



GRADE 12 EXAMINATION
NOVEMBER 2016

ADVANCED PROGRAMME MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 1 hour

100 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

MODULE 2 STATISTICS

QUESTION 1

1.1 (a)
$$P(X = 3) = \binom{10}{3} \left(\frac{1}{5}\right)^3 \left(\frac{4}{5}\right)^7$$

$$= 0,2013 \qquad (5)$$

(b)
$$15 \binom{1}{5} = 3 \qquad (2)$$

(c)
$$P(x \geq 1) > 0,95$$

$$1 - \binom{n}{0} \left(\frac{1}{5}\right)^0 \left(\frac{4}{5}\right)^n > 0,95$$

$$\left(\frac{4}{5}\right)^n < 0,05$$

$$n \log \frac{4}{5} < \log 0,05$$

$$n > 13,4251$$

$$14 \text{ people} \qquad (8)$$

1.2 (a)
$$3! \times 4! \times 8! \times 3! = 34\,836\,480 \qquad (5)$$

(b)
$$\frac{\binom{3}{2} \binom{4}{2} \binom{8}{2}}{\binom{15}{6}} = \frac{72}{715} \text{ or } 0,1007 \qquad (5)$$

(c)
$$\binom{8}{3} \binom{3}{1} \binom{4}{2} + \binom{8}{3} \binom{3}{2} \binom{4}{1} + \binom{8}{4} \binom{3}{1} \binom{4}{1}$$

$$= 2\,520 \qquad (8)$$

[33]

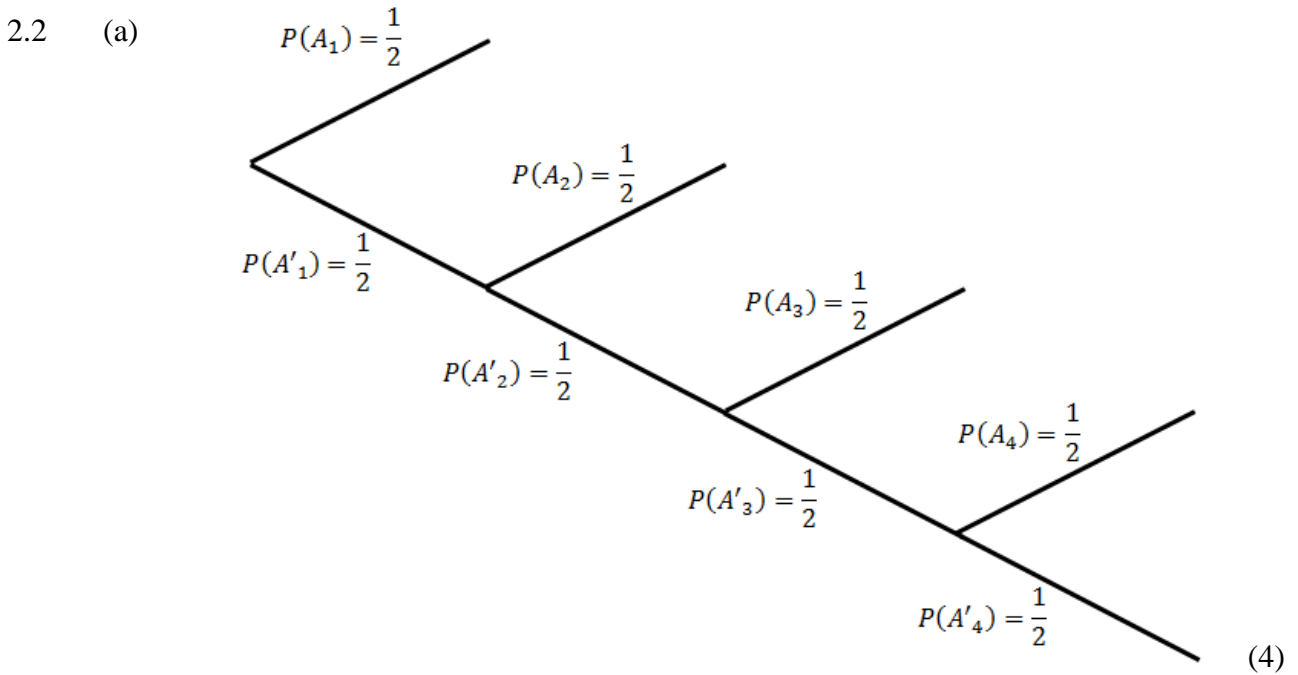
QUESTION 2

2.1 (a) $y = 10,38 - 1,54x$ (4)

(b) $r = -0,983$
Data lie close to the regression line. (2)

(c) $y = 10,38 - 1,54(35)$
 $= -43,52$ (2)

(d) Reliable but caution is required as although extrapolation occurs, correlation is very strong. (2)



(b)

x	0	1	2	3	4
$P(X = x)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{16}$

(4)

(c) $E[X] = 0\left(\frac{1}{2}\right) + 1\left(\frac{1}{4}\right) + 2\left(\frac{1}{8}\right) + 3\left(\frac{1}{16}\right) + 4\left(\frac{1}{16}\right) = 0,9375$ (2)

[20]

QUESTION 3

3.1 (a) A 96% confidence interval for p is:

$$0,8 \pm 2,05 \sqrt{\frac{(0,8)(0,2)}{250}}$$

$$(0,7481 ; 0,8519) \quad (6)$$

(b) 96% of the time the interval will contain the population proportion. (2)

3.2 (a) $\sim N(\mu ; \sigma^2)$

$$\sigma = \frac{3}{5} \mu$$

$$P(x < 2\mu) = P\left(z < \frac{2\mu - \mu}{\frac{3}{5}\mu}\right)$$

$$= P(z < 1,67)$$

$$= 0,5 + 0,4525$$

$$= 0,9525 \quad (8)$$

(b) $-0,84 = \frac{\frac{1}{3}\mu - \mu}{2}$

$$\frac{-42}{52} = -0,84 \times 2 = \frac{-2}{3} \mu$$

$$\mu = +2,52$$

(7)

[23]

QUESTION 4

4.1 A one-tailed test should be used as Gareth would like to see if there has been an increase in his travel time from last year to this year. (2)

4.2 $H_0 : \mu = 45,7$

$H_1 : \mu > 45,7$

Rejection region: reject H_0 if $z > 1,75$

Test Statistic:

$$z = \frac{47,4 - 45,7}{\frac{3,2}{\sqrt{8}}} = 1,5026$$

Conclusion: since $z < 1,75$, we fail to reject the H_0 at the 4% level of significance and suggest insufficient evidence to support the claim, hence no significant increase in Gareth's journey time. (10)

[12]

QUESTION 5

5.1

	1	2	3	4
Arnie	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
Michael	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
Connor	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{3}{7}$

For entrance 1:

$$\begin{aligned}
 P(x \geq 2) &= P(A \cap M \cap C') + P(A \cap M' \cap C) + P(A' \cap M \cap C) + P(A \cap M \cap C) \\
 &= \left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(\frac{6}{7}\right) + \left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\left(\frac{1}{7}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(\frac{1}{7}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(\frac{1}{7}\right) \\
 &= \frac{11}{56}
 \end{aligned}$$

(6)

5.2

$$\begin{aligned}
 &\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(\frac{1}{7}\right) + \left(\frac{1}{6}\right)\left(\frac{1}{4}\right)\left(\frac{1}{7}\right) + \left(\frac{1}{6}\right)\left(\frac{1}{4}\right)\left(\frac{2}{7}\right) + \left(\frac{1}{6}\right)\left(\frac{1}{4}\right)\left(\frac{3}{7}\right) \\
 &= \frac{3}{56}
 \end{aligned}$$

(6)

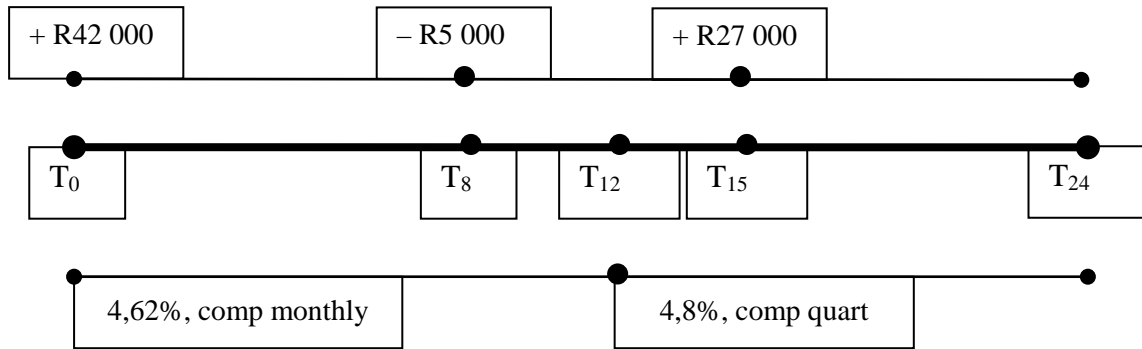
[12]

Total for Module 2: 100 marks

MODULE 3 FINANCE AND MODELLING

QUESTION 1

1.1



(6)

$$1.2 \quad 42\,000 \left(1 + \frac{0,0462}{12}\right)^{12} \left(1 + \frac{0,048}{4}\right)^4 = 46\,131,4622$$

$$5\,000 \left(1 + \frac{0,0462}{12}\right)^4 \left(1 + \frac{0,048}{4}\right)^4 = 5\,325,58533$$

$$27\,000 \left(1 + \frac{0,048}{4}\right)^3 = 27\,983,71066$$

$$46\,131,46 - 5\,325,58 + 27\,983,71 = \mathbf{68\,789,59}$$

OR

$$42\,000 \left(1 + \frac{0,0462}{12}\right)^8 = 43\,311,1661$$

$$38\,311,1661 \left(1 + \frac{0,0462}{12}\right)^4 = 38\,904,5740$$

$$38\,904,5740 \left(1 + \frac{0,048}{4}\right) = 39\,371,4289$$

$$66\,371,4289 \left(1 + \frac{0,048}{4}\right)^3 = \mathbf{68\,789,59}$$

(10)

[16]

QUESTION 2

$$1\ 000\ 000 - 30\ 000 = \frac{30\ 000 \left[1 - \left(1 + \frac{0,064}{4} \right)^{-n} \right]}{\frac{0,064}{4}}$$

$$0,482\ 666 = 1,016^{-n}$$

$n = 45,890$ quarters = **11 years, 5 or 6 months**

OR

$$1\ 000\ 000 = \frac{30\ 000 \left(1 + \frac{0,064}{4} \right) \left[1 - \left(1 + \frac{0,064}{4} \right)^{-(n+1)} \right]}{\frac{0,064}{4}}$$

$$0,475\ 066 = 1,016^{-(n+1)}$$

$\therefore n = 45,890$ quarters = **11 years, 5 or 6 months**

[12]

QUESTION 3

3.1 $33\ 000 = 30\ 400(1 + r)$ $r = 8,55\%$
 $30\ 400 = 28\ 000(1 + r)$ $r = 8,57\%$ (6)

3.2
$$\frac{270 \left[\left(1 + \frac{0,052}{12} \right)^{18 \times 12 + 1} - 1 \right]}{\frac{0,052}{12}} = 96\ 928,20$$
 (6)

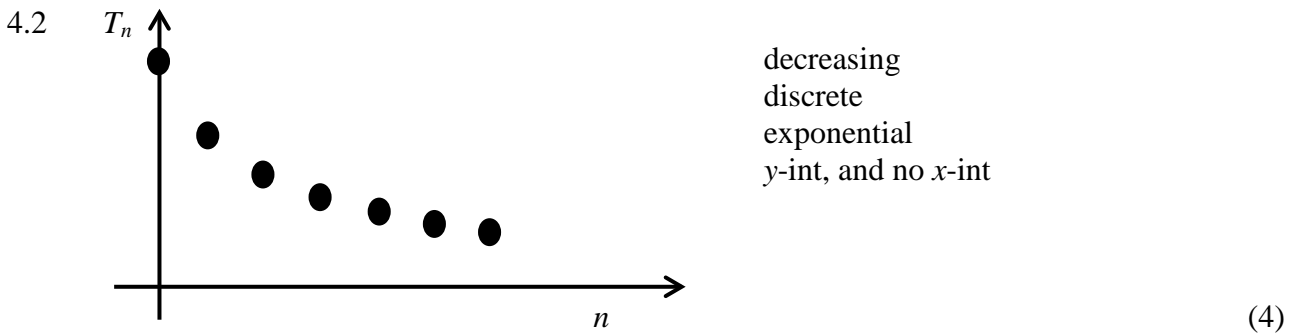
3.3 $96\ 928,20 - 270 \times (18 \times 12 + 1) = 38\ 338,20$ (4)

3.4 $33\ 000 + 30\ 400 \left(1 + \frac{0,052}{12} \right)^{12} + 28\ 000 \left(1 + \frac{0,052}{12} \right)^{24}$
 $= 96\ 080,86$
Yes, it will cover the costs (8)

[24]

QUESTION 4

4.1 $T_3 = 4(2) + 3(-2) - 4(-1)^{2-1} = 6$
 $T_4 = 4(-2) + 3(2) - 4(-1)^{3-1} = -6$
 $T_5 = 4(2) + 3(-2) - 4(-1)^{4-1} = 6$ (5)



4.3

$$P_{n+1} = \overbrace{54 + r \cdot 54} \left(1 - \frac{54}{120}\right) = 54 + 29,7r$$

$$P_{n+2} = P_{n+1} + r \cdot P_{n+1} \left(1 - \frac{P_{n+1}}{120}\right)$$

$$70 = (54 + 29,7r) + r(54 + 29,7r) \left(1 - \frac{54 + 29,7r}{120}\right)$$

$r = 0,27$ ('solve' function)

(9)
[18]

QUESTION 5

- 5.1 Single data points, not joined by a curve
Measurements taken at intervals; measurements between intervals not considered (2)
- 5.2 $1\ 800 < \text{prey} < 3\ 800$ (2)
- 5.3 35–40 (2)
- 5.4 rate of decrease : steeper gradient (2)
- 5.5 (a) b – rate of deadly interactions between predator and prey (3)
- (b) $1/c =$ life span of predator; hence predators live for shorter periods
OR $cF =$ how many predators die in each cycle; hence more predators die in each cycle.
Fewer predators implies increase in prey populations or slower decline. (3)
- 5.6 C prey initially increasing and predator initially decreasing with tendency to equilibrium. (4)
[18]

QUESTION 6

- 6.1 G (2)
- 6.2 31 (2)
- 6.3 $T_{n+1} = 2 \cdot T_n + 1, T_1 = 1$ (4)
- 6.4 $1\ 000\ 000 = 2^n - 1$ $n < 19,93 = 19$ (4)
[12]

Total for Module 3: 100 marks

MODULE 4 MATRICES AND GRAPH THEORY**QUESTION 1**

$$1.1 \quad \begin{array}{l} 2 \times 4 + 4 \times (-1) = z \\ x + 2y = 4 \quad \text{and} \quad 4x - y = -11 \end{array} \quad \begin{array}{l} z = 4 \\ x = -2, y = 3 \end{array} \quad (8)$$

$$1.2 \quad (a) \quad \text{false} \quad (2)$$

$$(b) \quad \text{false} \quad (2)$$

$$(c) \quad \text{true} \quad (2)$$

$$(d) \quad \text{true} \quad (2)$$

[16]**QUESTION 2**

$$2.1 \quad (a) \quad \text{translation 3 units right and 1 unit down} \quad (3)$$

$$(b) \quad \text{yes} \quad (1)$$

$$(c) \quad M = \begin{pmatrix} 4 & 2 & 5 \\ 3 & 2 & -1 \end{pmatrix} \quad (2)$$

$$2.2 \quad (a) \quad \text{shear of factor 3, y-axis invariant} \quad (3)$$

$$(b) \quad \text{no} \quad (1)$$

$$(c) \quad \text{determinant} \quad (1)$$

$$(d) \quad \det = 1 \text{ area of figure and image unchanged} \quad (3)$$

$$2.3 \quad \tan A = 3 \quad A = 71,565^\circ \\ \begin{pmatrix} \cos 143,13 & \sin 143,13 \\ \sin 143,13 & -\cos 143,13 \end{pmatrix} \begin{pmatrix} \cos 30 & -\sin 30 \\ \sin 30 & \cos 30 \end{pmatrix} = \begin{pmatrix} -0,39 & 0,92 \\ 0,92 & 0,39 \end{pmatrix} \quad (8)$$

[22]

QUESTION 3

$$3.1 \quad \begin{pmatrix} 6 & 2 & -3 \\ 6 & 3 & 1 \\ 9 & 3 & -2 \end{pmatrix}^{-1} = \frac{1}{15} \begin{pmatrix} -9 & -5 & 11 \\ 21 & 15 & -24 \\ -9 & 0 & 6 \end{pmatrix} \text{ OR } \begin{pmatrix} -0,6 & -0,333 & 0,733 \\ 1,4 & 1 & -1,6 \\ -0,6 & 0 & 0,4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{15} \begin{pmatrix} -9 & -5 & 11 \\ 21 & 15 & -24 \\ -9 & 0 & 6 \end{pmatrix} \begin{pmatrix} 2 \\ 15 \\ 8 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{15} \begin{pmatrix} -5 \\ 75 \\ 30 \end{pmatrix} = \begin{pmatrix} -\frac{1}{3} \\ 5 \\ 2 \end{pmatrix} \tag{8}$$

$$3.2 \quad \begin{aligned} 6(3t - 3) - 2(6t - 9) - 3(18 - 27) &= 0 \\ 6t + 27 &= 0 \end{aligned} \qquad t = -4,5 \tag{6}$$

$$3.3 \quad \text{scale factor of 1,5:} \qquad w = 3 \tag{2}$$

[16]

QUESTION 4

$$4.1 \quad VW \tag{2}$$

$$4.2 \quad \text{triangular inequality does not hold} \tag{2}$$

$$4.3 \quad W V R Q U P S T W \qquad \text{or} \qquad W V R U Q P S T W \tag{4}$$

$$4.4 \quad \begin{aligned} PSTW &= 15 \\ 2 + 3 + x + 3 + 2 &< 15 \qquad x < 5 \qquad \text{so } x = 4 \end{aligned} \tag{6}$$

[14]

QUESTION 5

5.1 English, Swahili (2)

5.2 $E \rightarrow X (48) \rightarrow Z (45) \rightarrow S (72) \rightarrow F (64) \rightarrow P (58) \rightarrow A (60) \rightarrow E (85)$
 = **432 min** (8)

5.3 $E \rightarrow P (65) \rightarrow A (60) \rightarrow F (62) \rightarrow S (64) \rightarrow X (62) \rightarrow Z (45) \rightarrow E (50)$
 = **408 min**

OR

$E \rightarrow Z (50) \rightarrow X (45) \rightarrow S (62) \rightarrow F (64) \rightarrow A (62) \rightarrow P (60) \rightarrow E (65)$
 = **408 min**

OR any other acceptable circuit < 412 min (10)
[20]

QUESTION 6

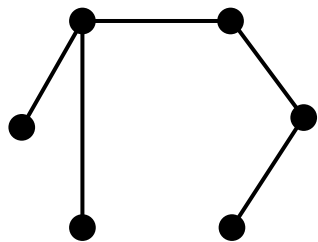
6.1 5 edges (2)

6.2 $2 \times 5 = 10$ (2)

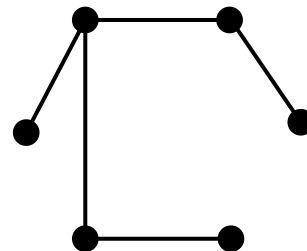
6.3  (8)

Vertices 1, 1, 1, 1, 3, 3

Vertices 1, 1, 1, 1, 2, 4



Vertices 1, 1, 1, 2, 2, 3



Vertices 1, 1, 1, 2, 2, 3

[12]

Total for Module 4: 100 marks

Total: 100 marks