\ m)$ $=64m^2\ +\ 4(2m\ \times\sqrt{5{,}1}\ m)$ $= 82{,}07m^2$	✓Slant height ✓Correct substitution into area of a cube ✓Correct substitution into area of pyramid ✓Simplification ✓Total area (5)
11.1.3	Cost of paint = 82,07 × R30,50 = R2503,14	✓82,07 × R30,50 ✓R2503,14 (2)

11.2	$A_T = a\left(\frac{o_1+o_n}{2} + o_2 + o_2 + o_3 + ... o_{n-1}\right)$ $=4\left(\frac{6{,}2+2}{2} + y+5{,}1+4{,}9\right)$ $=4(14{,}1+y)$ $=56{,}4+4y$ $\therefore 72=56{,}4+4y$ $\Rightarrow y=\frac{15{,}6}{4}$ $=3{,}9m$ **OR** $A_T = a(m_1 + m_2 + m_3 + ... m_n)$ $72=4\left(\frac{6{,}2+y}{2}+\frac{y+5{,}1}{2}+\frac{51+4{,}9}{2}+\frac{4{,}9+2}{2}\right)$ $18=\frac{11{,}3+2y}{2}+8{,}45$ $9{,}55=\frac{11{,}3+2y}{2}$ $19{,}1=11{,}3+2y$ $\therefore y=3{,}9m$	✓Formula ✓Substitution ✓Simplification ✓ 3,9*m* **OR** ✓Formula ✓Substitution ✓Simplification ✓ 3,9*m* (4)
		[17]

TOTAL: 150