NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2016

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours | 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A

QUESTION 1

(a) (1) $\frac{4x}{2} - \frac{2x+1}{3} = 5$

$\frac{12x - 4x - 2}{6} = \frac{30}{6}$ **OR** $12x - 2(2x + 1) = 30$

$8x = 32$

$x = 4$ (2)

(2) $(x-5)(x-6) \leq 56$

$x^2 - 11x + 30 \leq 56$

$x^2 - 11x - 26 \leq 0$

$(x-13)(x+2) \leq 0$

Critical Values: 13 ; -2

$-2 \leq x \leq 13$ (5)

(b) TP (–2 ;–8) Shape

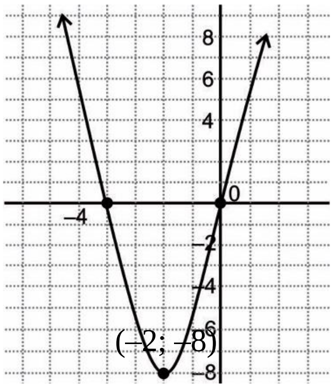
Y – Int: (0 ; 0)

X – Int. Let $y = 0$

$(x + 2) = \pm 2$ OR $x(x + 4) = 0$

$\therefore x = 0$ **OR** $x = -4$

$\therefore x$-int: (0;0) and (–4;0)



(5)

(c) (1) $x = -1$

and

$y = 2$ (2)

(2) $\frac{4}{x+1} + 2 = x$

$\therefore 4 + 2(x + 1) = x(x + 1)$

$\therefore 4 + 2x + 2 = x^2 + x$

$\therefore x^2 - x - 6 = 0$

$\therefore (x - 3)(x + 2) = 0$

$\therefore$ (3; 3) (2; 2)

NB: (For $x = 3$ **OR** $x = 2$ award 3 out of 4) (4)

(d) $c = -1$ or $c = -\frac{1}{4}$ (other answers possible) (2)

(e) $3 - k < 0 \;\therefore k > 3$ (2)

[22]

QUESTION 2

(a) (1) LHS = $3\left(\frac{1}{3}\right) = 1$

RHS = $\sqrt{6\left(\frac{1}{3}\right) - 1} = -1$

LHS $\neq$ RHS $\therefore$ $x = \frac{1}{3}$ is incorrect (2)

(2) $3x = -\sqrt{6x-1}$

$(3x)^2 = 6x - 1$

$9x^2 - 6x + 1 = 0$

$x = \frac{1}{3}$

from (1), no solution

Alternate: Let x be a solution

Then $3x < 0$ so $x < 0$

But $x \geq \frac{1}{6}$

$\therefore$ No solution (4)

(b) $7^{x+a}(1 + 3) = 28\left(7^{a^2}\right)$

$7^{x+a} = \frac{28\left(7^{a^2}\right)}{4}$

$7^{x+a} = 7\left(7^{a^2}\right)$

$7^{x+a} = 7^{1+a^2}$

$x = a^2 - a + 1$ (3)

[9]

QUESTION 3

(a) $4\,800 - \left(4\,800 \times \frac{13,5}{100}\right)$

$= \text{R}4\,152$ (2)

(b) $415\,200 = x\left[\frac{1-\left(1+\frac{7}{1\,200}\right)^{(-5\times 12)}}{\frac{7}{1\,200}}\right]$ Use of correct formula

$x \approx \text{R}8\,221,46$ (4)

[6]

QUESTION 4

(a) Amount paid for all 110 laptops: $6\,000 \times 110 = 660\,000$

Depreciation over 5 years: $A = 660\,000\left(1 - \frac{15}{100}\right)^5$

$\approx 292\,845,51$

Inflation: $A = P(1+i)^n$

$A = 660\,000\left(1 + \frac{6}{100}\right)^5$

$A = 883\,228,881$

Amount required in 5 years less "buy-back" = 883 228,88 – 292 845,51

= R590 383,37 (4)

(b) Sinking Fund: $F = x\left[\frac{(1+i)^n - 1}{i}\right]$

$590\,383,37 = x\left[\frac{\left(1 + \frac{12}{1200}\right)^{(5\times12)} - 1}{\frac{12}{1200}}\right]$ Use of correct formula

$x \approx \text{R}7\,228,92$ (4)

[8]

QUESTION 5

(a) $T_1 = 5(1) + 2 = 7$

$T_2 = 5(2) + 2 = 12$

$T_3 = 5(3) + 2 = 17$

$\therefore$ Since $T_1 + T_2 + T_3 = 36$

Then $y = 3$

OR Alternate:

$7 + 12 + 17 + \ldots + (5y + 2)$

$\therefore$ sequence is arithmetic with $a = 7$ and $d = 5$

$\therefore \frac{y}{2}[7 + 5y + 2] = 36$

$\therefore 9y + 5y^2 = 72$

$\therefore 5y^2 + 9y - 72 = 0$

$\therefore (5y + 24)(y - 3) = 0$

$\therefore y = 3$ (3)

(b) (1) $3p - (2p + 14) = (p + 7) - 3p$

$3p - 2p - 14 = p + 7 - 3p$

$3p = 21$

$p = 7$ (2)

(2) $a = 28$ and $d = -7$

$S_{38} = \frac{38}{2}\left[2(28) + (38-1)(-7)\right]$

$S_{38} = -3\,857$ (3)

(c) $T_n = an^2 + bn + c$

$a + b + c = 7$ … eq ①

$4a + 2b + c = 13$ … eq ②

$9a + 3b + c = 21$ … eq ③

② – ①: $3a + b = 6$

③ – ②: $5a + b = 8$

Sub. $b = 6 - 3a$

Into: $5a + b = 8$

$5a + 6 - 3a = 8 \quad \therefore 2a = 2$

$a = 1, b = 3, c = 3$

$\therefore T_n = n^2 + 3n + 3$

Alternate 1

$$T_n = \frac{(n+1)^3 - 1}{n}$$

$$= \frac{n^3 + 3n^2 + 3n + 1 - 1}{n}$$

$$= n^2 + 3n + 3$$

Alternate 2

7 13 31

```
7    13    21    31
  V     V     V
  6     8     10
     V     V
     2     2
```

$\therefore$ Quadratic sequence

$$T_n = T_1 + (n-1)\cdot f + \frac{(n-1)(n-2)}{2}\cdot s$$

f = first term of the first difference = 6

s = second difference = 2

$T_n = n^2 + 3n + 3$ (4)

(d) $r = \frac{2}{3}$

The sequence of sums is:

$9,\ 15;\ 19;\ 21\frac{2}{3};\ 23\frac{4}{9};\ \frac{665}{27};\ \frac{2\,059}{81}$

$T_6 \approx 24{,}6$ and $T_7 = 25{,}4$

$\therefore n = 7$ is the smallest.

OR

$$Sn = 27\left(1 - \frac{2}{3}\right)^n$$

$Sn > 25$ leads to $\left(\frac{2}{3}\right)^n < \frac{2}{27}$

Try $n = 6$, it does not work,

But $n = 7$ works.

Alternate

$a = 9$ and $r = \frac{2}{3}$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$25 = \frac{9\left[\left(\frac{2}{3}\right)^n - 1\right]}{\frac{2}{3} - 1}$$

$$\left(\frac{2}{3}\right)^n = \frac{2}{27}$$

$$n = \log_{\frac{2}{3}} \frac{2}{27}$$

$n = 6{,}41\ldots$

$\therefore$ Smallest value: $n = 7$

(6)

(e) $V_1 = 729 \text{ cm}^3$

$V_{n+1} = \frac{1}{3} A_{n+1} \cdot h_{n+1}$

$= \frac{1}{3}\left(\frac{1}{3}\right) An \left(\frac{1}{3}h_n\right)$

$= \frac{1}{9}V_n$

$\therefore$ Sequence is geometric

with $r = \frac{1}{9}$

$\therefore S\infty = \frac{729}{1-\frac{1}{9}} = 820{,}1 \text{ cm}^3$

Alternate

Volume of pyramid ① $= \frac{1}{3} \times (9 \times 9) \times 27 = 729 \text{ cm}^3$

Volume of pyramid ② $= \frac{1}{3} \times \left(\frac{81}{3}\right) \times \frac{27}{3} = 81 \text{ cm}^3$

Volume of pyramid ③ $= \frac{1}{3} \times \left(\frac{27}{3}\right) \times \frac{9}{3} = 9 \text{ cm}^3$

The sequence is geometric

$a = 729$; common ratio is $\frac{1}{9}$

$S\infty = \frac{a}{1-r}$

$= 820\frac{1}{8} \text{ cm}^3$

(5)
[23]

QUESTION 6

(a) $$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Working:

$$f(x)=3x^2+2x$$

$$f(x+h)=3(x+h)^2+2(x+h)$$

$$f(x+h)=3x^2+6xh+3h^2+2x+2h$$

$$f'(x)=\lim_{h\to 0}\frac{3x^2+6xh+3h^2+2x+2h-(3x^2+2x)}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{6xh+3h^2+2h}{h}$$

$$f'(x)=\lim_{h\to 0}(6x+3h+2)$$

$$f'(x)=6x+2$$

(5)

(b) $$y=-x^{-1}+x^{\frac{1}{2}}$$

$$\frac{dy}{dx}=+x^{-2}+\frac{1}{2}x^{-\frac{1}{2}}$$

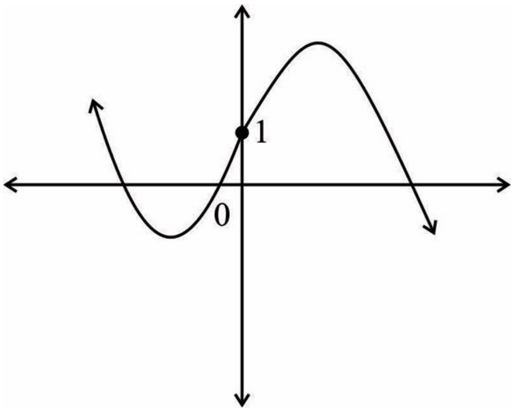
(4)

[9]

77 marks

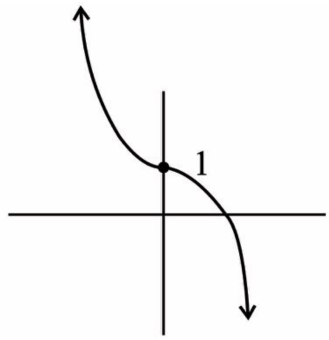
SECTION B

QUESTION 7



Shape
y-int and Pt of Inflection (0;1)
Concave down for $x > 0$

OR



Shape
y-int and Pt of Inflection (0;1)
Concave down for $x > 0$

[3]

QUESTION 8

(a) Axis of symm: $x = \dfrac{-3+1}{2} = -1$

$\therefore f'(x) > 0$ and $g(x) < 0$
OR $f'(x) < 0$ and $g(x) > 0$
$\therefore x < -1$ **OR** $x > 0$ (4)

(b) $g(x) = d^x + q$ sub. $(0;0)$

$0 = d^0 + q$
$q = -1$
Sub. $(1;2)$
$2 = d^1 - 1$
$d = 3$
$\therefore g(x) = 3^x - 1$ (4)

(c) Inverse of g :

$x = 3^y - 1$

$3^y = x + 1$

$y = \log_3(x+1)$ (3)

(d) Domain: $x > -1$

Alternative

Domain of g^{-1} = Range of g

= $(-1; \infty)$ (2)

(e) $f(x) = a(x + 3)(x - 1)$

$6 = a(3)(-1)$

$a = -2$

$\therefore f(x) = -2(x + 3)(x - 1)$

$= -2x^2 - 4x + 6$

$\therefore a = -2,\ b = -4,\ c = 6$

Alternate: Given y-int (0;6)

$y = ax^2 + bx + 6$ sub (-3;0)

$0 = a(-3)^2 + b(-3) + 6$

$b = \frac{9a+6}{3}$ eq. 1

sub (1;0) $0 = a(1)^2 + b(1) + 6$

$a + b + 6 = 0$ eq. 2

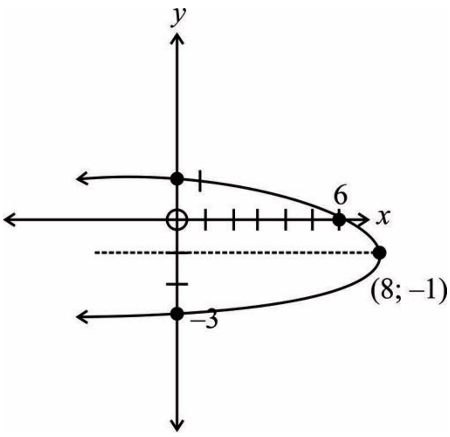
Sub. Eq. 1 in Eq. 2

$-24 = 12a \quad \therefore \quad a = -2$

$b = -4$

(4)

(f)



Turning point of f: $f(-1) = -2(-1 + 3)(-1 - 1)$

$= 8$

$\therefore (8; -1)$ is the T.P of g

Shape

y-int: –3 and 1

x-int: 6

TP (8;–1)

(5)

(g) $k > -6$ **OR** $k \in (-6 ; \infty)$ (2)

[24]

QUESTION 9

(a) $f(1) = a(1)^3 + b(1)^2 \therefore a + b$
$f(2) = a(2)^3 + b(2)^2 \therefore f(2) = 8a + 4b$

$$\therefore 5{,}5 = \frac{8a + 4b - (a+b)}{2-1}$$

$7a + 3b = 5{,}5$ …… (1)

$f'(x) = 3ax^2 + 2bx$
$-18 = 3a(6)^2 + 2b(6)$
$-18 = 108a + 12b$ …… (2)
$4(1)–(2)$: $\begin{cases} 28a + 12b = 22 \\ 108a + 12b = -18 \end{cases}$
$\therefore -80a = 40$
$\therefore a = -\frac{1}{2}$
$\therefore b = 3$

Note: No marks for answer only. (8)

(b) $f(x)$ is increasing when $f'(x) \geq 0$

$-\frac{3}{2}x^2 + 6x \geq 0$

$-3x^2 + 12x \geq 0$

$-3x(x-4) \geq 0$

$0 \leq x \leq 4$

Alternate

$f'(x) = -\frac{3}{2}x^2 + 6x$

$= \frac{-3x}{2}(x-4)$

$\therefore x_c = 4$

f is increasing on $0 \leq x \leq x_c = 4$ (4)

(c) f is concave down when

$f''(x) < 0$

$-6x + 12 < 0$

$x > 2$

Alternate

Point of inflection is: $x = \frac{0+4}{2} = 2$

From graph, f is concave down when $x > 2$ (3)

[15]

QUESTION 10

$$h + r = 9$$
$$\therefore h = 9 - r$$

$$V = \pi r^2 h$$
$$V = \pi r^2 (9 - r)$$
$$V = 9\pi r^2 - \pi r^3$$

$$V = 9\pi r^2 - \pi r^3$$
$$V' = 18\pi r - 3\pi r^2$$
$$0 = 18\pi r - 3\pi r^2$$
$$3\pi r(6 - r) = 0$$
$$r \neq 0 \quad \therefore \quad r = 6 \text{ units}$$

[7]

QUESTION 11

(a) (1) $\frac{46}{80} \times \frac{45}{79} = 0,3$ (3)

(2) $\left(\frac{9}{80} \times \frac{25}{79}\right) + \left(\frac{25}{80} \times \frac{9}{79}\right)$

$= \frac{45}{1264} + \frac{45}{1264}$

$= \frac{45}{632} \approx 0,07$ (4)

(b) $\frac{8!}{2!2!}$

$= 10\,080$ (3)

(c) $P(\text{Khanya will win}) = P(RB) + P(RRRB) + P(RRRRRB) + ...$

$P(\text{Khanya will win}) = \left(\frac{6}{7} \times \frac{1}{7}\right) + \left[\left(\frac{6}{7}\right)^3 \times \frac{1}{7}\right] + \left[\left(\frac{6}{7}\right)^5 \times \frac{1}{7}\right] + \ldots$

This is an infinite geometric series since $-1 < r < 1$

$a = \frac{6}{7^2}$ and $r = \left(\frac{6}{7}\right)^2$

For $\frac{1}{6}$ and $\frac{1}{7}$

Tree diagram

$P\ (\text{Khanya will win}) = \frac{a}{1 - r}$

$P\ (\text{Khanya will win}) = \frac{\frac{6}{7^2}}{1 - \left(\frac{6}{7}\right)^2} = \frac{6}{13} \approx 0,46$ (7)

(7)

[17]

QUESTION 12

$$\left[\frac{1}{8}(4\pi x^2)+(x^2-\frac{1}{4}\pi x^2)\times 3\right]\times 8+30x^2=28$$

$$\therefore 24x^2-2\pi x^2+30x^2=28$$

$$x^2(54-2\pi)=28$$

$$\therefore x^2=\frac{28}{54-2\pi}$$

$$\therefore x=0,766$$

$$\therefore x\approx 0,8$$

Marks allocated as follows:

$54x^2$
$30x^2$
$4\pi x^2$
$6(24x^2-6\pi x^2)$
sum of the three parts = 28
$\therefore x \approx 0,8$

[7]

73 marks

Total: 150 marks