

### NATIONAL SENIOR CERTIFICATE EXAMINATION NOVEMBER 2016

#### **MATHEMATICS: PAPER II**

**EXAMINATION NUMBER** 

Time: 3 hours

150 marks

### PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

- 1. This question paper consists of 27 pages and an Information Sheet of 2 pages (i–ii). Please check that your paper is complete.
- 2. Read the questions carefully.
- 3. Answer ALL the questions on the question paper and hand this in at the end of the examination. Remember to write your examination number on the space provided.
- 4. Number your answers exactly as the questions are numbered.
- 5. Diagrams are not necessarily drawn to scale.
- 6. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
- 7. Ensure that your calculator is in **DEGREE** mode.
- 8. Round off your answers to one decimal digit where necessary, unless otherwise stated.
- 9. All the necessary working details must be clearly shown.
- 10. It is in your own interest to write legibly and to present your work neatly.

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	TOTAL
8	24	11	16	8	8	12	10	16	11	9	17	/150

#### FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

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### SECTION A

## **QUESTION 1**

In the diagram below, straight line AB makes an angle of  $135^{\circ}$  with the y-axis. AB cuts the y-axis at A.

B is joined to C, a point on the x-axis, so that BC is parallel to the y-axis.



(a) Explain why ABCO is not a cyclic quadrilateral.

(b) If OA = 8 units determine the equation of the line AB.

(1)

(3)

# (c) If OC = 6 units then determine

(1) the equation of BC.

(1)

## (2) the area of OCBA.

(3) [**8**]

If 
$$M = \frac{2\sin^2(180^\circ - \theta) + \sin 2\theta}{\cos 2\theta}$$
 and  $P = \frac{2\sin \theta}{\cos \theta - \sin \theta}$ 

(a) (1) Prove that M = P.

(5)

(2)

(5)

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(b) If 
$$\sin \beta = \frac{\sqrt{2}}{\sqrt{11}}$$
 and  $\cos \beta < 0$  then

(1) In which quadrant is angle  $\beta$ ? (Circle the correct number) (1)



(2) Without the use of a calculator, determine the value of  $\tan \beta$ .

(4)

c)	(1)	Prove that $\cos(\alpha - 30^\circ) - \cos(\alpha + 30^\circ) = \sin \alpha$ .	
			( 1
	(2)	Hence, determine the general solution to the equation: $\cos(\alpha - 30^\circ) - \cos(\alpha + 30^\circ) = 2\sin^2 \alpha.$	(4
			(5)

In the diagram below:

- Circle P touches the *x*-axis and the *y*-axis.
- Circle P touches circle Q at one point.
- Circle Q touches circle R at one point.
- Circle P has a radius of 5 units.
- PQ is parallel to the *x*-axis and RQ is parallel to the *y*-axis.
- PR cuts the circles with centres P and R at A and B respectively.



(a) If PQ = 9 units, determine the equation of circle centre Q.

(b) Find the length of line QR if the equation of circle centre R is  $(x-p)^2 + y^2 - 22y = -117$ .

(3) Determine the length of AB, correct to two decimal digits. (4) [11]

(c)

(a) Use the diagram below to prove the theorem that states that the opposite angles of a cyclic quadrilateral are supplementary.



### (b) In the diagram below:

- O is the centre of the circle.
- A, B and C lie on the circumference of the circle.
- Line DAE is a tangent to the circle at point A.
- $\hat{A}_4 = 62^{\circ}$ .
- $\hat{A}_2 = 25^\circ$ .



Calculate the size of angle  $\hat{C}_1$ .

- (c) (1) What single statement can be deduced from both of the following statements?
  - $$\begin{split} M + N &= D \\ M + Q &= D \end{split}$$

(1)

- (2) In the diagram below:
  - A, B, C and D lie on the circumference of the circle.



Prove that  $\hat{B} = \hat{A}_1 + \hat{C}_2$ .

The graph of  $f(x) = 3\sin x + 1$  with  $x \in [0^\circ; 360^\circ]$  is drawn below. Line AB is parallel to the x-axis. A is a point on the y-axis.



(a) Write down the coordinates of point B.

(2)

(b) Calculate the value(s) for x where f(x) = -1 if  $x \in [0^\circ; 360^\circ]$ .

(4)

(c) If g(x) = k and k is a constant term, then for what value(s) of k will f(x) = g(x) have no real solutions if  $x \in [0^\circ; 180^\circ]$ ?

(2) [**8**]

The manager of a hardware store records the number of staff that are in the store on a given day as well as the sales generated that day.

Staff	20	5	17	7	4	8	15	1	10	12	23	14
Sales in rand	35 200	9 200	32 000	15 600	9 200	17 200	31 200	3 000	19 600	26 800	39 200	20 800

(a) Calculate the correlation coefficient for the data above and then comment on the strength of the relationship.

(b) Find the equation for the line of best fit in the form y = ...

ar day : Explain your answer.

[8]

(3)

**Total for Section A: 75 marks** 

ages of people who say they will buy the product.

#### **SECTION B**

#### **QUESTION 7**

Cumulative Age **Mid-point** Frequency Frequency  $5 < x \le 15$ 10 200 200  $15 < x \le 25$ 20 450 Α 470  $25 < x \le 35$ 30 20  $35 < x \le 45$ 40 32 B  $45 < x \le 55$ 23 525 50 300 825  $55 < x \le 65$ 60  $65 < x \le 75$ 70 475 1300

You develop a product and do some market research. The table below is a summary of the

(a) Calculate the values of A and B in the table above.

(2)

(b) (1) Calculate the estimated mean age of the people who say they will buy the product.

(2) Find the modal class interval.

(1)

(2)

(c) Sketch the Ogive on the grid below.



(d)

In the diagram below, circle centre T touches the *x*-axis at R.

- AO is a tangent to the circle at P.
- OT = 5 and TP = 3.



(a) Determine the coordinates of T.

(5)

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Determine	$y_p$ , the y-coo	rdinate of poi	nt P.		
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(2)

## **QUESTION 9**

In the diagram below, C and A are points on the *x*-axis and *y*-axis respectively.

- OB  $\perp$  AC.
- OB has a length of  $\sqrt{20}$  units.
- BC has a length of  $\sqrt{80}$  units.



(a) Determine the length of OC.

Calculate the coordinates of point B.	alculate the coordinates of point B.		
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		Calculate the coordinates of point B.	

(d) Prove that  $\Delta ABO / / / \Delta OBC$  and hence deduce that  $AB = \frac{OB^2}{BC}$ .



The diagram below is the top view design of a new railway system. There are eight stations being built and these are labelled with letters from A–H. You have been asked to do some calculations for the railway company.

As the engineer you know that:

• AF//BE and AC//GD.



If the straight line distance of the track from F to C is 374 kilometres and it takes (c) 50 hours to build one kilometre of the track, determine the number of hours it will take to build the section from E to D.

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In the diagram below:

- B, C, D, E and F lie on the circle centre O.
- Lines AB and AF are tangents to the circle at B and F respectively.
- Line BE passes through O.



(a) Prove that  $\hat{C}_1 + \hat{D} = 90^\circ$ .

(4)

(b) If  $\hat{D} = 38^\circ$ , determine the size of BÅF.

In the diagram below:

- A, B, C and D lie on the circle centre O.
- P is a point on BC so that  $OP \perp BC$ .
- AD = DC = CP = 6 units.
- $\hat{ADC} = 130^{\circ}$  and  $\hat{BCD} = \theta$ .



(a) Determine the area of  $\triangle ADC$ .

(b) Show that  $D\hat{B}C = 25^{\circ}$ .

(2)

(c) Calculate the value of  $\theta$  if  $\theta < 90^{\circ}$ .

Given that A, D	and C remain fixed points on the circle and that point B is lifted off
the plane conta vertically above	aining the circle and positioned at a point, T, which is 9 units point A. Determine TĈA.
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**Total for Section B: 75 marks** 

Total: 150 marks